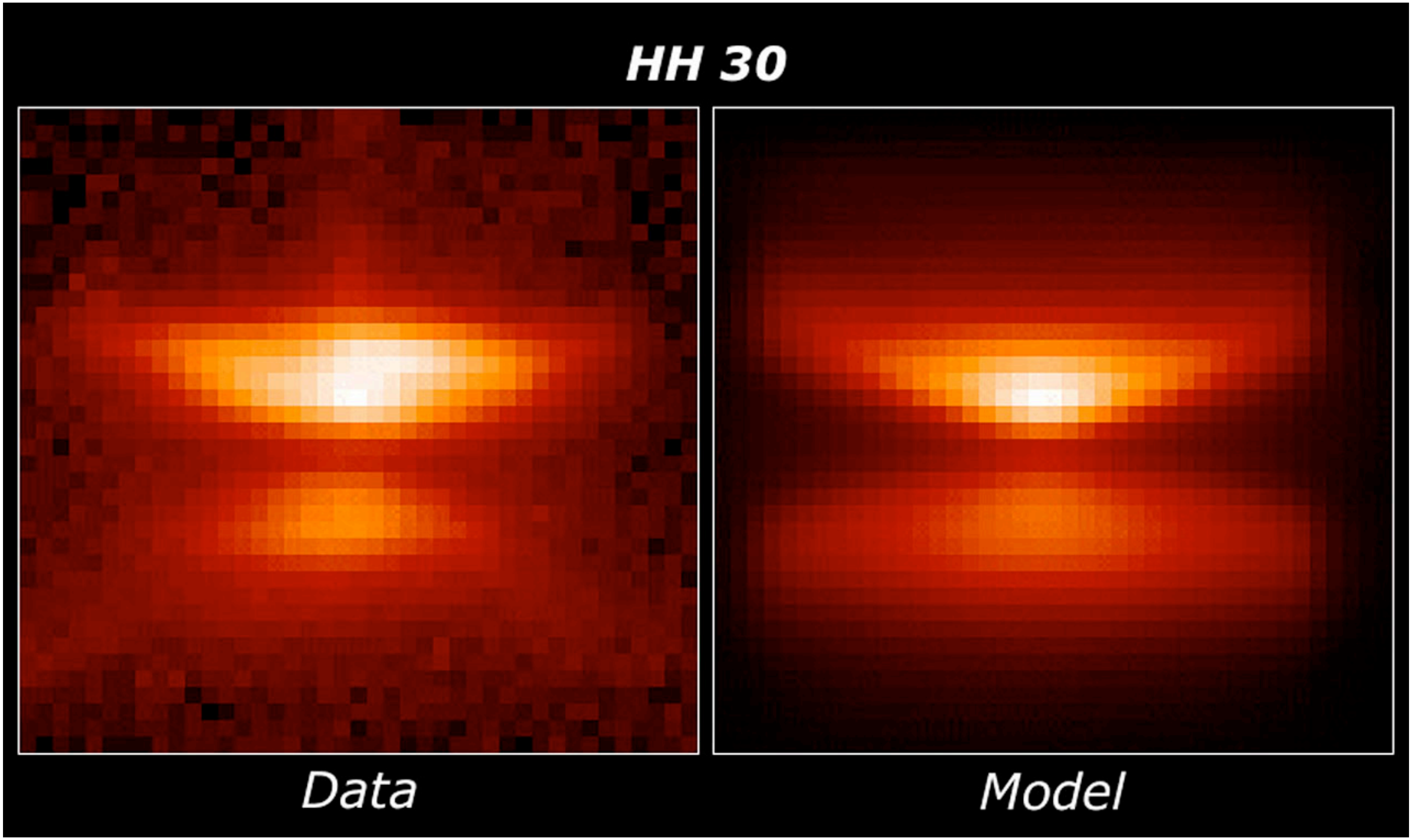
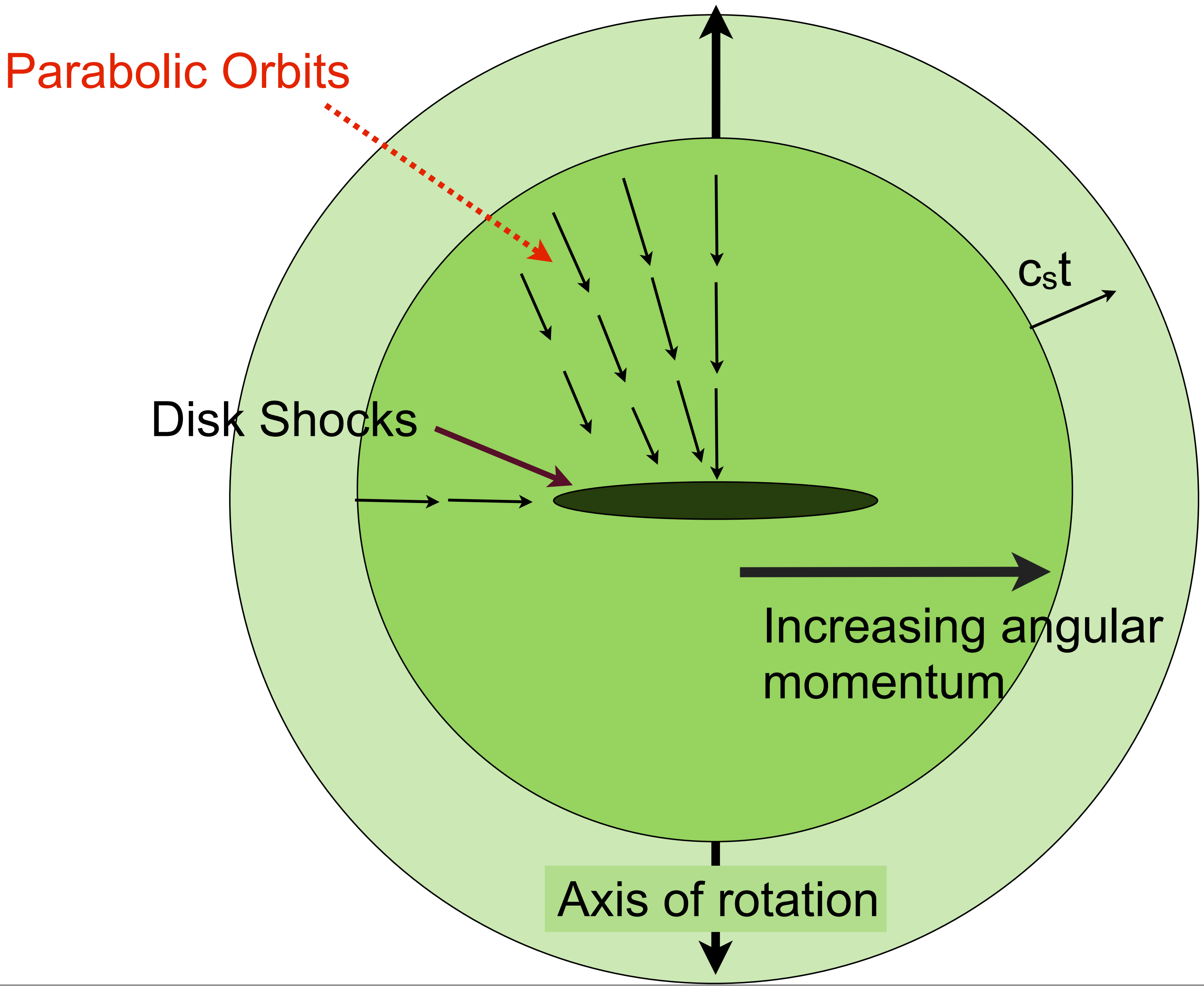
Lecture 10: The Spectral Energy Distributions of Passively Heated Disks

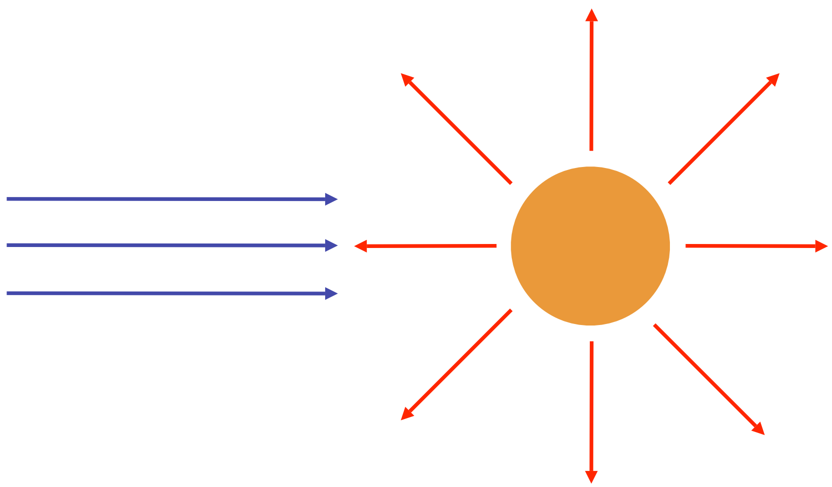


Angular Momentum leads to Disk



Temperature of a dust grain

$$\int B_\nu(T)\kappa_\nu \, d\nu = \frac{1}{4\pi}\int F_\nu \kappa_\nu \, d\nu$$



<u>Assume</u> grey opacity:

$$T^4 = \frac{1}{4\sigma}\frac{L_*}{4\pi r^2}$$

$$T = \sqrt{\frac{r_*}{2r}}\, T_*$$

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Reprocessing of Starlight and Dust Photospheres

Imagine a star with a radius $R_\star$ and temperature $T_\star$ surrounded by an optically thick shell of dust at a radius R_{shell}. Assuming that the shell is in temperature equilibrium, i.e. it is emitting as much power as it is absorbing, then.

$$L_{shell} = L_\star \tag{3}$$

which can be written as

$$4\pi R_\star^2 \sigma T_\star^4 = 4\pi R_{shell}^2 \sigma T_{shell}^4 \tag{4}$$

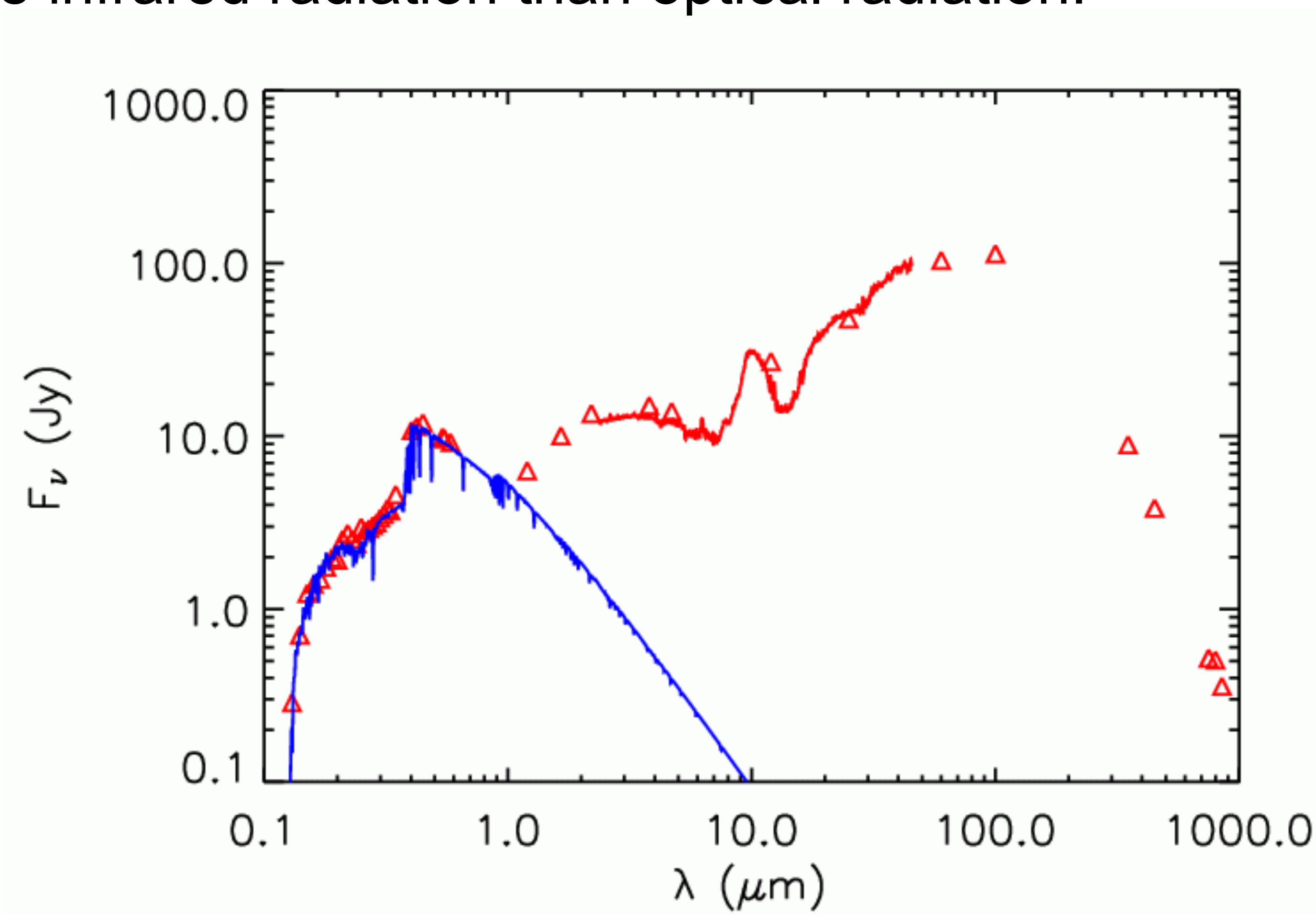
where

$$\frac{T_{shell}}{T_\star} = \left(\frac{R_\star}{R_{shell}}\right)^{1/2} \tag{5}$$

Thus, the shell will appear as a cool blackbody

Spectral Energy Distributions (SEDs)

Plotting normal flux makes it look as if the source emits much more infrared radiation than optical radiation:



This is because energy is: $F_\nu \, d\nu = F_\nu \, \Delta\nu$

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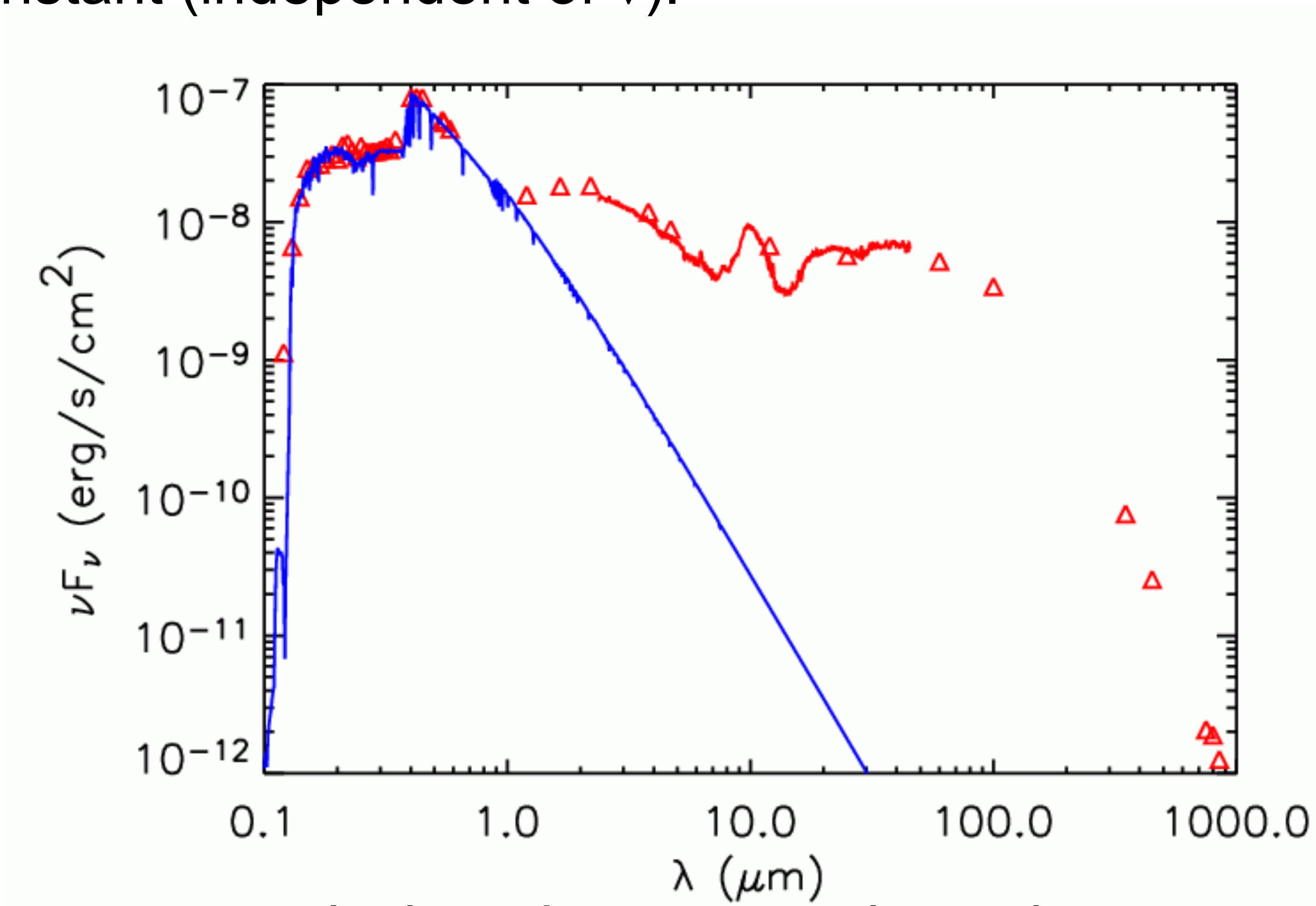
Spectral Energy Distributions (SEDs)

Typically one can say: $\Delta\nu = \nu\,\Delta(\log\nu)$ and one takes $\Delta(\log\nu)$ a constant (independent of ν).



In that case νF_ν is the relevant quantity to denote energy per interval in logν. NOTE: $\nu F_\nu \equiv \lambda F_\lambda$

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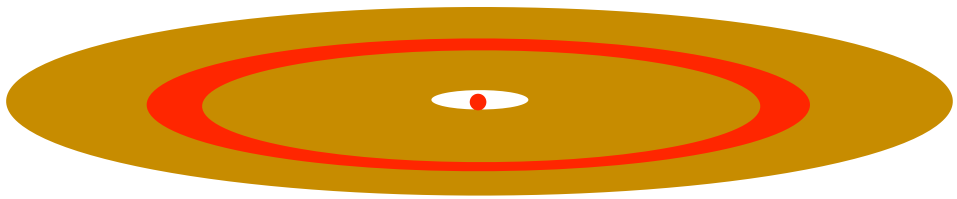
Disks

Calculating the SED from a flat disk

Assume here for simplicity that disk is vertically isothermal: the disk emits therefore locally as a black radiator.



$$I_\nu(r) = B_\nu(T(r))$$

Now take an annulus of radius r and width dr. On the sky of the observer it covers:

$$d\Omega = \frac{2\pi\, r dr}{d^2} \cos i$$

and flux is:

$$F_\nu = I_\nu d\Omega$$

Total flux observed is then:

$$F_\nu = \frac{2\pi \cos i}{d^2} \int_{r_{\rm in}}^{r_{\rm out}} B_\nu(T(r))\, r dr$$

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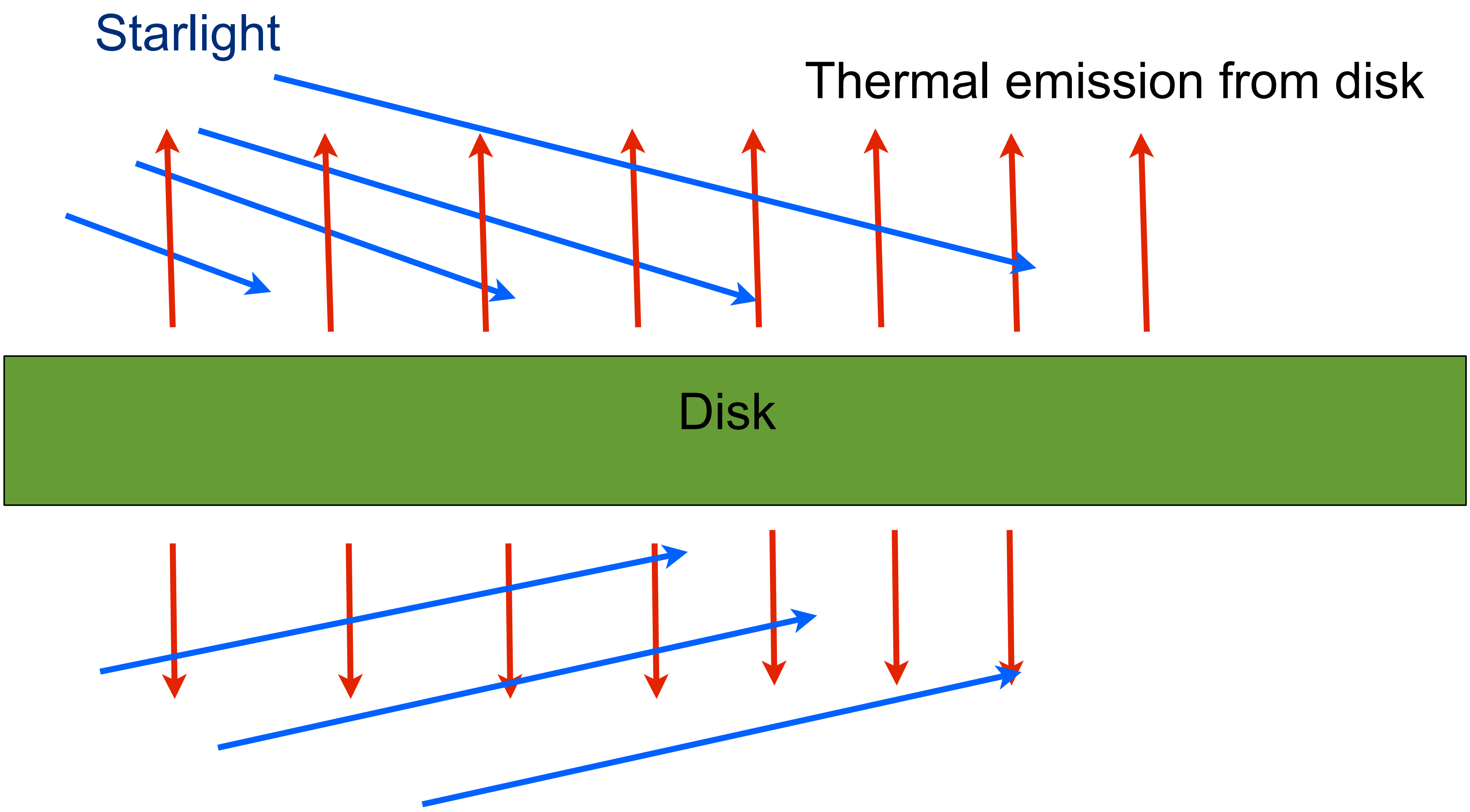
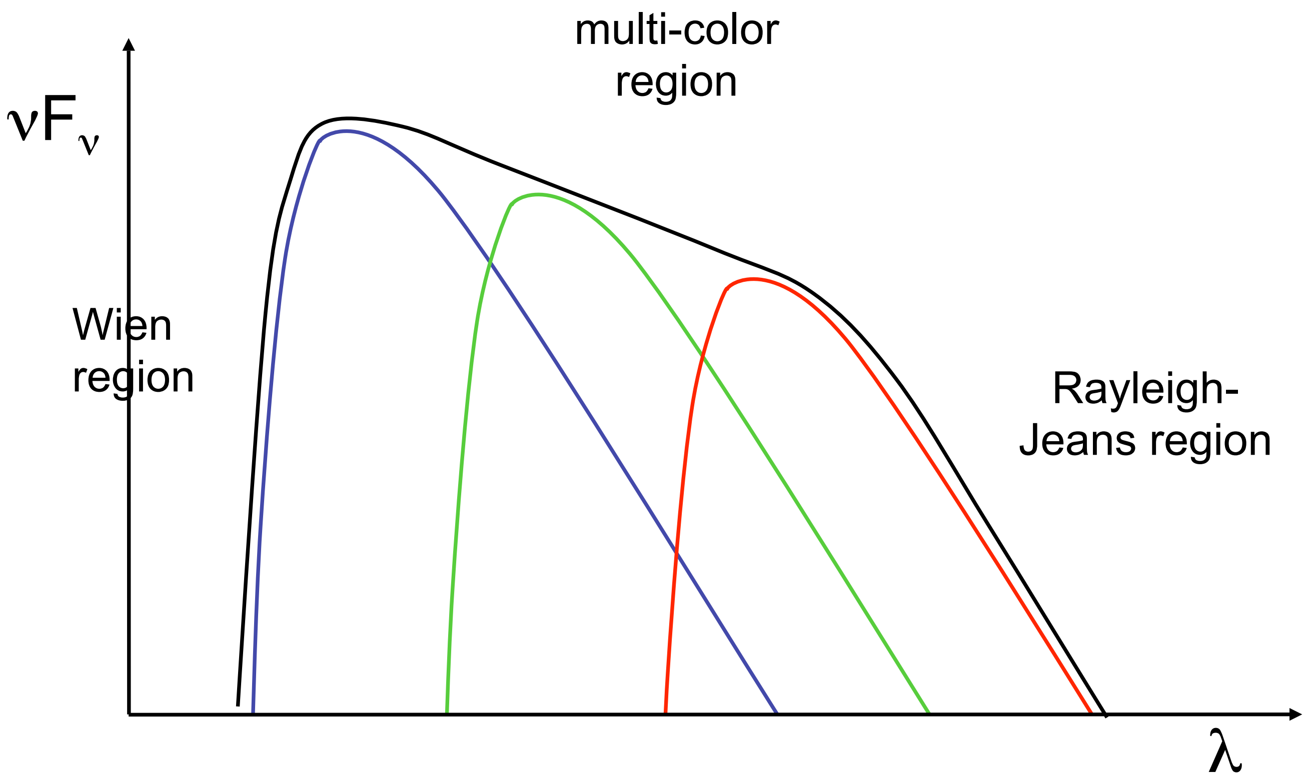
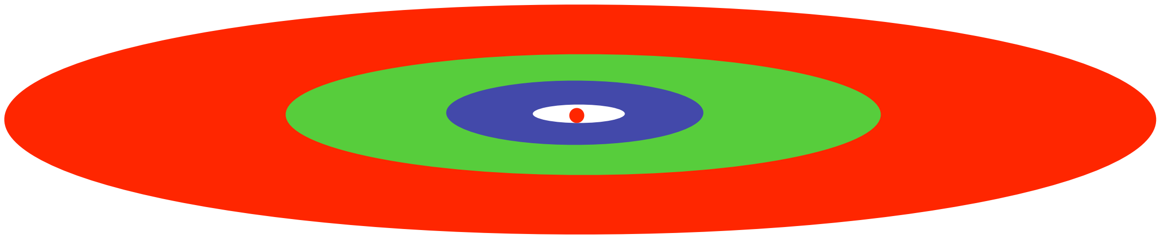
Heating and Coolings of Disks



Multi-color blackbody disk SED





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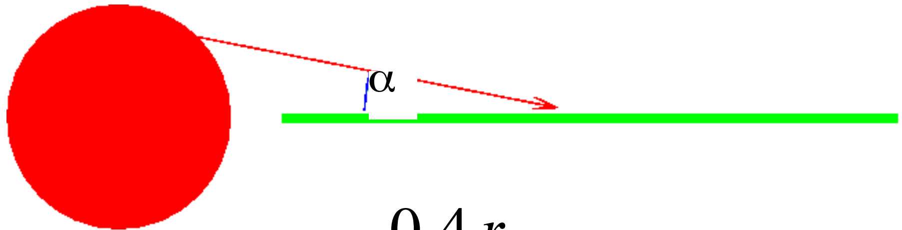
Flat irradiated disks



$$\alpha \cong \frac{0.4\, r_*}{r}$$

Irradiation flux:

$$F_{\rm irr} = \alpha \frac{L_*}{4\pi\, r^2}$$

Cooling flux:

$$F_{\rm cool} = \sigma T^4$$

$$T = \left(\frac{0.4\, r_* L_*}{4\pi\sigma r^3} \right)^{1/4}$$

$$T \propto r^{-3/4}$$

Similar to active accretion disk, but flux is fixed.
Similar problem with at least a large fraction of HAe and T Tauri star SEDs.

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SED of disks disk

According to our derived SED rule (4q-2)/q=4/3 we obtain:

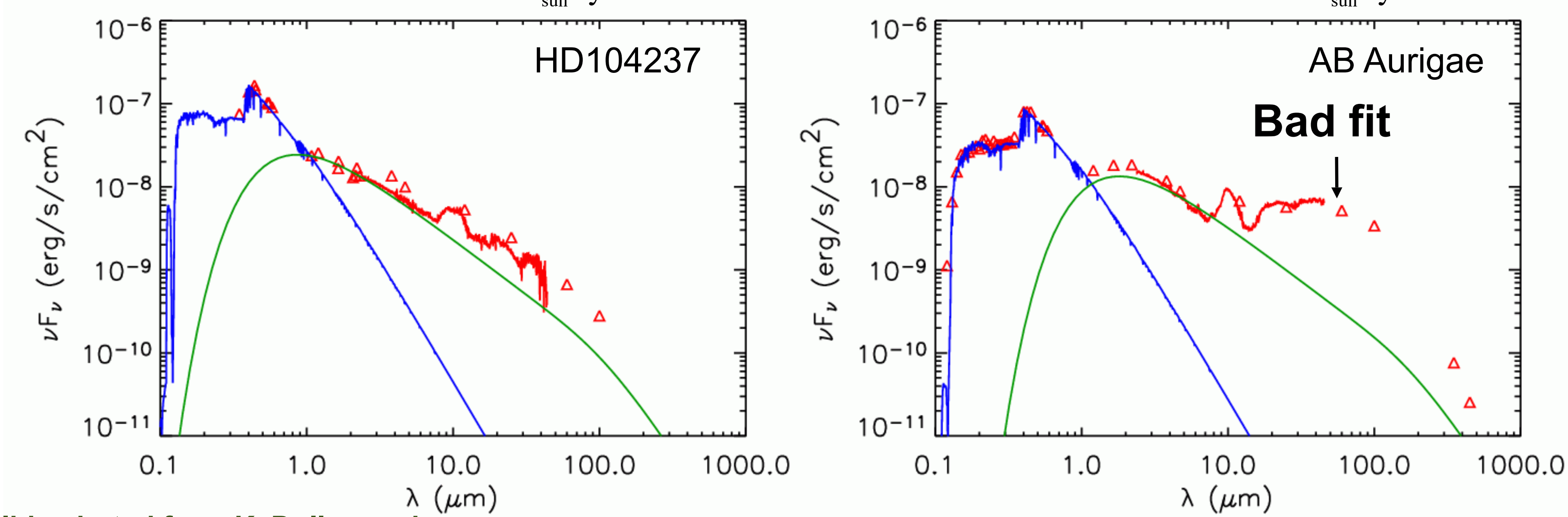
$$\nu F_\nu \propto \nu^{4/3}$$

Does this fit SEDs of Herbig Ae/Be stars?

Higher than observed from veiling (see later)

need $\dot{M} = 2 \times 10^{-7} M_{\rm sun}/{\rm yr}$

need $\dot{M} = 7 \times 10^{-7} M_{\rm sun}/{\rm yr}$



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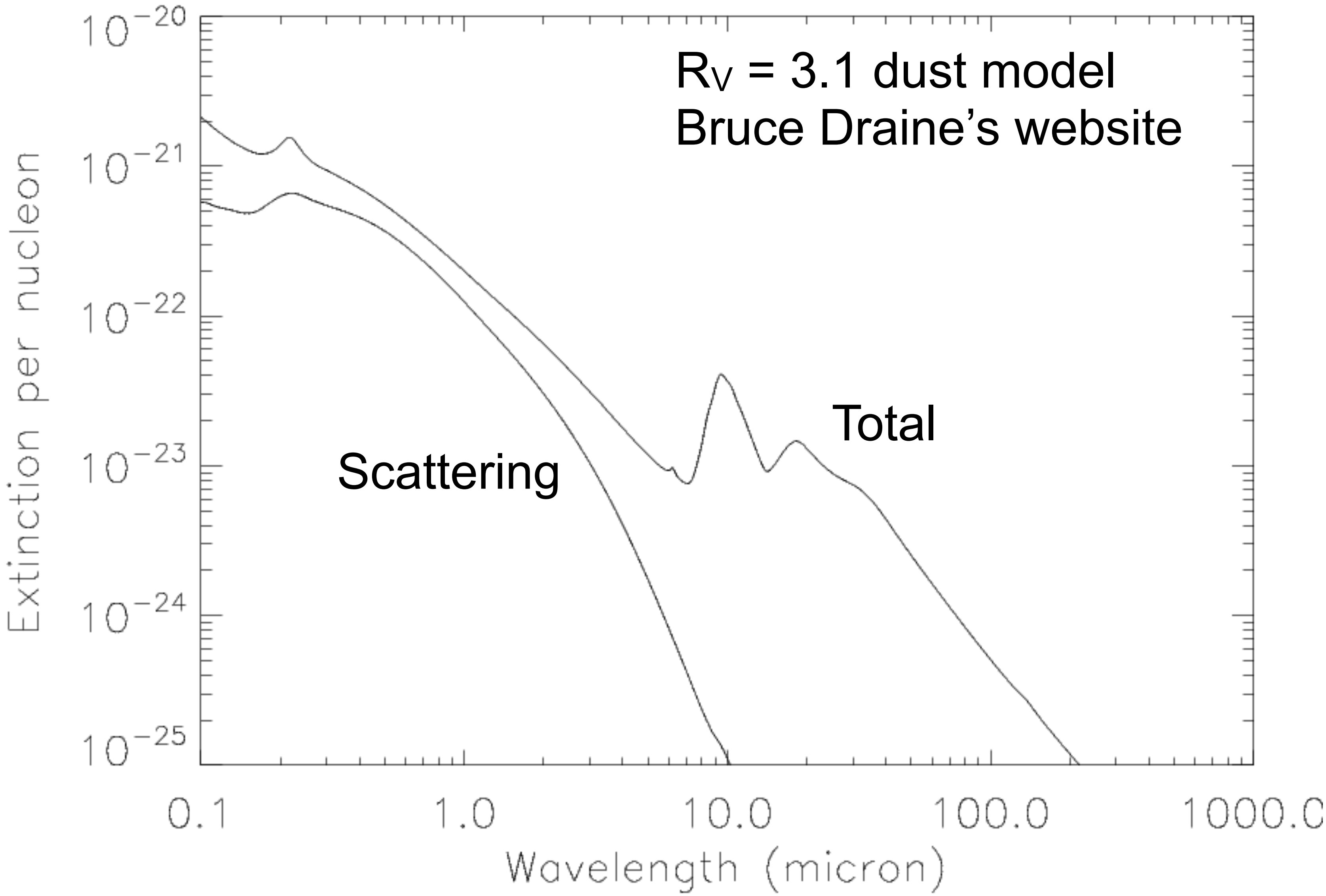
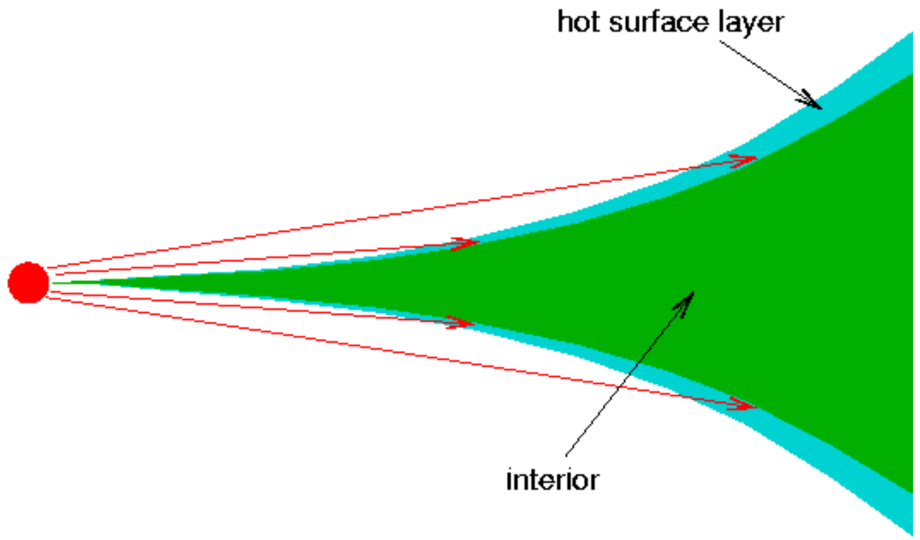
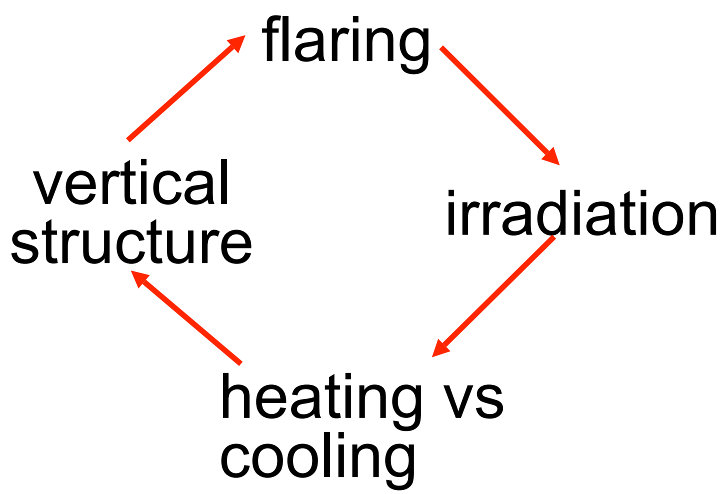
The Contribution from Scattering



Flared disks





- Kenyon & Hartmann 1987
- Calvet et al. 1991; Malbet & Bertout 1991
- Bell et al. 1997;
- D'Alessio et al. 1998, 1999
- Chiang & Goldreich 1997, 1999; Lachaume et al. 2003

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Why a Flared Disk?

Consider the force on a particle orbiting in a disk which is sitting a height z above a disk at a radius R. From the perspective of the central star, the angle between the particle and disk (with the central star at the apex) is given $\alpha \approx z/R$ (assuming $z << R$). The force can be considered two components are in cylindrical coordinates:

$$F = -[sin(\alpha)\hat{\mathbf{z}} + cos(\alpha)\hat{\mathbf{r}}]\frac{GmM}{R^2} \tag{1}$$

where M is the mass of the central star. In the case of $z << R$, this becomes:

$$F = -\left(\frac{z}{R}\hat{\mathbf{z}} + \hat{\mathbf{r}}\right)\frac{GmM}{R^2} \tag{2}$$

where $sin(\alpha) = z/R$. The radial component is balanced by centrifugal force due to the Keplerian rotation of the disk. In the vertical direction, we assume that the disk is locally supported by pressure, and it could be considered to be in hydrostatic equilibrium. In this case:

Flared Disk

Keplerian rotation of the disk. In the vertical direction, we assume that the disk is locally supported by pressure, and it could be considered to be in hydrostatic equilibrium. In this case:

$$\frac{dP}{dz} = -\frac{z}{R}\frac{G\rho M}{R^2} \tag{3}$$

where z is now the distance of above the midplane. we have replaced the mass of the particle m by the density of the gas ρ. Assuming isothermal gas ($P = c_s^2\rho$):

$$\frac{d\rho}{dz} = -\frac{G\rho z M}{c_s^2 R^3} \tag{4}$$

Flared Disks

The solution of this equation is:

$$\rho(z) = \rho(0)e^{-(GMz^2)/(2c_s^2R^2)} = \rho(0)e^{-\frac{z^2}{2H^2}} \tag{5}$$

where the scale height is

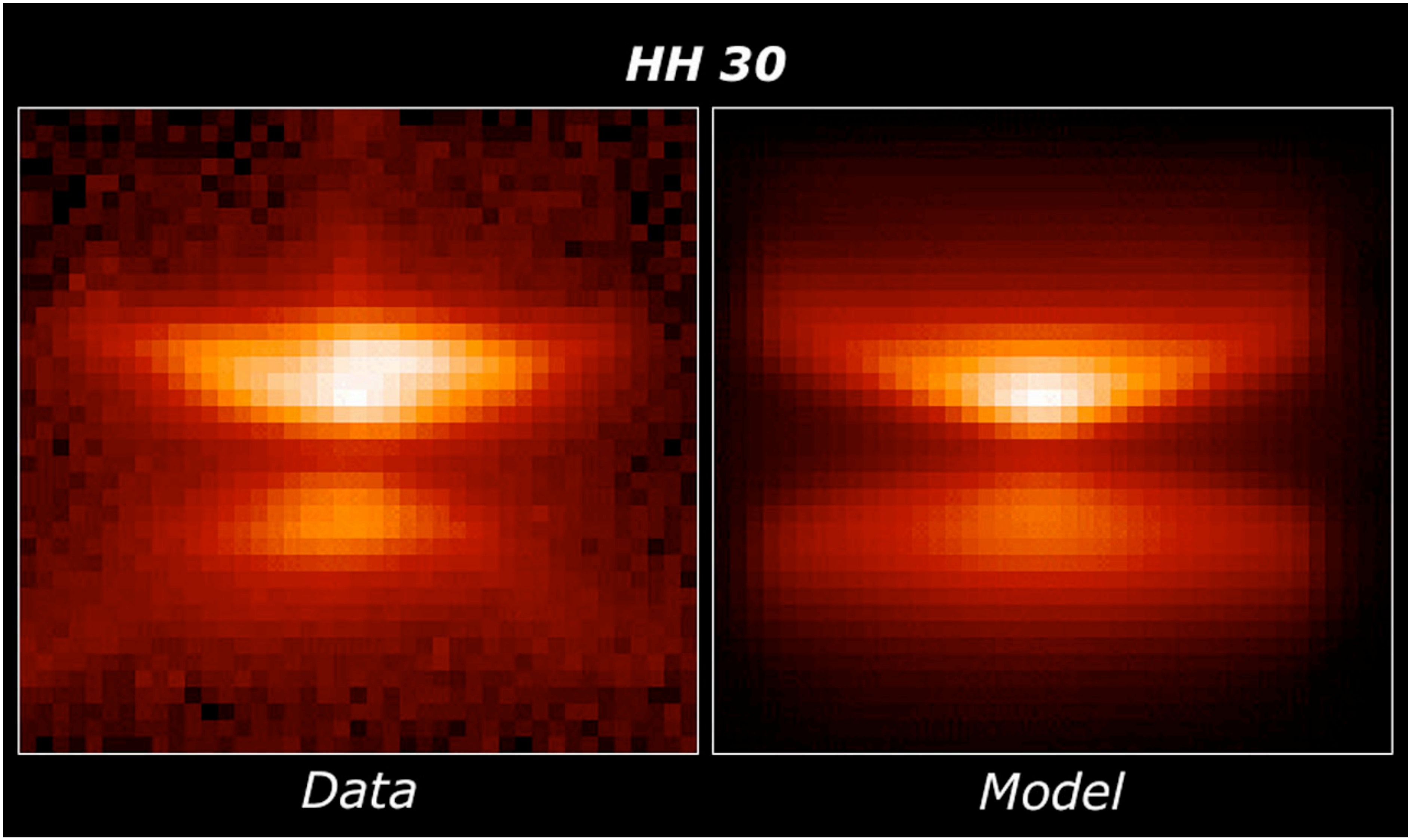
$$H = \left(\frac{R^3c_s^2}{GM}\right)^{\frac{1}{2}} \tag{6}$$

we can also write the scale height in terms of the Keplerian rotation speed $v_\phi = \sqrt{GM/R}$.

$$\frac{H}{R} = \frac{c_s}{v_\phi}, \text{ or } H = \frac{c_s}{\omega} \tag{7}$$

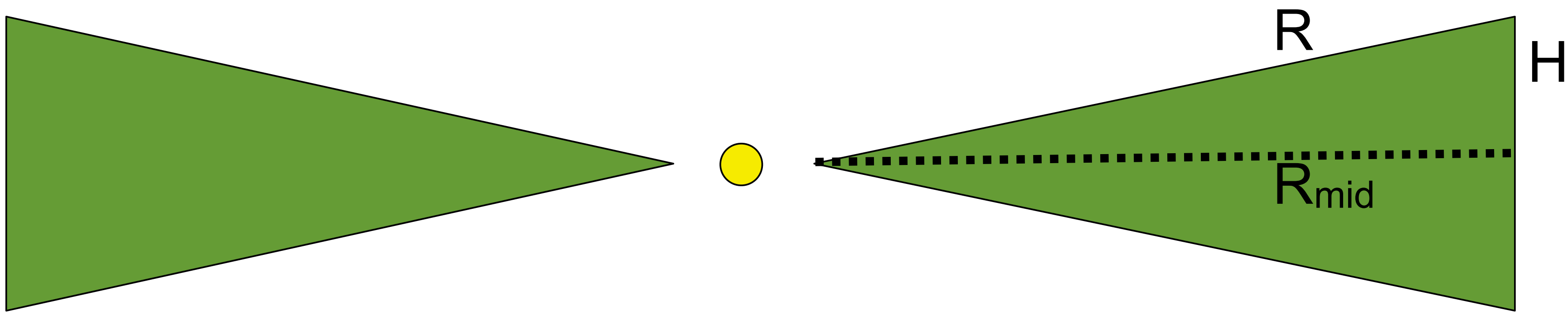
where $v_\phi = \omega R$,

HH30 - a flared disk seen in scattered light



Thermal Emission From Flared Disk

Imagine a wedge shaped disk. In limit that R is large, this is no different than a flat disk.



Thus, we expect $T = k\ R^{-3/4}$

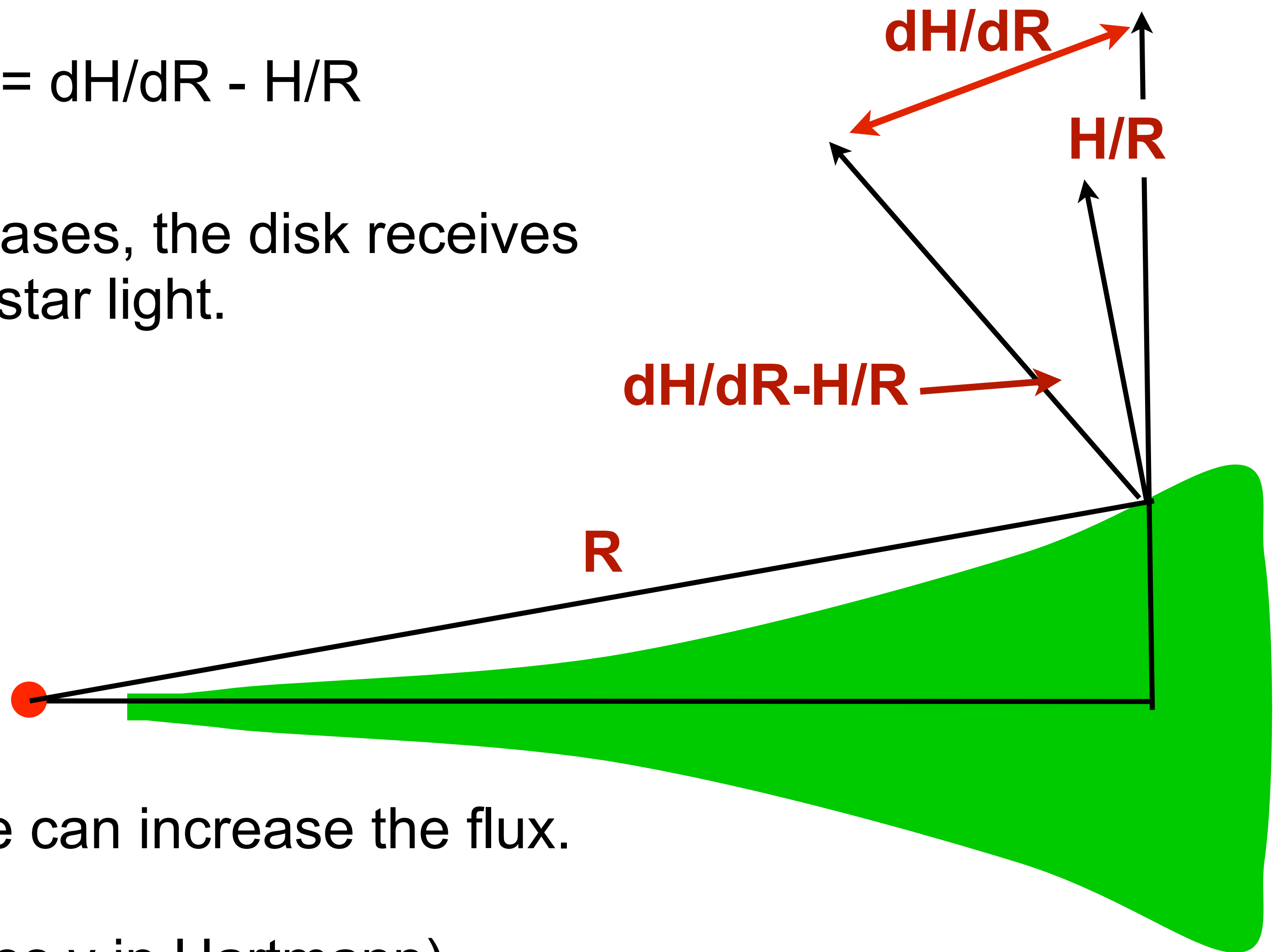
Note: $R_{mid} \sim R$ since $H << R$

Flared Disk Geometry

sin α = dH/dR - H/R

As sin α increases, the disk receives more “direct” star light.

Flaring geometry:



In this case, we can increase the flux.

(Note sin α = cos γ in Hartmann)

Flared Disk Geometry

Consider the region $R >> R_\star$ where a flat disk which show be irradiated by a very weak radiation field. Now consider a small square on a flared disk surface titled at angle α relative to the line of sight to the star. That angle α is related to the scale height by

$$sin(\alpha) = \frac{dH}{dR} - \frac{H}{R} \tag{8}$$

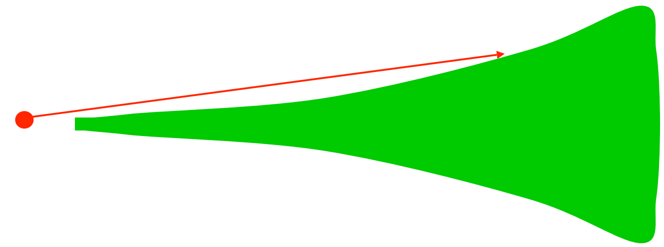
where H is the height of the disk surface (i.e photosphere) at radius R. We can also write $sin(\alpha)$ as:

$$sin(\alpha) = R\frac{\partial}{\partial R}\left(\frac{H}{R}\right) \tag{9}$$

Flared disks: Chiang & Goldreich model

The flaring angle:

$$\alpha = r\frac{\partial}{\partial r}\left(\frac{h_s}{r}\right) \rightarrow \xi\frac{h_s}{r}$$



Irradiation flux:

$$F_{\rm irr} = \alpha\frac{L_*}{4\pi r^2}$$

$$T^4 = \frac{\xi}{\sigma}\frac{h_s L_*}{4\pi r^3}$$

Cooling flux:

$$F_{\rm cool} = \sigma T^4$$

Express surface height in terms of pressure scale height:

$$h_s = \chi h \qquad \chi = 1\ldots6$$

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Flared disks: Chiang & Goldreich model

$$T^4 = \frac{\xi}{\sigma}\frac{h_s L_*}{4\pi r^3} \longleftarrow \qquad h_s = \chi h$$

$$T^4 = \frac{\xi}{\sigma}\frac{\chi h L_*}{4\pi r^3}$$

Remember formula for pressure scale height:

$$h = \sqrt{\frac{kTr^3}{\mu m_p G M_*}} \qquad h^8 = \left(\frac{k}{\mu m_p G M_*}\right)^4 r^{12} T^4$$

We obtain

$$h^7 = \left(\frac{k}{\mu m_p G M_*}\right)^4 r^9 \frac{\xi}{\sigma}\frac{\chi L_*}{4\pi}$$

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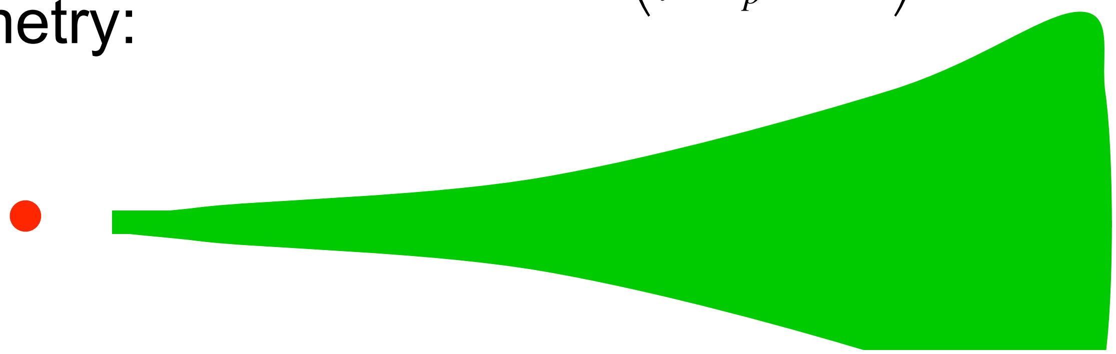
Flared disks: Chiang & Goldreich model

$$h^7 = \left(\frac{k}{\mu m_p G M_*}\right)^4 r^9 \frac{\xi}{\sigma} \frac{\chi L_*}{4\pi}$$

We therefore have:

$$h = C^{1/7} r^{9/7} \quad \text{with} \quad C = \left(\frac{k}{\mu m_p G M_*}\right)^4 \frac{\xi}{\sigma} \frac{\chi L_*}{4\pi}$$

Flaring geometry:

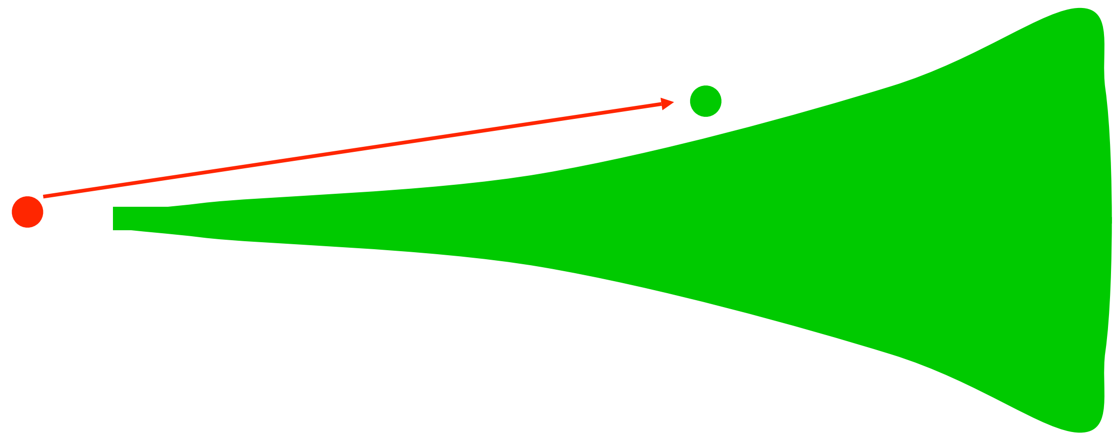


$$h \propto r^{9/7}, T \propto r^{-3/7} \tag{18}$$

Thus, the temperature gradient is much less steep than $T \propto r^{-3/4}$ for flat disks.

Remark: in general χ is not a constant (it decreases with r). The flaring is typically <9/7

The surface layer



A dust grain in (above) the surface of the disk sees the direct stellar light. Is therefore much hotter than the interior of the disk.

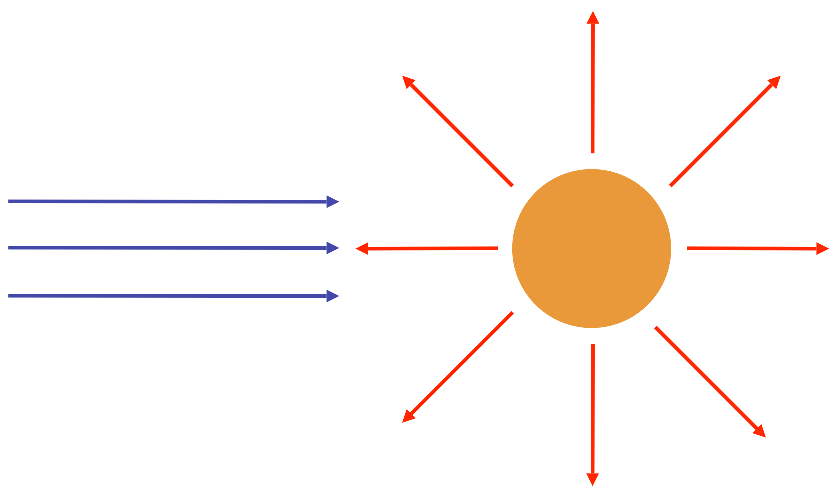
$$T^4 = sin(\alpha)\frac{L_\star}{4\pi\sigma R^2}$$

$$T^4 = \frac{L_\star}{16\pi\sigma R^2}$$

Temperature of a dust grain

<u>Optically thin case:</u>

Heating:



$$Q_+ = \pi a^2 \int F_\nu \, \varepsilon_\nu \, d\nu$$

a = radius of grain

ε_ν= absorption efficiency (=1 for perfect black sphere)

Cooling:

$$Q_- = 4\pi a^2 \int \pi B_\nu(T) \varepsilon_\nu \, d\nu$$

$$\kappa_\nu = \frac{\pi a^2 \varepsilon_\nu}{m}$$

Thermal balance:

$$\int B_\nu(T) \kappa_\nu \, d\nu = \frac{1}{4\pi} \int F_\nu \, \kappa_\nu \, d\nu$$

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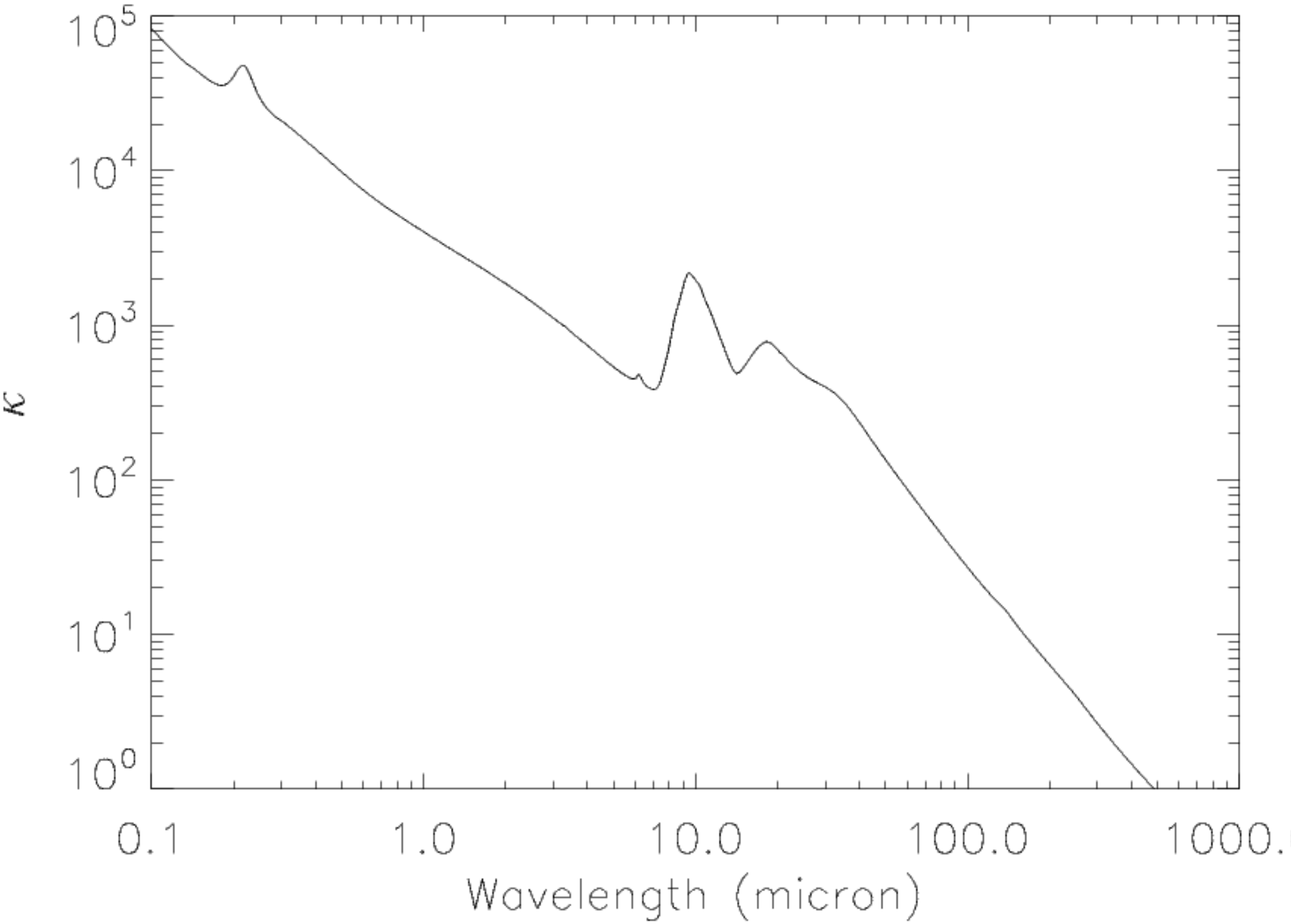
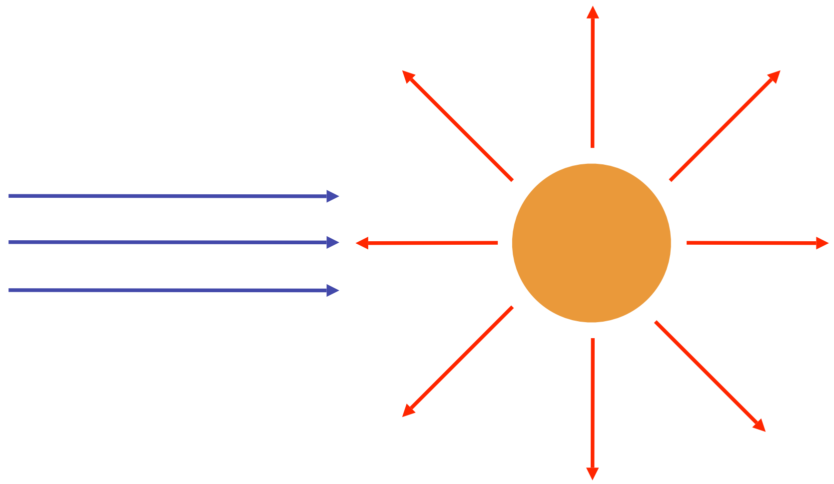
Dust Absorption in the IR



Temperature of a dust grain

$$\int B_\nu(T)\kappa_\nu\, d\nu = \frac{1}{4\pi}\int F_\nu\, \kappa_\nu\, d\nu$$



Big grains, i.e. grey opacity:

$$T^4 = \frac{1}{4\sigma}\frac{L_*}{4\pi r^2}$$

$$T = \sqrt{\frac{r_*}{2r}}\, T_*$$

Small grains: high opacity at short wavelength, where they absorb radiation, low opacity at long wavelength where they cool.

$$T > \sqrt{\frac{r_*}{2r}}\, T_*$$

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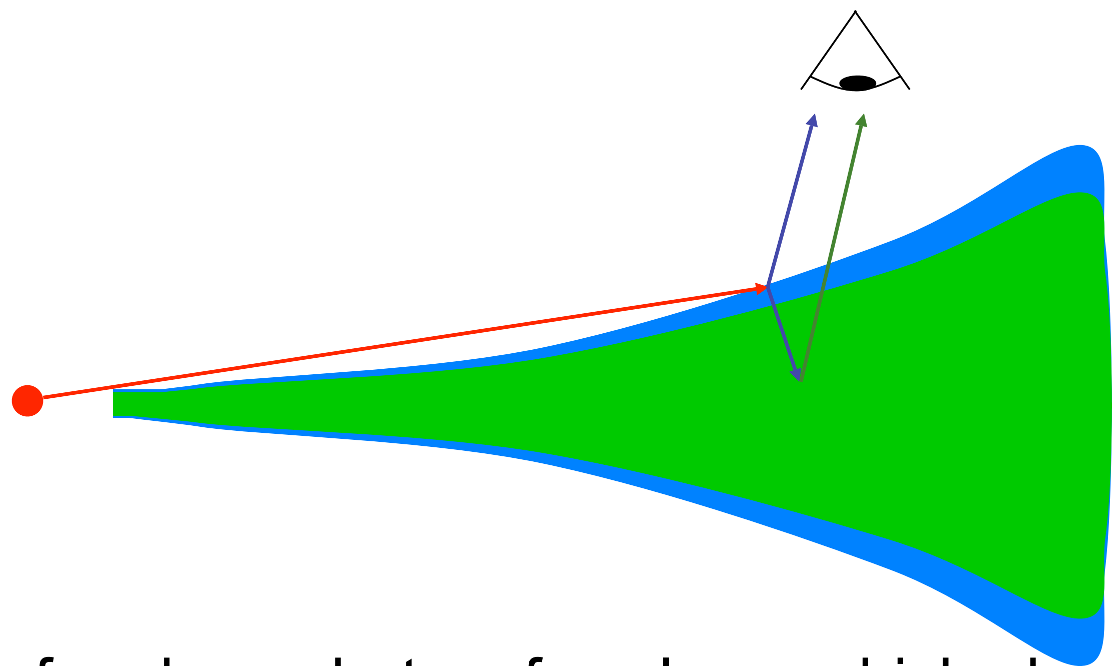
The Superthermal Surface Layer



Disk therefore has a hot surface layer which absorbs all stellar radiation. It is optically thick at visible wavelengths, but optically thin in the infrared.

The hot surface emits in the infrared. Half of it is re-emitted upward (and escapes); half of it is re-emitted downward (and heats the interior of the disk).

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Why is the atmosphere superthermal?

A few reasons:

- High opacity for visible and UV light, lower opacity for IR light. Creates high temperature for individual grains in optically thin case.
- Geometric: grain cross-section to emitting area is 1/4. Cross section to emitting area for disk in sin(α).

Disk

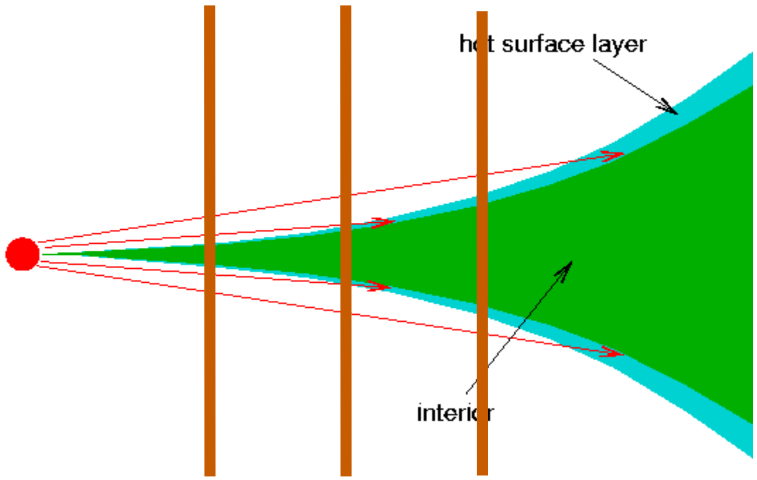
$$T^4 = sin(\alpha)\frac{L_\star}{4\pi\sigma R^2}$$

Grey grain

$$T^4 = \frac{L_\star}{16\pi\sigma R^2}$$

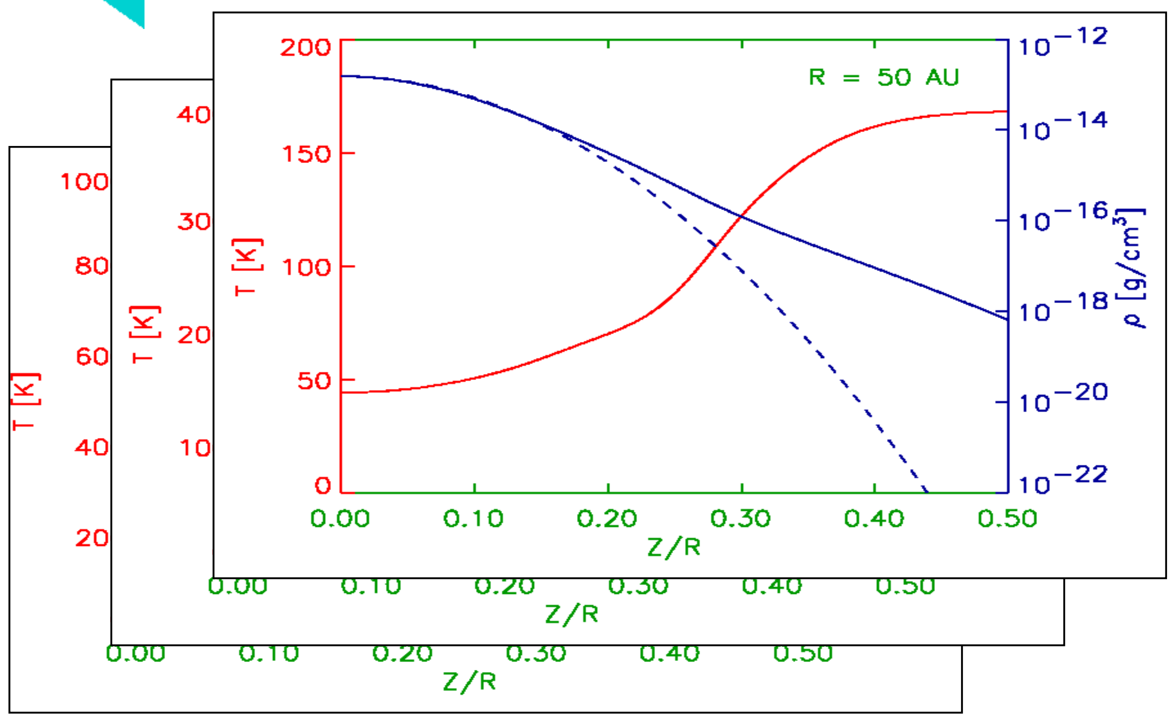
- Atmosphere is optically thin to IR radiation from disk (inner disk can radiate into cold space)

Flared disks: detailed models



Global disk model...

... consists of vertical slices, each forming a 1D problem. All slices are independent from each other.



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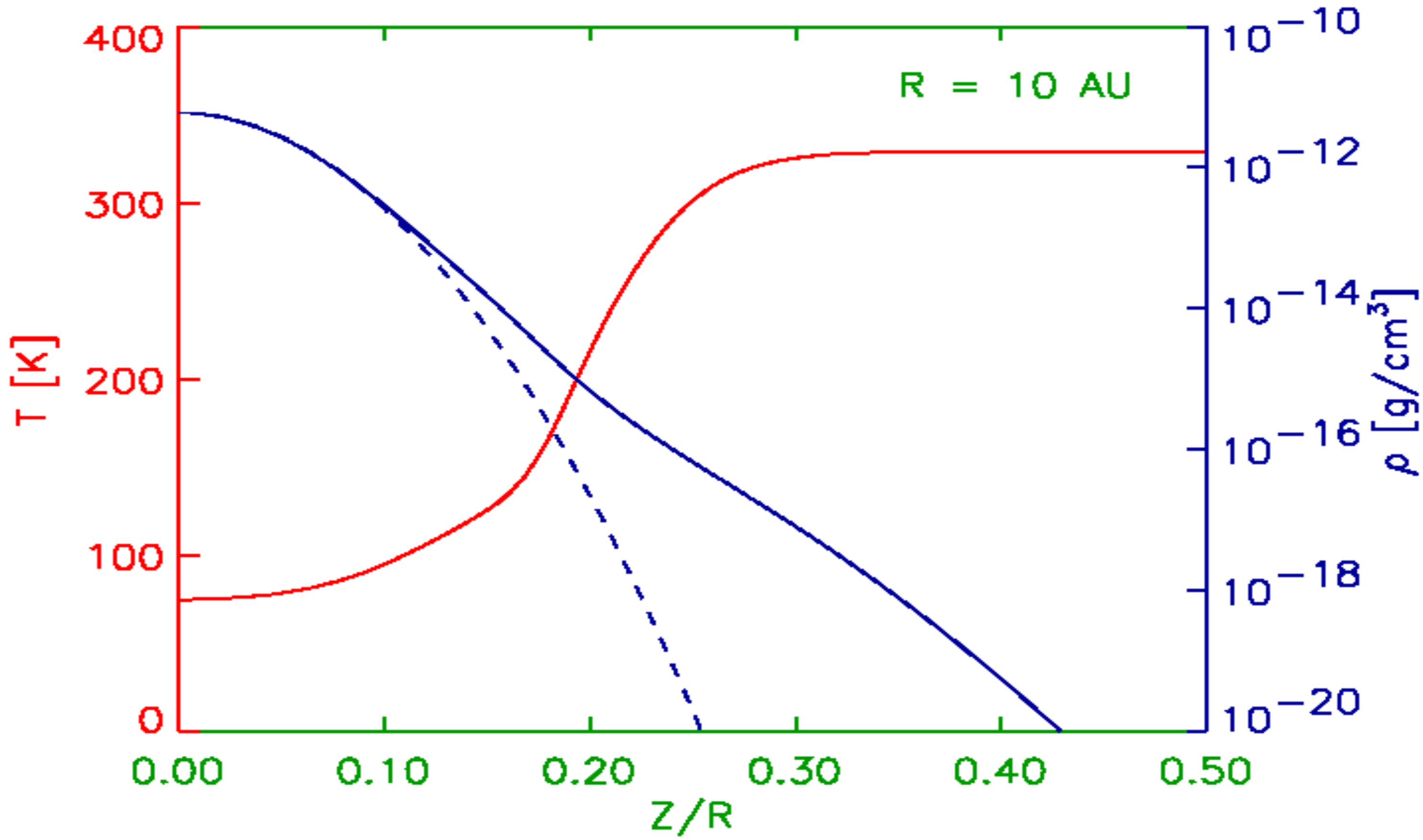
Flared disks: detailed models

A closer look at one slice:



Malbet & Bertout, 1991, ApJ 383, 814
D'Alessio et al. 1998, ApJ 500, 411
Dullemond, van Zadelhoff & Natta 2002, A&A 389, 464

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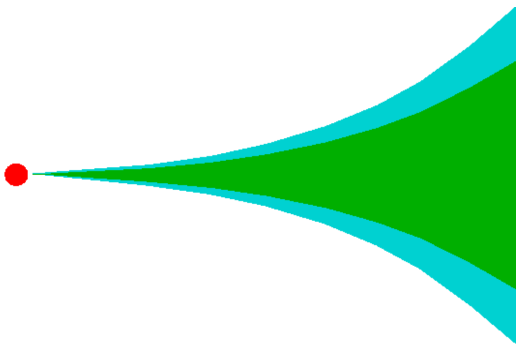
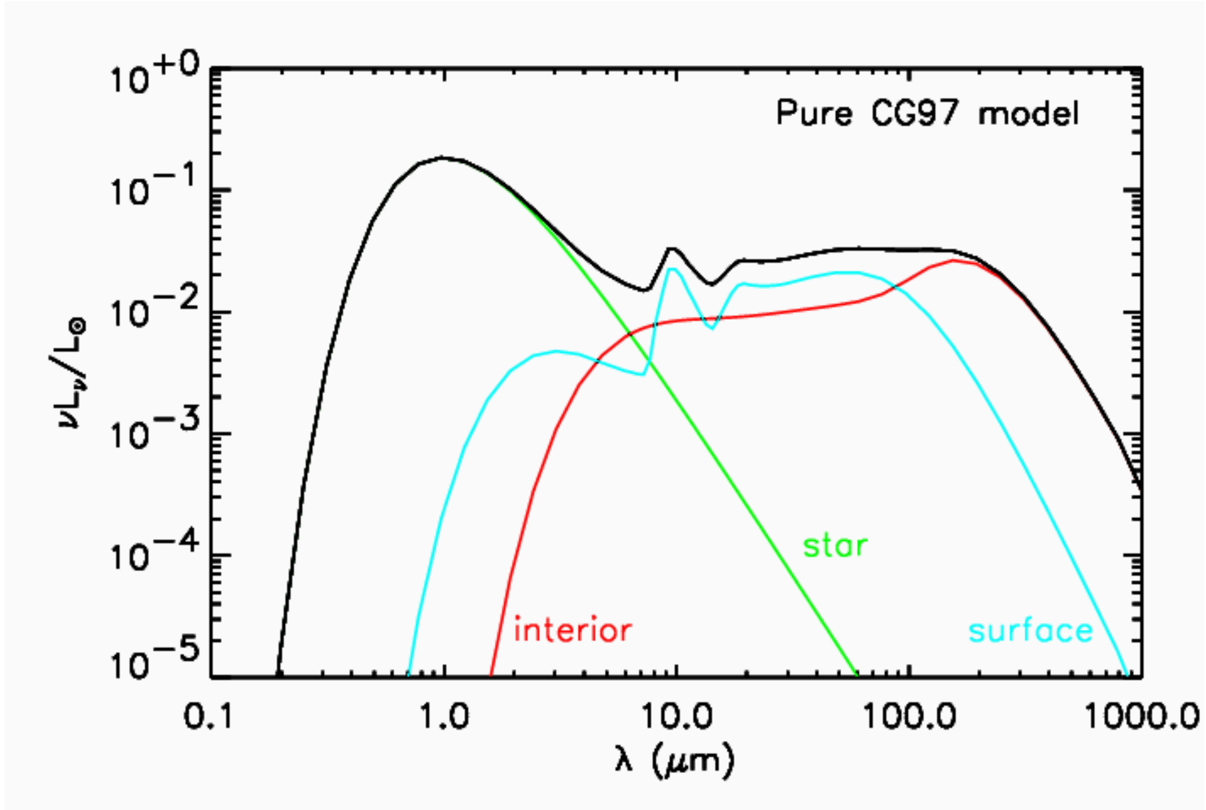
Chiang & Goldreich: two layer model

Model has two components:

- Surface layer
- Interior





Chiang & Goldreich (1997) ApJ 490, 368

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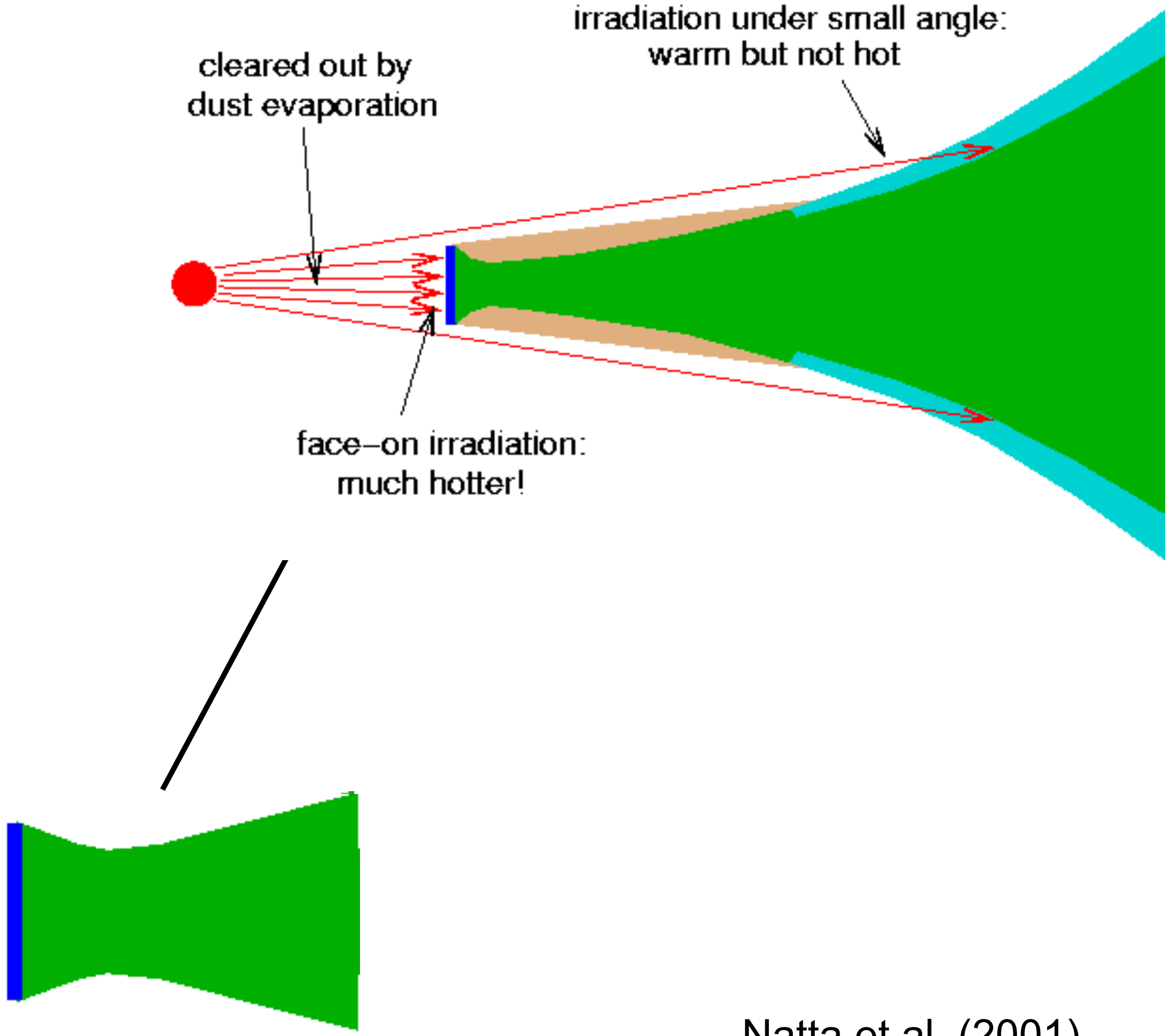
Dust evaporation and disk inner rim



Natta et al. (2001)
Dullemond, Dominik & Natta (2001)

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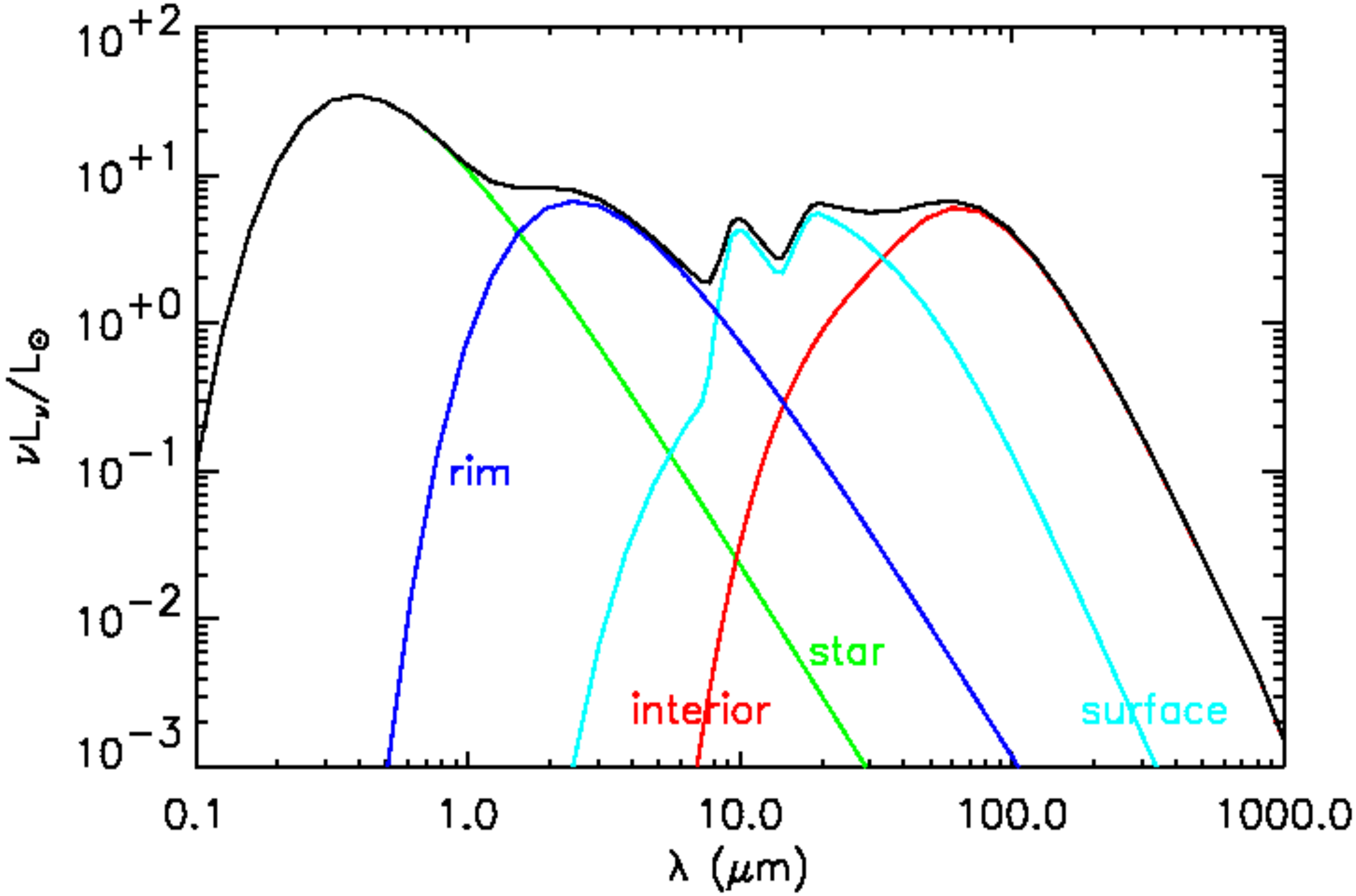
SED of disk with inner rim



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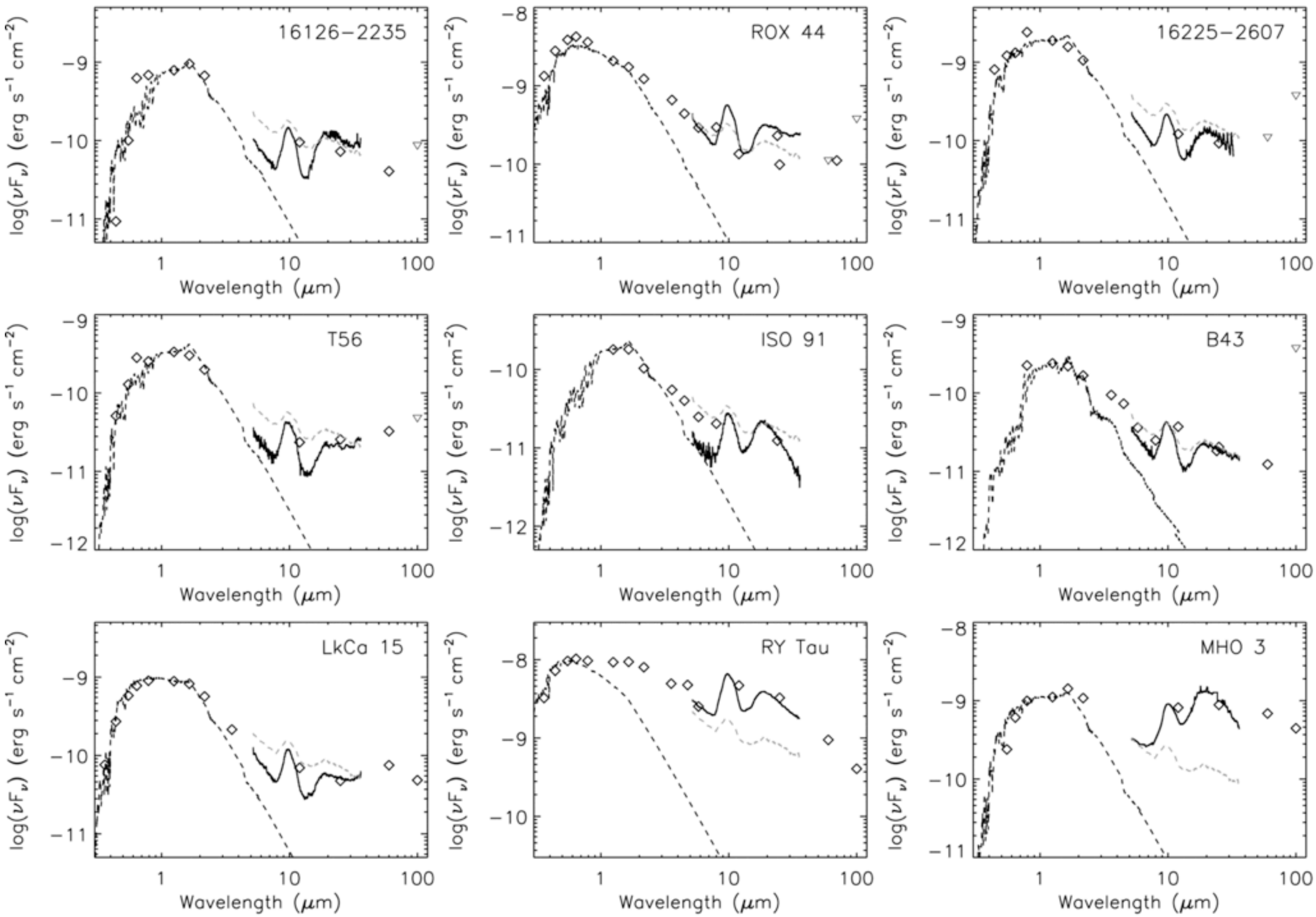
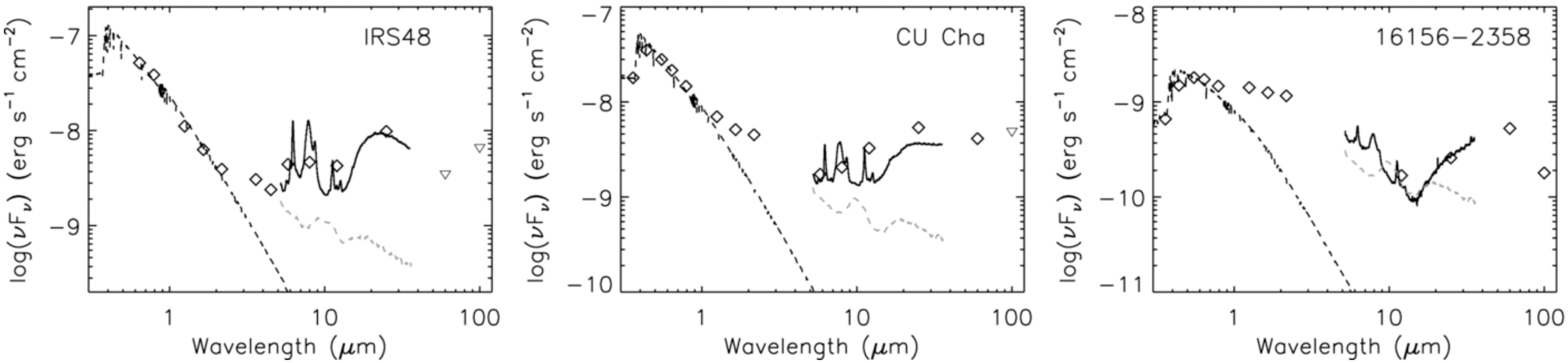
T-Tauri Star Disks (Furlan et al. 2009)



Herbig Ae/Be Star Disks (Furlan et al. 2009)



Summary

- Flat disk has a temperature of $T = k R^{-3/4}$
- Flaring of disk can occur if decrease in gravity beats out decrease in temperature with increasing disk radius.
- Scale height of flared disk $H = k R^{9/7}$
- Temperature of flared disk $T = k R^{-3/7}$
- Hot disk atmosphere will form by absorption of visible and UV by dust grains - this hot surface layer will produce hot emission from 8-30 microns - including the silicate features at 9 and 18 microns in emission
- Inner rim will form where dust is sublimated by central star - this produces a 1800 K blackbody that produces emission primarily from 2-8 microns.

Summary

Spectral Energy Distributions: distribution of power over large portion of the electromagnetic spectrum. Usually constructed from a mixture of photometry and spectroscopy from many different instruments.

SEDs are a major source of information on protostars and stars with disks.

Dust temperature for a grain being heated directly by a star decreases $T = k\ r^{-1/2}$

An optically thick shell (were the primary form of opacity is dust) can reprocess radiation to a lower wavelength, creating an effective low temperature dust photosphere. $Tshell = k\ Rshell^{-1/2}$

The radius of the dust photosphere depends on the wavelength and opacity - this pushes the peak of the protostellar SED into the far-IR

Scattering of light from the inner star and disk by the envelope may also fill in protostellar SEDs at wavelengths < 10 μm.

Disks can be modeled as a series of concentric annuli each heated to a different temperature.

For a passively heated flat disk, the temperature goes as $T = k\ r^{-3/4}$