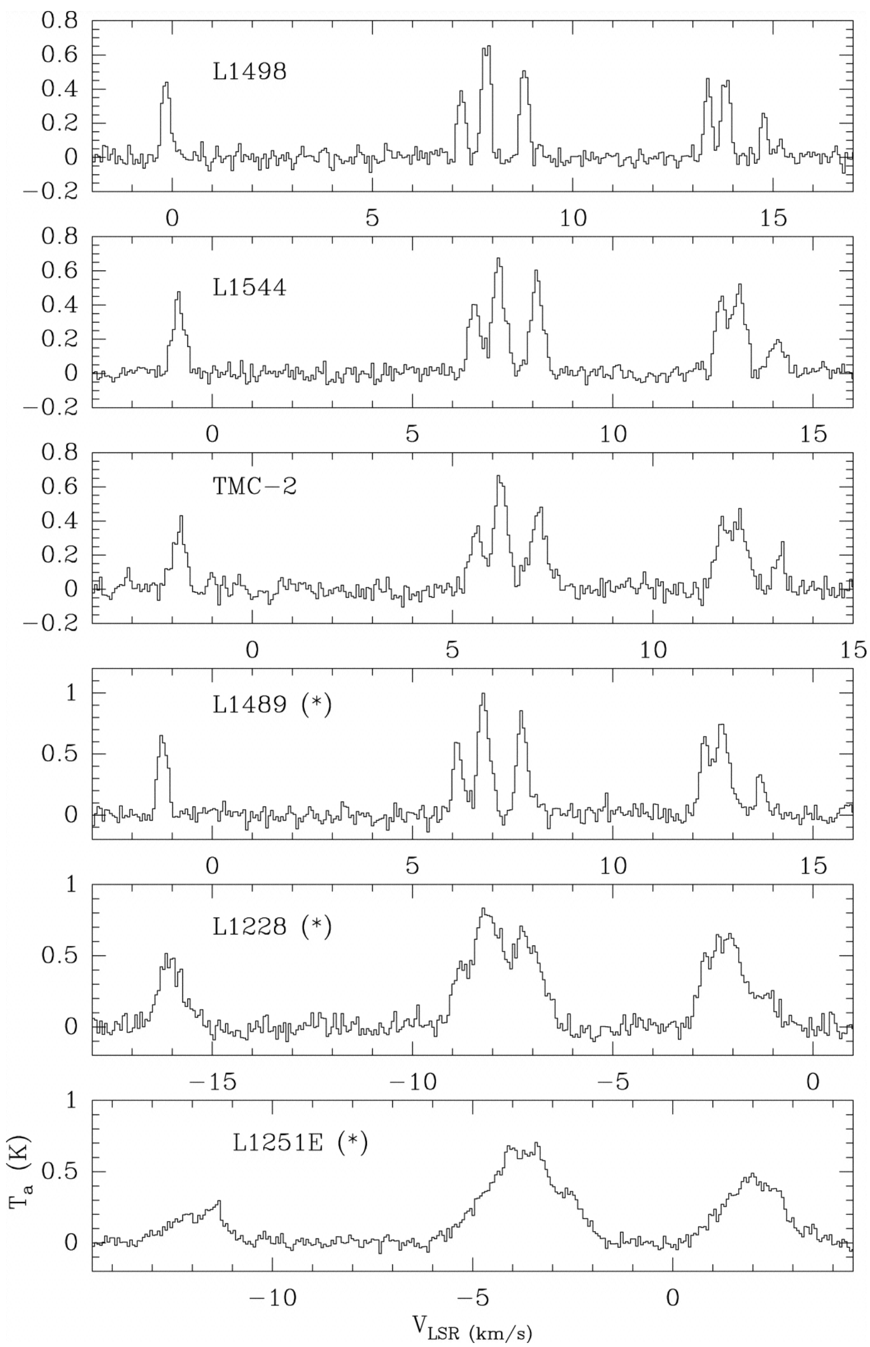
Homework 2 for Ph6820, due February 8

1. Below are the six hyperfine components of the J = 1 -> 0 transition of the molecular ion N_2H^+ [I have included more information on N_2H^+ at the end of the homework if you are curious]. This transition is shown for four dense cores (Caselli, Bense, Myers & Tafalla, 2002). The spectra show intensity (given as Antenna Temperature in units of K) vs frequency. The frequency has been converted into velocity using the Doppler shift formula and the rest frequency of the N_2H+ (1 -> 0) components.

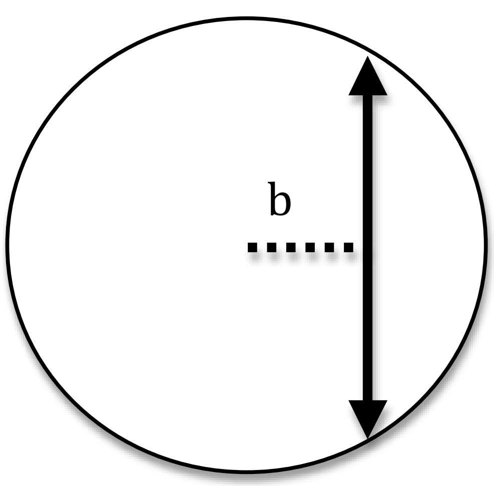


a. Measure the internal 1-D velocity dispersion by using the leftmost component (F_1F = 01 -> 12, see last page), which is isolated and not blended with a nearby hyperfine component. First, measure the Full Width at Half Maximum (FWHM) velocity for that component (just the width of the line at half the maximum intensity level). Do this for all 6 cores. You will need to do this by counting bins or using a ruler.

b. Assuming the lineshape is a Gaussian function, convert the FWHM into the velocity dispersion (i.e. the standard deviation of the Gaussian). You need to first derive a simple relationship between the FWHM and the σ of the Gaussian.

c. Now adopt a kinetic temperature of 10 K for all cores. Assume the non-thermal and thermal velocities have a Gaussian shaped distribution, giving $\sigma_{tot}^2 = \sigma_{NT}^2 + \sigma_{TH}^2$ where σ_{tot} is the total velocity distribution, σ_{NT} is the non-thermal velocity distribution, and σ_{TH} is the thermal velocity distribution. (the non-thermal velocity is a combination of all motions not generated by the thermal motions of the individual molecules: rotation, infall, outflow and turbulence). Using your derivation of σ_{TH} vs temperature from the last HW, calculate σ_{NT} and σ_{NT}/σ_{TH} for all 6 cores.

2. Isothermal spheres and the outer regions of Bonner-Ebert spheres both show a volume density that scales as the 1/radius^2 : $n(H_2) = a/r^2$, where n is the number density of H_2 molecules by volume. However, when we observe cores, we do not measure the volume density, we measure the column density. The column density is the integral of the volume density along a line-of-site (solid line with double arrow). The distance of the line-of-site from the center of the core is given by the impact parameter b (dotted line) from the center of the core. Find the dependence of column density $N(H_2)$ as a function of a and b.

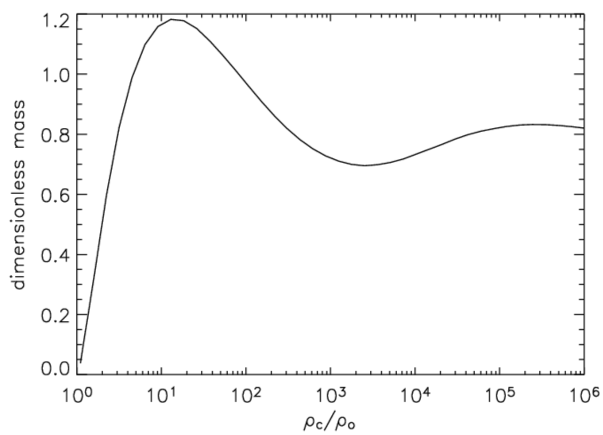


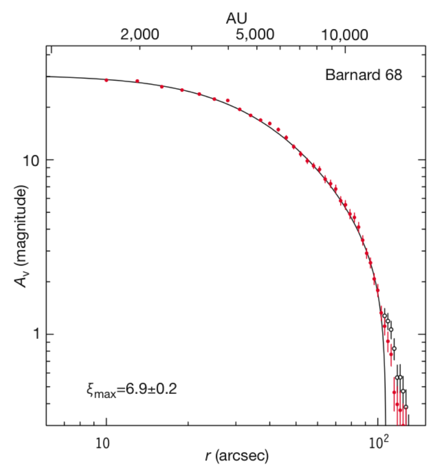
3. Experience the thrill of working with a real Bonner Ebert sphere!!! Consider the dark globule B68, where we know ξ_{max} = 6.9. Using the graphs below:

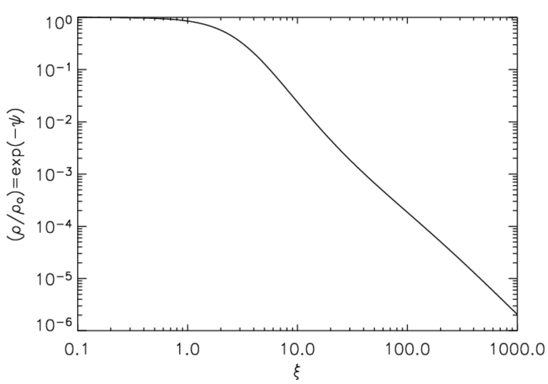
   a. Is this core stable or unstable?

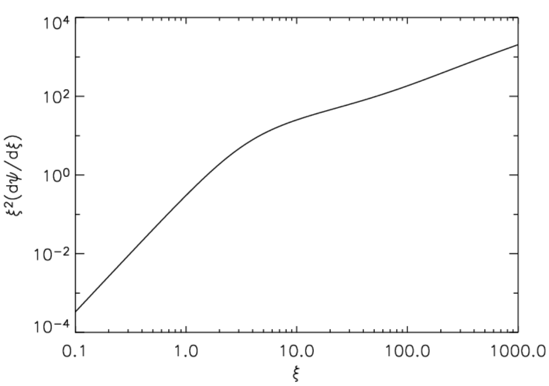
   b. What is the value of ρ_c? Use the distance to B68 of 128 pc, its outer diameter in arcseconds (from the figure), its kinetic temperature of 10.5 K, and the value for $\xi_{max.}$ (hint: how is r related to ξ?)

   c. How does the external pressure compare to the interstellar medium average value of 2 x 10^4 cm^{-3} K? Use the cloud mass of 1.6 solar masses to determine the external pressure, P/k (i.e. the pressure over Boltzman's constant, giving pressure in units of cm^{-3} K).









Addendum: From Caselli, Myers and Thaddeus 1995 ApJ 455, L77.

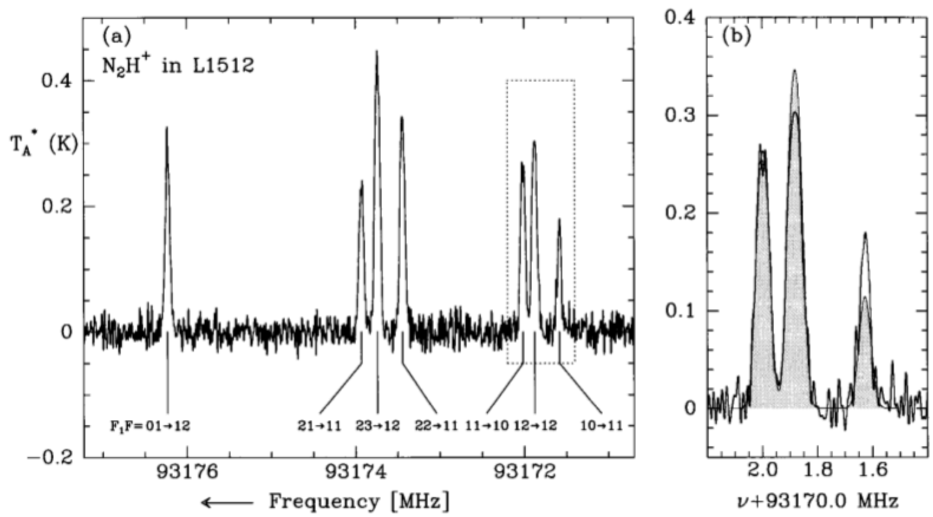


FIG. 1.—(*a*) N_2H^+ (1–0) spectrum toward L1512 obtained at the Haystack antenna. Each hyperfine component has been labeled with the corresponding quantum numbers F_1F. The spectrum is centered at the frequency of the main component ($\nu_{2,3\rightarrow1,2}$ = 93173.809 MHz; Cazzoli et al. 1985). (*b*) The $F_1 = 1 \rightarrow 1$ hyperfine group of N_2H^+ in L1512 superposed with the seven-component hyperfine model-fit spectrum (gray shading) which assumes a single excitation temperature. The F_1, F = 1, 0 → 1, 1 and 1, 2 → 1, 2 components present excitation anomalies.

TABLE 1

OBSERVED LSR VELOCITIES AND DERIVED FREQUENCIES

Transition (1)		V_{LSR}[a] (km s^{-1}) (2)	$\sigma_{V_{LSR}}$ (km s^{-1}) (3)	ν (MHz) (4)	σ_ν[b] (MHz) (5)	ν_{lab}[c] − ν (kHz) (6)
			N_2H^+			
$J F_1 F \rightarrow J' F_1' F'$	1 0 1 → 0 1 2	−0.7951	0.0005	93176.2650	0.0011	45.0
	1 2 1 → 0 1 1	6.6004	0.0016	93173.9666	0.0012	49.4
	1 2 3 → 0 1 2	7.2113	0.0009	93173.7767	0.0012	32.3
	1 2 2 → 0 1 1	8.1673	0.0011	93173.4796	0.0012	25.4
	1 1 1 → 0 1 0	12.7565	0.0014	93172.0533	0.0012	24.7
	1 1 2 → 0 1 2	13.1955	0.0013	93171.9168	0.0012	30.2
	1 1 0 → 0 1 1	14.1473	0.0021	93171.6210	0.0013	−2.0
			C_3H_2			
$J_{K_aK_c} \rightarrow J'_{K_a'K_c'}$	$2_{1,2} \rightarrow 1_{0,1}$	7.1074	0.0037	85338.905[d]	0.006[d]	···

[a] Apparent V_{LSR} values, used to derive hyperfine frequencies ν, are based on rest frequency 93173.809 MHz and corresponding LSR velocity of the center of the band 7.0 km s^{-1}.

[b] Quoted errors do not take into account the uncertainty on the C_3H_2 ($2_{1,2}$–$1_{0,1}$) frequency (see text).

[c] Laboratory measurement (Cazzoli et al. 1985).

[d] Vrtilek, Gottlieb, & Thaddeus 1987.