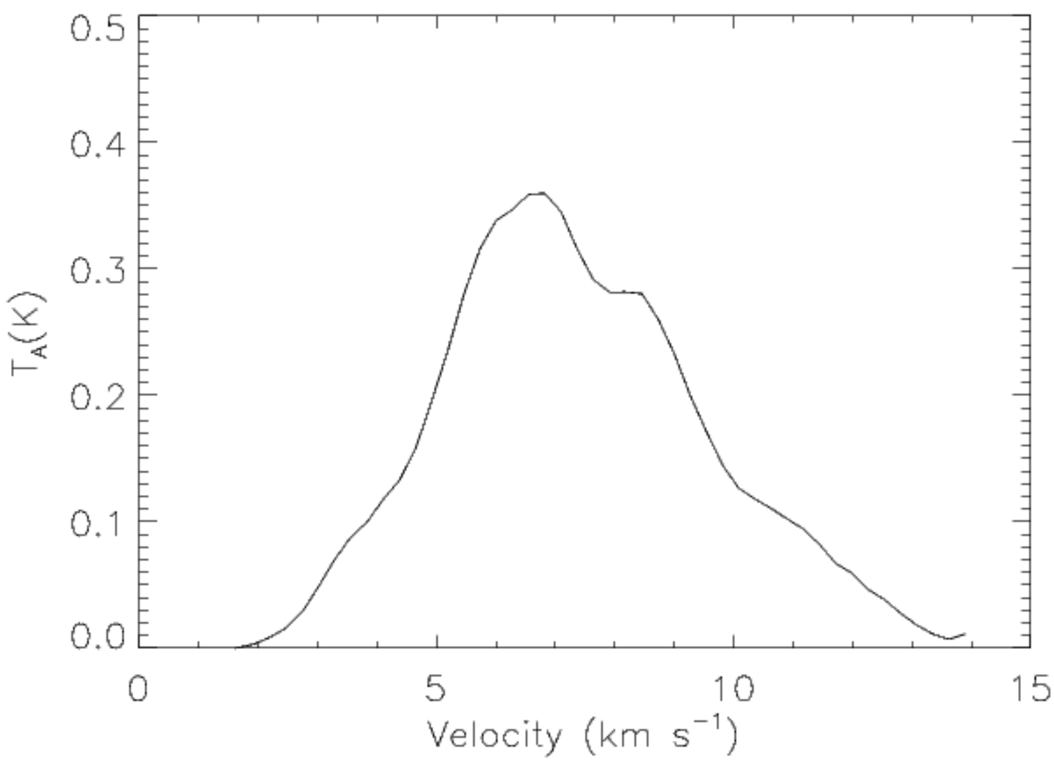
Homework 1 for Ph6820, due September 27

1. Below is the spectrum of the ^{13}CO (1 -> 0) rotational line averaged over the entire Orion cloud. It plots intensity (given as Temperature in units of K) vs frequency. The frequency has been converted into velocity using the Doppler shift formula and the rest frequency of the ^{13}CO 1->0 transition. This can be done since most of the linewidth comes from the motions of the CO molecules in the clouds (i.e. doppler broadening).



The linewidth, σ_v, of a spectral line gives us a measure of the velocity dispersion in the line of sight. The linewidth can be thought of as the standard deviation of the line:

$$\sigma_v^2 = \frac{\sum \langle T(v) \rangle (v - v_\odot)^2}{\sum \langle T(v) \rangle} \text{km}^2 \text{ s}^{-2}. \qquad (3)$$

a. Use the Maxwellian distribution to calculate σ_v for a gas of pure molecular hydrogen at a temperature T_K. Assume that there are only thermal motions of the molecules (no turbulence, no rotation, etc.). You can replace the sum in the above equation with an integral. Put the answer in terms of the mass of a hydrogen atom, m_H, the kinetic temperature of the gas, T_K, and k (Boltzmann constant). What is the sound speed for T_K = 20 K?

b. Given the answer of part a., what value does σ_v need to be for the linewidth to be in excess of the thermal sound speed, i.e. supersonic? Adopt a temperature of T_K = 20 K. Does the line shown above indicate supersonic motions?

c. A typical spectral resolution for a millimeter wave receiver is Δf = 50 MHz. For observations of ^{13}CO (1 -> 0) at 110.2 GHz, what is the resolution in velocity, Δv? Can you resolve purely thermal motions in a 20 K gas?

The Jeans mass for an average density molecular cloud is several hundred solar masses. The next two questions provide some insight in how molecular clouds can fragment into objects with masses close to one solar mass (i.e. fragment into objects that will collapse into single stars).

2. An interesting implication of the Jeans instability was first described by Fred Hoyle in 1953. Imagine a clump of gas with $M = M_J$ = 100 Msun becomes unstable and collapses (M_J is the Jeans mass). What happens to the Jeans mass of the clump as it collapses? Describe what can happen as the density increases by a factor of 4, 16, 64 , … and so on. Do you ever reach the point where $M_J \sim 1$ Msun?

3. Shocks in a supersonically turbulent gas can affect the Jeans mass. Imagine the collision of two clumps of gas moving relative to each other supersonic velocities. A slab of shock wave compressed gas is created between the colliding clumps. In the case that the gas is isothermal (i.e. once the shock compresses and heats the gas, the gas quickly cools back to its initial temperature), the resulting compression is given by formula for an isothermal shock:

$$\rho_{shock}/\rho_{pre\text{-}shock} = M^2 = (v_{shock}/c_s)^2$$

where ρ_{shock} is the density of the shocked gas in the dense slab, $\rho_{pre\text{-}shock}$ is the initial, pre-shock density, M is the mach number, c_s is the sound speed, and v_{shock} is the shock speed (essentially the speed between the two clumps divided by 2). Now imagine that the slab of shocked gas can fragment into collapsing clumps through the Jeans instability. Calculate the Jeans mass as a function of the shock speed, the initial density and the temperature. Describe then how shocks may affect the Jeans mass. How fast must the shock be to obtain Jeans masses of 1 Msun in an initially 1000 cm^{-3}, T_K = 20 K gas? Is this at all consistent with the linewidth in problem 1? To convert between the mass density and number density, use μ = 2.7.