

*Homework 2 Solutions*

1. Below are the six hyperfine components of the  $J = 1 \rightarrow 0$  transition of the molecular ion  $N_2H^+$ .

a. *Measure the internal 1-D velocity dispersion:* Everyone did well on this - no need to put any further description.

b. *Assuming the lineshape is a Gaussian function, convert the FWHM into the velocity dispersion:*

The full width at half maximum, or FWHM, is given by:

$$FWHM = 2a \tag{1}$$

where  $a$  is related to  $\sigma$  by the equation:

$$\frac{1}{2} = e^{-\frac{a^2}{2\sigma^2}} \tag{2}$$

Taking the natural log of both sides:

$$\ln(2) = \frac{a^2}{2\sigma^2} \tag{3}$$

$$\sigma = \frac{a}{\sqrt{2\ln(2)}} \tag{4}$$

$$\sigma = \frac{FWHM}{2\sqrt{2\ln(2)}} \tag{5}$$

$$\tag{6}$$

c. *calculate  $\sigma_{NT}$  and  $\sigma_{NT}/\sigma_{TH}$  for all 6 cores:*

The relevant equations are:

$$\sigma_{NT} = \sqrt{\sigma_{obs}^2 - \sigma_{N_2H^+}^2} \tag{7}$$

where  $\sigma_{N_2H^+} = \sqrt{kT/29m_h} = 0.05 \text{ km s}^{-1}$ . To determine if something is supersonic, the following ratio must exceed 1:

$$\frac{\sigma_{NT}}{c_s} = \sqrt{\frac{\sigma_{obs}^2 - \sigma_{N_2H^+}^2}{c_s^2}} \quad (8)$$

where  $c_s = \sqrt{kT/\mu m_H}$  where  $\mu = 2.7$ ,  $T=20$  k and  $c_s = 0.18$  km s<sup>-1</sup>

2. *Isothermal spheres and the outer regions of Bonner-Ebert spheres both show a volume density that scales as the 1/radius<sup>2</sup> :  $n(H_2) = a/r^2$ , where  $n$  is the number density of  $H_2$  molecules by volume. The distance of the line-of-sight from the center of the core is given by the impact parameter  $b$ . Find the column density  $N(H_2)$  as a function of  $a$  and  $b$ .*

Let  $\hat{z}$  be the vector aligned with the line of sight. We then integrate along  $\hat{z}$ . We set the limits of the integration to 0 and  $\infty$ , this later limit is a good approximation since the density drops rapidly with radius.

$$N(H_2) = 2 \int_0^\infty \frac{a}{r^2} dz \quad (9)$$

$$N(H_2) = 2 \int_0^\infty \frac{a}{b^2 + z^2} dz = \frac{2a}{b} \int_0^\infty \frac{1}{1 + x^2} dx \quad (10)$$

the solution of the integral is given by:

$$\int \frac{1}{1 + x^2} dx = \tan^{-1}(x) \quad (11)$$

which given the limits of 0 and  $\infty$  is:

$$N(H_2) = \frac{2a \pi}{b} \frac{1}{2} = \frac{\pi a}{b} \quad (12)$$

Note that by measuring column density as a function of  $b$ , we can determine  $a$  and hence the volume density as a function of radius.

3. *Experience the thrill of working with a real Bonner Ebert sphere!!! Consider the dark globule B68, where we know  $\xi_{max} = 6.9$ .*

a. *Is this core stable or unstable?*

Looking at the plot,  $\xi_{max} = 6.9$  implies that  $\rho_c/\rho_0 = 16$ . This core is unstable. (Everything with  $\rho_c/\rho_0 > 14.1$  or  $\xi_{max} > 6.5$  is unstable).

b. What is the value of  $\rho_c$ ?

It's outer radius is between 100'' and 200'', we will call this  $\theta_r = 150''$ . At 128 pc this corresponds to:

$$d = 150'' \left( \frac{2\pi}{360 \cdot 3600} \right) 128pc = 0.1 pc \quad (13)$$

We now take the equation

$$\xi_{max} \equiv \left( \frac{4\pi G \rho_c}{c_s^2} \right)^{1/2} r_{max} \quad (14)$$

and solve for  $\rho_c$

$$\rho_c = \left( \frac{c_s^2}{4\pi G} \right) \left( \frac{\xi_{max}}{r_{max}} \right)^2 \quad (15)$$

If we then calculate the sound speed

$$c_s = \sqrt{\frac{kT}{\mu m_H}} = \sqrt{\frac{10.5 K \cdot 1.38 \times 10^{-16} erg K^{-1}}{2.7 \cdot 1.66 \times 10^{-24} g}} = 0.18 km s^{-1} \quad (16)$$

and substitute it into equation 15 we get

$$\rho_c = 2 \times 10^{-19} gm cm^{-3} = 4.3 \times 10^4 cm^{-3} \quad (17)$$

c. How does the external pressure compare to the interstellar medium average value of  $2 \times 10^4 cm^{-3} K$ ?

We can just use the ratio of the internal and external pressure to get:

$$P = \rho_0 T = \frac{\rho_c}{16} T = 6250 cm^{-3} \cdot 10.5 K = 2.8 \times 10^4 cm^{-3} K \quad (18)$$

Alternatively,  $\rho_c/\rho_0 = 16$  implies  $m = 1.1$ . Putting this into the equation

$$m \equiv \frac{P_0^{1/2} G^{3/2} M}{c_s^4} \quad (19)$$

or (dividing by the Boltzmann constant to put the pressure in units of  $cm^{-3} K$ )

$$P_0 = \frac{1}{k} \left( \frac{mc_s^4}{G^{3/2}M} \right)^2 \quad (20)$$

$$P_0 = \frac{1}{1.38 \times 10^{-16} \text{ erg } K^{-1}} \left( \frac{1.1 \cdot (17890 \text{ cm } s^{-1})^4}{(3.2 \times 10^{33} \text{ g}) \cdot (6.8 \times 10^{-8} \text{ cm}^3 \text{ g}^{-1} \text{ s}^{-2})^{3/2}} \right)^2 \quad (21)$$

resulting in

$$P_0 = 2.9 \times 10^4 \text{ cm}^{-3} K \quad (22)$$

Thus, the pressure is similar to (but 40% higher than) the typical interstellar medium pressure.