1. The Salpeter Initial Mass Function

The initial mass function is a function describing the distribution of stellar masses in a newly formed population (i.e. none of the stars have had a chance to loose mass or undergo supernova). The initial mass function, IMF, was first derived by Ed Salpeter in 1955, who found that:

$$\xi(logM) = \frac{dN}{dlog(M)} = k_1 M^{-\Gamma} = k_1 M^{-1.35} \tag{1}$$

A similar function is the mass spectrum

$$\frac{dN}{dM} = k_2 M^{-\alpha} = k_2 M^{-2.35} \tag{2}$$

where $\alpha = \Gamma + 1$.

The total mass is then the integral of this:

$$M_{tot} = \int_{M_{min}}^{M_{max}} M k_2 M^{-2.35} dM = \frac{k_2}{0.35}(M_{min}^{-0.35} - M_{max}^{-0.35}) \tag{3}$$

This shows that most of the stellar mass is in low mass stars. On the other hand, if we calculate the total luminosity (and assuming $L \propto M^3$), then

$$L_{tot} = \int_{M_{min}}^{M_{max}} k_3 M^3 k_2 M^{-2.35} dM = \frac{k_2 k_3}{1.65}(M_{max}^{1.65} - M_{min}^{1.65}) \tag{4}$$

which shows that the total luminosity is driven by the most massive stars. We know now that the IMF is not a strict power law, and we will examine the variations.

2. The Field Star IMF

One way of deriving the IMF is to use field stars. To show how this is done, we follow the seminal work of Miller & Scalo (1979). This goes in two steps. First, a present day mass function, PDMF or $\phi(logM)$ for main sequence stars is found. This is the number of stars per mass per unit area in the galaxy; it is integrated over the "vertical" dimension of the disk. The PDMF is given by

$$\phi_{MS}(logM) = \phi(M_V)\left|\frac{dM_V}{dlogM}\right| 2H(M_V)f_{MS}(M_V) \tag{5}$$

where $\phi_{MS}(logM)$ is the present day mass function, $\phi(M_V)$ is the luminosity function as a function of the absolute magnitude M_V, $H(M_V)$ is the galactic scale height for a given M_V and $f_{MS}(M_V)$ is the fraction of stars with M_V.

To get the IMF, we must make an assumption about the birthrate of stars. Miller Scalo relate the IMF, $\xi(logM)$ to the PDMF, $\phi(logM)$, and the birthrate, $b(t)$, using the equation.

$$\phi_{MS}(logM) = \frac{\xi(logM)}{T_0}\int_{T_0-T_{MS}}^{T_0} b(t)dt,\ T_{MS} < T_0 \tag{6}$$

$$\phi_{MS}(logM) = \frac{\xi(logM)}{T_0}\int_{0}^{T_0} b(t)dt,\ T_{MS} \geq T_0 \tag{7}$$

Thus, if the birthrate is constant (an assumption first made by Ed Salpeter, and almost certainly wrong):

$$\phi_{MS}(logM) = \xi(logM)\left(\frac{T_{MS}}{T_0}\right),\ T_{MS} < T_0 \tag{8}$$

$$\phi_{MS}(logM) = \xi(logM),\ T_{MS} \geq T_0 \tag{9}$$

Using various input data, a number of researchers have found different versions of the IMF. Miller & Scalo overviewed these forms:

A quadratic fit:

$$log\xi(logM) = A_0 + A_1 logM + A_2(logM)^2 \tag{10}$$

A log-normal IMF:

$$\xi(logM) = C_1 e^{C_2(logM - C_3)} = ke^{-\frac{(logM - logM_c)^2}{2\sigma^2}} \tag{11}$$

or a multi-power-law form where over a given range of masses:

$$\xi(logM) = D_0 M^{D_1} \tag{12}$$

where values given are by Miller & Scalo (1979) and have been updated by numerous more recent works.

3. Bondi-Hoyle Accretion

Bondi-Hoyle accretion rate (which was actually first presented by Hoyle and Lyttleton in 1942) is for a star moving through an infinite medium, where the density converges to a value ρ_∞ ($\rho \rightarrow \rho_\infty$ as $r \rightarrow \infty$).

$$\dot{M} = 4\pi\rho \frac{(GM_\star)^2}{(v^2 + c_s^2)^{3/2}} \tag{13}$$