Homework 3 for Ph6820, due March 1st

1. We showed that in a flow of infalling gas, where the gas is in free fall toward the central protostar, the continuity equation (i.e. conservation of mass) requires that the gas density has the following dependency on radius:

$$\rho \propto r^{-3/2} \tag{1}$$

Now consider a constant velocity flow where there is no acceleration (either infall or outflow, the direction of the flow is irrelevant). The density is given by:

$$\rho \propto r^{-\alpha} \tag{2}$$

Using the equation of continuity, what is α?

2. For an isothermal sphere, the mass infall rate, $\dot{M}$, is a constant in time and approximately equal to c_s^3/G. In contrast, I showed in class that for the collapse of a Bonnor-Ebert sphere there is an initial ramp-up in infall rate. What is the time dependance for the this initial phase of a Bonnor-Ebert sphere collapse? Approximate the inner region of a Bonner-Ebert sphere as a constant density sphere. Assume a wave of infalling gas moving out at $c_s t$. Note that the solution for free fall for a constant density sphere is that each concentric shell takes an equal time to collapse. The infall of each concentric region is triggered by the passage of the rarefraction wave. Write the answer in terms of the gas density, ρ_c, the sound speed c_s, and time.

3. Derive the spectral slope of a flat disk. Make the following assumptions

$$T = T_0(r/r_0)^{-q} \tag{3}$$

where T_0, r_0 and q are constants (T_0 is the temperature at r_0).

i.) Use the equation for the flux density from a disk inclined at an angle θ,

$$F_\nu = \frac{2\pi cos(\theta)}{d^2} \int_{R_{inner}}^{R_{outer}} B_\nu(T(r)) r dr \tag{4}$$

to determine νF_ν. In this equation, d is distance to the disk, R_{inner} is the inner radius of the disk and R_{outer} is the outer radius of the disk. For the purpose of this exercise, you can let $R_{inner} \to 0$ and $R_{outer} \to \infty$. Write the answer in in terms of constants given in equations 3 and 4, a constant value which is given by an integral between 0 and ∞, and a power-law of ν where the exponent is a function of q. You can do this by substituting

$$x = \left(\frac{h\nu}{kT_0}\right)^{1/q} \frac{r}{r_0} \tag{5}$$

into the integral.

ii.) What is the spectral index:

$$\alpha = dlog(\nu F_\nu)/dlog(\nu) \tag{6}$$

as a function of q? What is α for a passively heated flat disk where $T \propto r^{-3/4}$?

4. Write down the radiative transfer equation of a two layer disk, the two layers being a dense inner layer centered on the mid-plane of the disk and the outer layer being a thinner disk atmosphere. Consider each layer as an infinite slab of constant thickness. Consider only radiative transfer perpendicular to the slab (reducing this to a 1-D problem). First, write the equation for the intensity of light emitted perpendicular to the slabs in terms of the temperature of the inner slab T_2, outer slab temperature T_1, the opacity per mass κ_ν, the density of the outer slab ρ_1, and the thickness of the outer slab, h_1. Assume that the inner slab has $\tau_2 >> 1$.

In which of the following cases do you see the silicate features in emission, in absorption, or no silicate feature?

$$T_1 > T_2, \tau_1 << 1, \tau_2 >> 1 \tag{7}$$

$$T_1 > T_2, \tau_1 >> 1, \tau_2 >> 1 \tag{8}$$

$$T_1 < T_2, \tau_1 << 1, \tau_2 >> 1 \tag{9}$$