
Lecture 12: The Initial Mass Function (and a little more about debris disks)

Summary

- Young pre-main sequence stars are surrounded by disks of gas and dust. Small dust grains make disks optically thick.
- Disappearance of gas disks happens quickly, 1/2 of stars lose their disks in 2-3 Myr, almost all in 10 Myr.
- Evolution of disks includes settling of dust and collision of dust to form larger grains.
- Evidence of dust settling and growth apparent in SEDs, silicate features, and sub-millimeter part of the spectrum.
- Holes and gaps are observed in disks, these may be created by planets, binaries or photoevaporation.
- After the gas disk is dispersed, we are left with debris disks, which are disks of small dust grains (which produce IR-excess) produced in collisions of large planetesimals (which aren’t observed).

The Size of Dust Grains

Imagine you have a constant mass of dust in a disk, but the size of grains are growing. Let's assume that all the grains have the same radius, a, and the same density ρ. Then the number of grains is simply:

$$N = \frac{3M_{dust}}{4\pi a^3} \tag{7}$$

The geometric cross section of those same grains will be

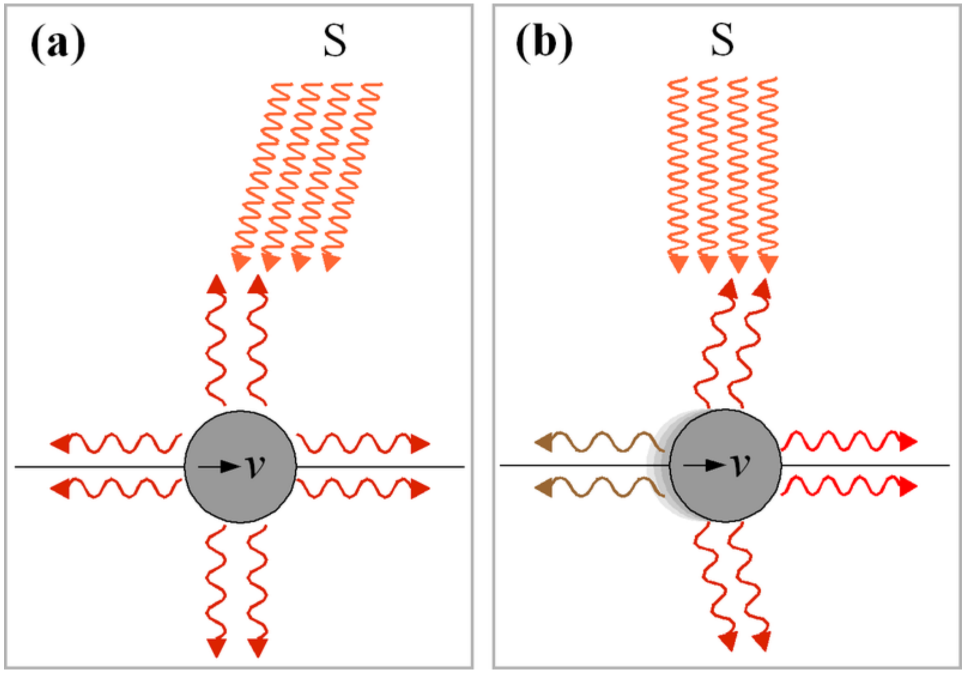
$$\sigma_{geo} = \frac{3\pi a^2 M_{dust}}{4\pi \rho a^3} = \frac{3M_{dust}}{4\rho a} \tag{8}$$

Thus the cross section of the grains decrease with size. In other words, as the grains collide and coagulate into larger bodies, the cross-section drops. As grains go from 1 μm to 1 m, the cross section drops by 10^{-6}. Consider an optically thin dist made out of grains with size a. The therma emission is:

$$L_\nu = 4\pi R^2 \pi B_\nu(T)\sigma_{geo} = \frac{3M_{dust}B_\nu(T)}{4\rho a} \tag{9}$$

Thus the luminosity from the dust grains drops by 10^{-6} as grains go from 1 μm to 1 m in size.

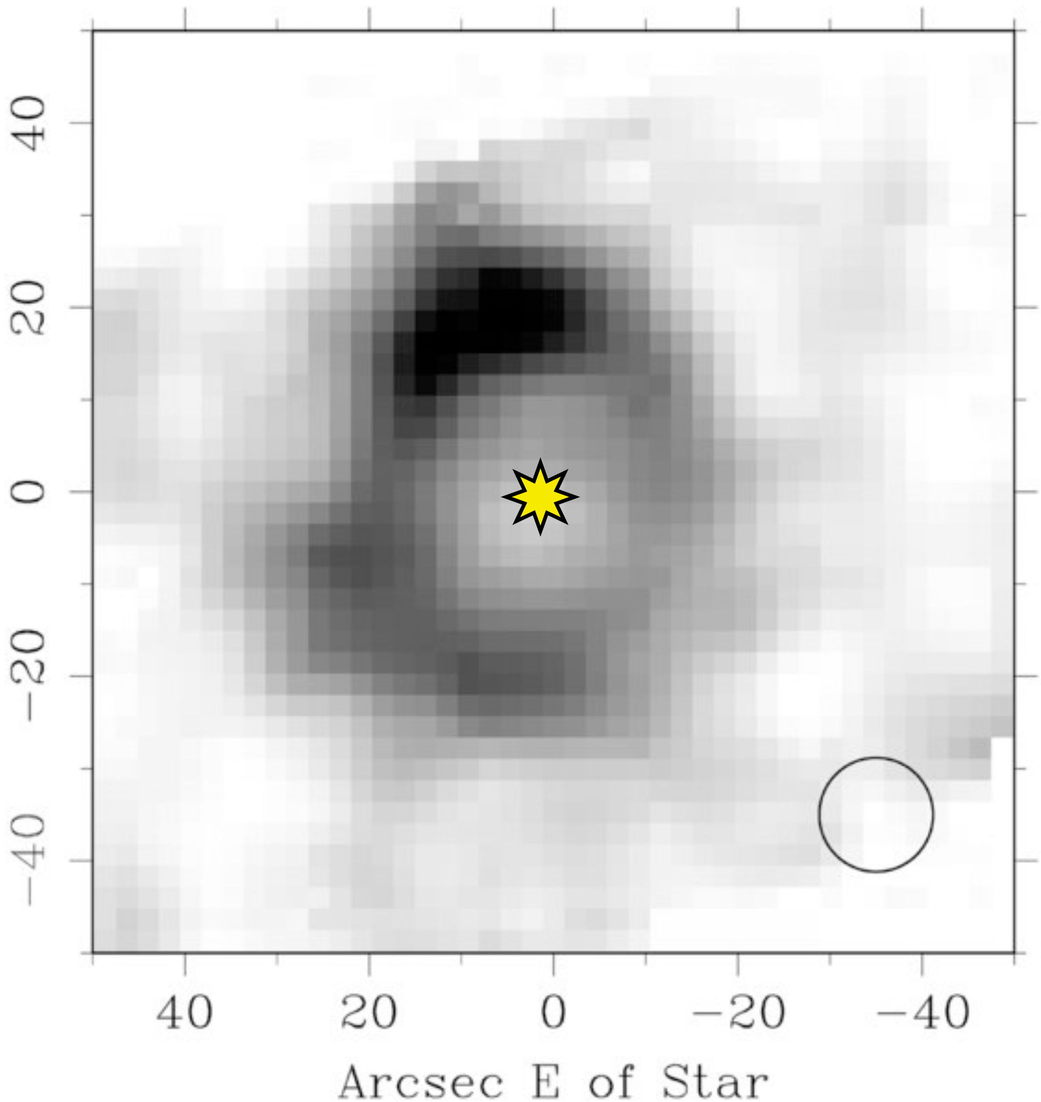
Poynting Robertson Drag



Drag due to photon momentum. Dust grains in orbit around star will have a small torque placed on them.

$$t_{PR}[yr] \approx 2.2 \times 10^3 \left(\frac{a}{\mu m}\right)\left(\frac{\rho}{3\ g\ cm^{-3}}\right)\left(\frac{r^2}{AU^2}\right)\left(\frac{L_\star}{L_\odot}\right) \quad (15)$$

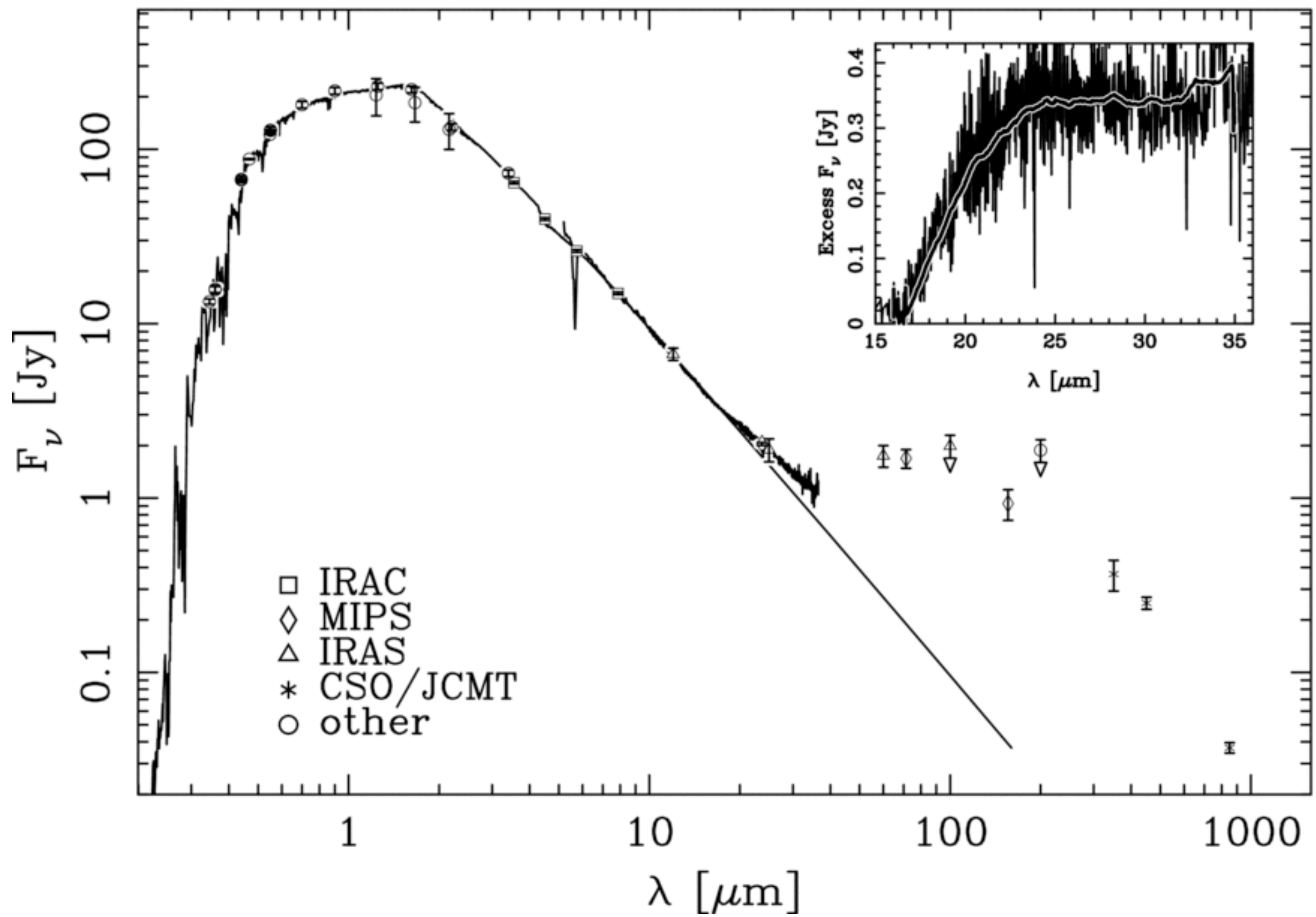
The Epsilon Eridani Debris Disk



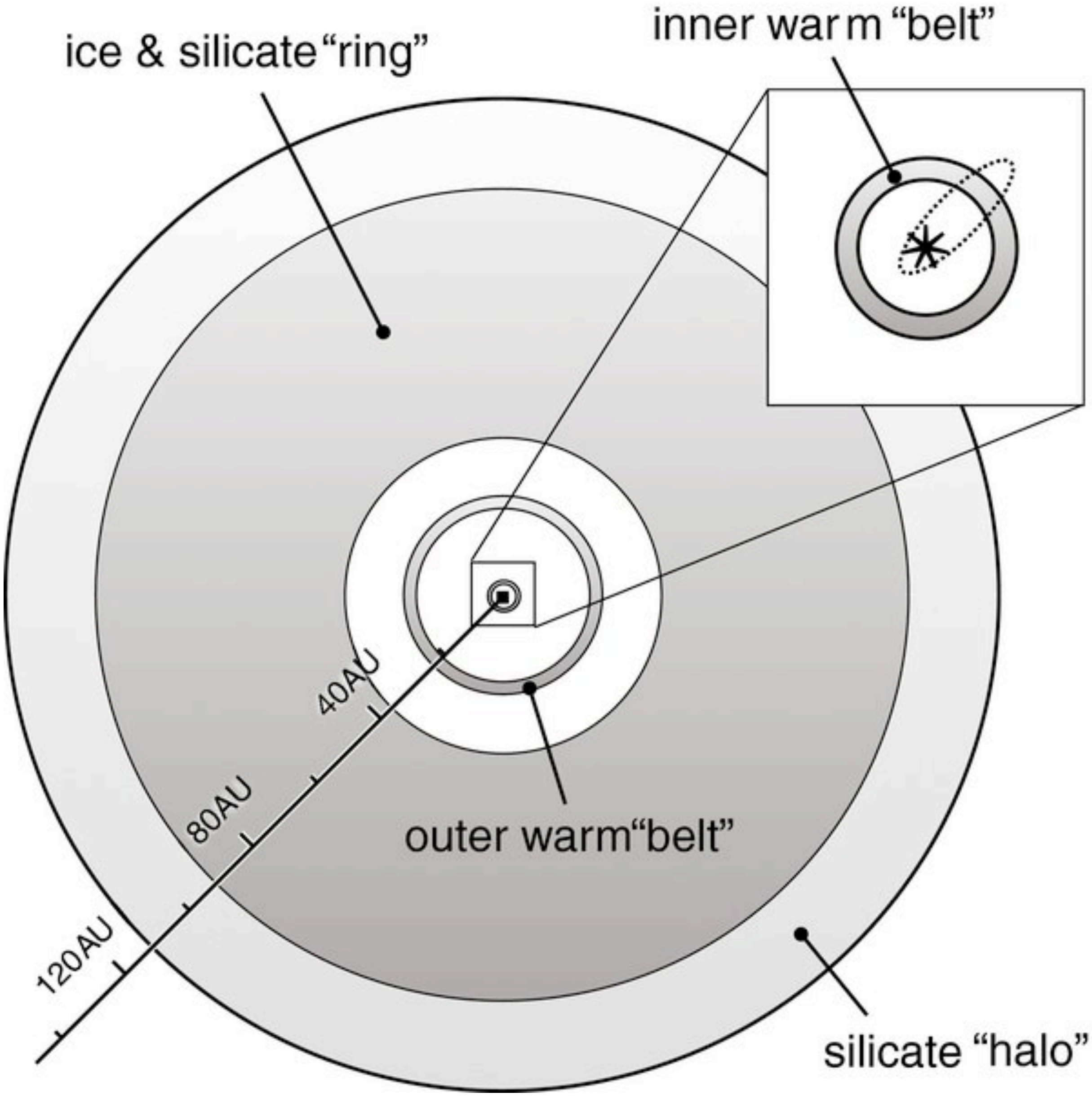
The disk appears as a ring at 350 μm

Backman et al. 2009

SED of the K2V Star Epsilon Eridani



Model for Epsilon Eridani



The Salpeter IMF

The initial mass function is a function describing the distribution of stellar masses in a newly formed population (i.e. none of the stars have had a chance to loose mass or undergo supernova). The initial mass function, IMF, was first derived by Ed Salpeter in 1955, who found that:

$$\xi(logM) = \frac{dN}{dlog(M)} = k_1 M^{-\Gamma} = k_1 M^{-1.35} \tag{1}$$

A similar function is the mass spectrum

$$\frac{dN}{dM} = k_2 M^{-\alpha} = k_2 M^{-2.35} \tag{2}$$

where $\alpha = \Gamma + 1$.

What Stellar Masses dominate the Integrated Mass and Luminosity?

The total mass is then the integral of this:

$$M_{tot} = \int_{M_{min}}^{M_{max}} M k_2 M^{-2.35} dM = \frac{k_2}{0.35}(M_{min}^{-0.35} - M_{max}^{-0.35}) \tag{3}$$

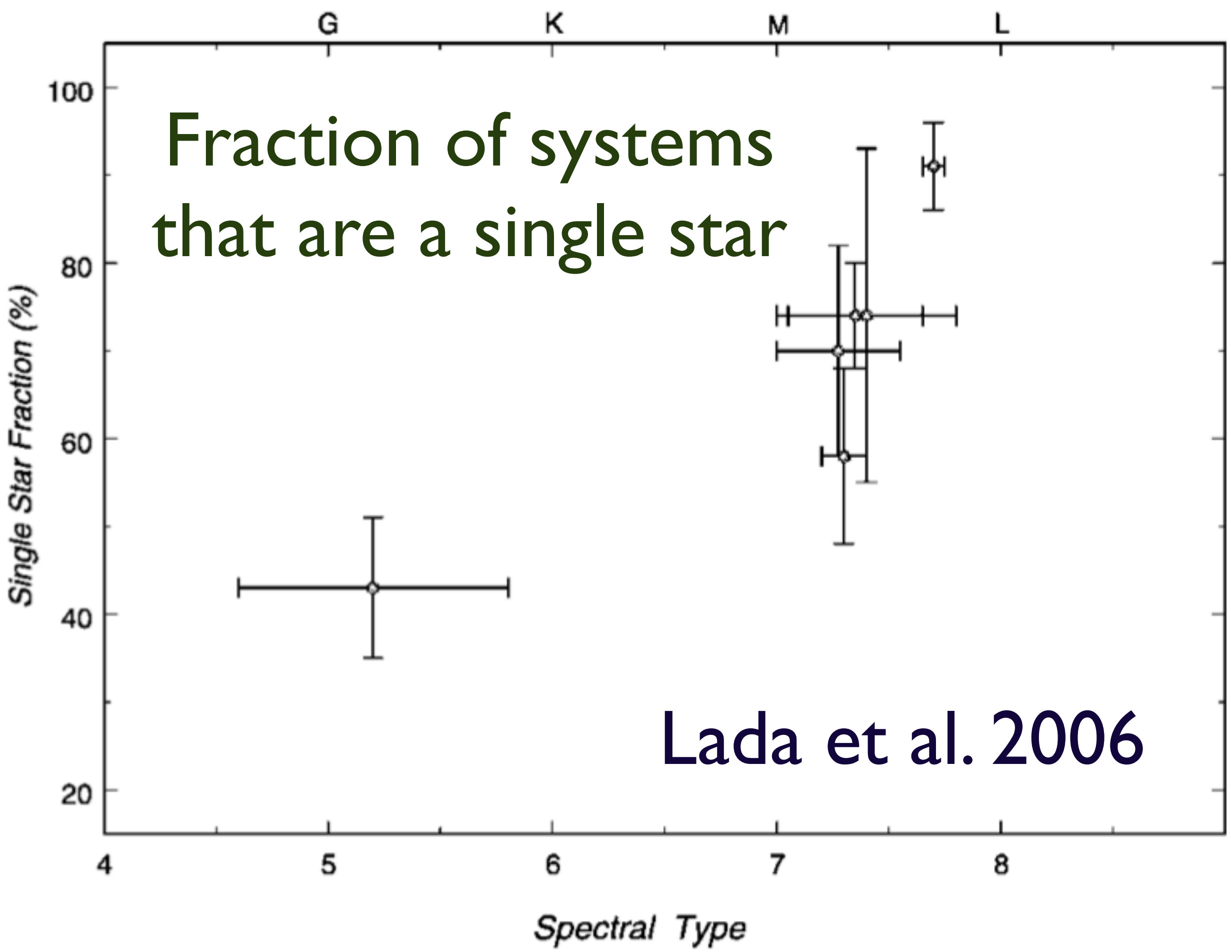
This shows that most of the stellar mass is in low mass stars. On the other hand, if we calculate the total luminosity (and assuming $L \propto M^3$), then

$$L_{tot} = \int_{M_{min}}^{M_{max}} k_3 M^3 k_2 M^{-2.35} dM = \frac{k_2 k_3}{1.65}(M_{max}^{1.65} - M_{min}^{1.65}) \tag{4}$$

which shows that the total luminosity is driven by the most massive stars. We know now that the IMF is not a strict power law, and we will examine the variations.

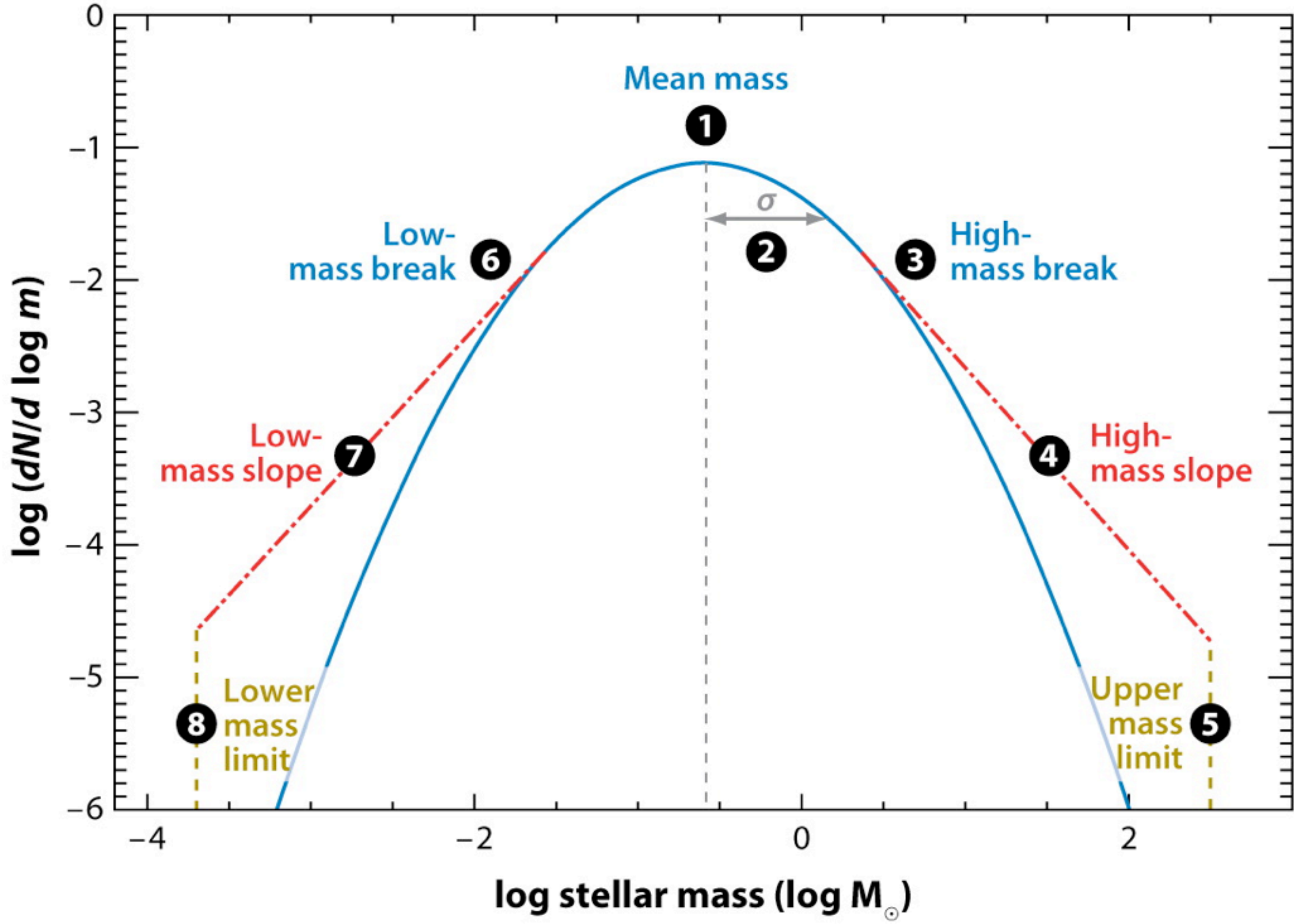
Systems or Single Stars?

Most IMFs are actually system IMFs.



Multiplicity hard to correct for, since need to know fraction of stars that are systems and the mass ratio of that systems.

A general view of the IMF



Bastian N, et al. 2010.
Annu. Rev. Astron. Astrophys. 48:339–89

Determination of Initial Mass Function from Field Stars

One way of deriving the IMF is to use field stars. This goes in two steps. First, a present day mass function for main sequence stars is found. This is the number of stars per mass per unit area in the galaxy. It is integrated over the "vertical" dimension of the disk (Miller & Scalo 1979)

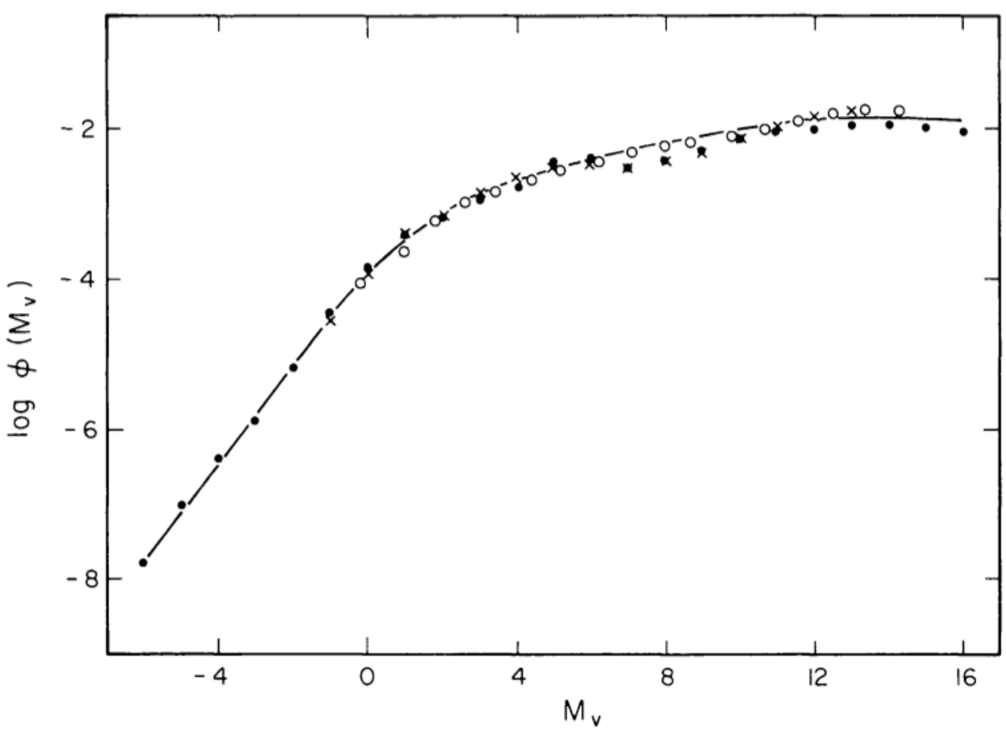
$$\phi_{MS}(logM) = \phi(M_V)\left|\frac{dM_V}{dlogM}\right| 2H(M_V)f_{MS}(M_V) \tag{5}$$

where $\phi_{MS}(logM)$ is the present day mass function, $\phi(M_V)$ is the luminosity function as a function of the absolute magnitude M_V, $H(M_V)$ is the galactic scaleheight for a given M_V and $f_{MS}(M_V)$ is the fraction of stars with M_V.

MILLER AND SCALO

Vol. 41

Luminosity Function



Mass vs Luminosity

No. 3, 1979

BIRTHRATE AND IMF IN

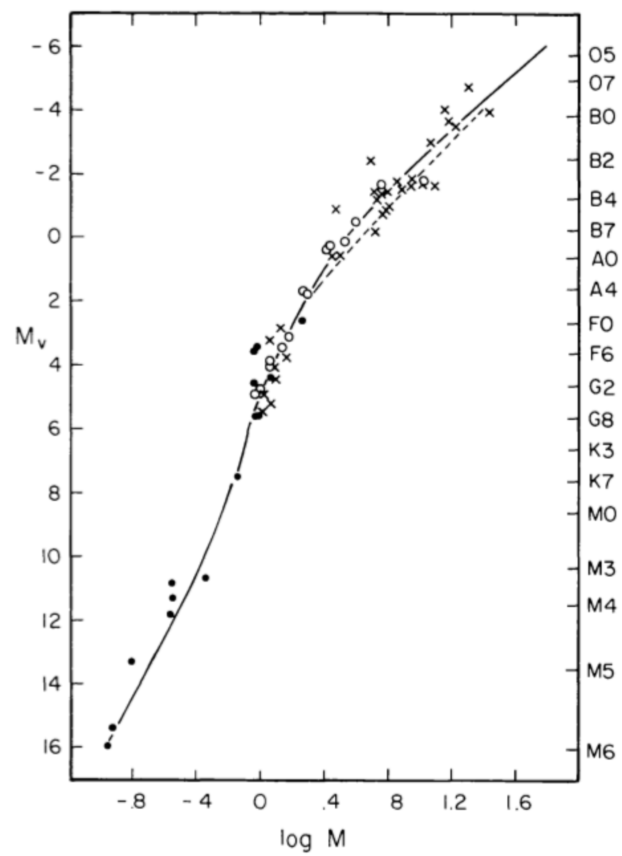
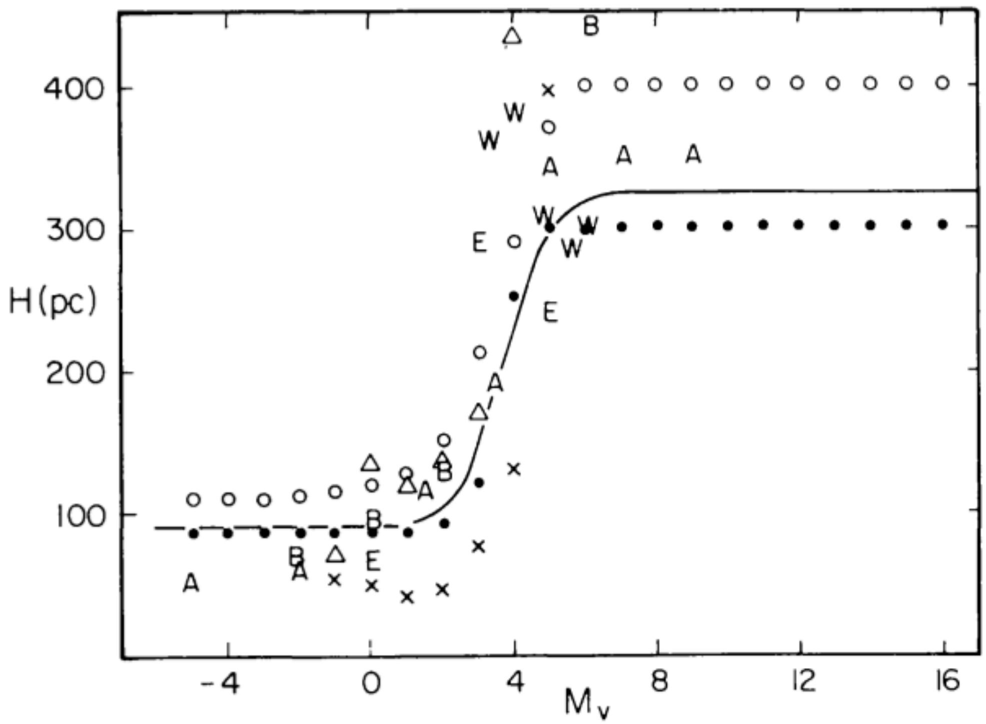


FIG. 2.—Relationship between absolute visual magnitude M_v and mass M ($M_\odot$) for main-sequence stars. ●, visual binaries (Lacy 1977); ○, eclipsing binaries (Lacy 1977); ×, spectroscopic binaries (Cester 1965); *dashed line*, theoretical mass-luminosity relation (Stothers 1974); *solid line*, adopted relation.



M_V vs H

$f(M_V)$

TABLE 2

FRACTION (f_{ms}) OF STARS OF A GIVEN ABSOLUTE MAGNITUDE ON THE MAIN SEQUENCE

M_v	Salpeter 1955	Sandage 1957	Schmidt 1959	Upgren 1963	McCuskey 1966
−6......	...	0.46	...	...	0.40
−5......	...	0.48	0.41	...	0.42
−4......	0.18	0.48	0.41	...	0.43
−3......	0.36	0.50	0.46	...	0.44
−2......	0.50	0.51	0.48	...	0.45
−1......	0.47	0.53	0.52	0.12	0.47
0......	0.41	0.56	0.46	0.29	0.51
+1......	0.47	0.62	0.33	0.62	0.56
+2......	0.65	0.71	0.69	0.91	0.66
+3......	0.76	0.86	0.87	1.00	0.82
+4......	0.95	1.00	1.00	1.00	0.98
+5......	1.00	1.00	1.00	1.00	1.00

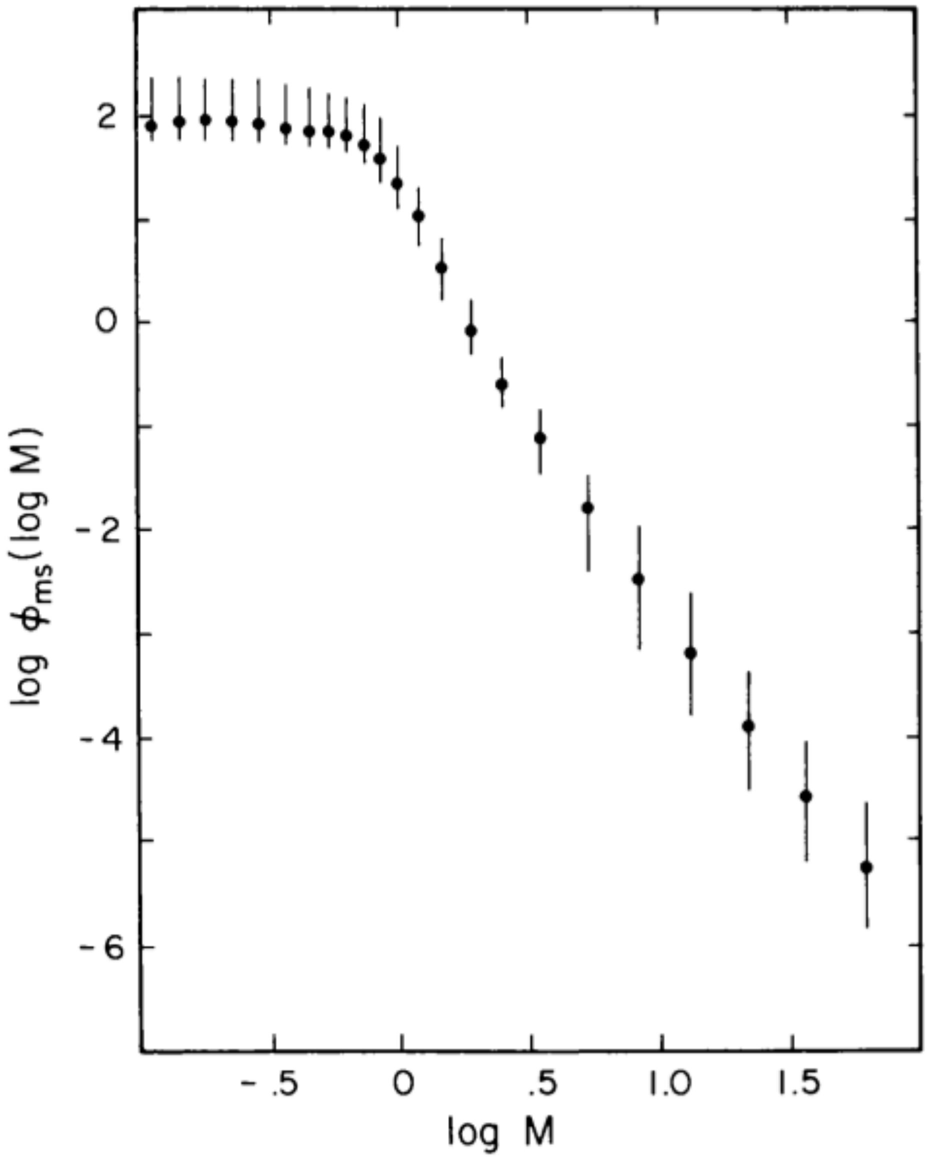


FIG. 4.—The PDMF of main-sequence field stars $\phi_{ms}(\log M)$ given as the number of stars $pc^{-2} \log M^{-1}$ in the solar neighborhood. Uncertainty bars are from Table 5.

The Field Star PDMF

Converting the PDMF into an IMF

To get the IMF, we must make an assumption about the birthrate of stars. Miller Scalo relate the IMF, $\xi(logM)$ to the PDMF, $\phi(logM)$, and the birthrate, $b(t)$, using the equation.

$$\phi_{MS}(logM) = \frac{\xi(logM)}{T_0} \int_{T_0 - T_{MS}}^{T_0} b(t)dt, \ T_{MS} < T_0 \tag{6}$$

$$\phi_{MS}(logM) = \frac{\xi(logM)}{T_0} \int_{0}^{T_0} b(t)dt, \ T_{MS} \geq T_0 \tag{7}$$

Thus, if the birthrate is constant (an assumption first made by Ed Salpeter, and almost certainly wrong):

$$\phi_{MS}(logM) = \xi(logM) \left(\frac{T_{MS}}{T_0} \right), \ T_{MS} < T_0 \tag{8}$$

$$\phi_{MS}(logM) = \xi(logM), \ T_{MS} \geq T_0 \tag{9}$$

The Field Star IMF

1979ApJS...41..

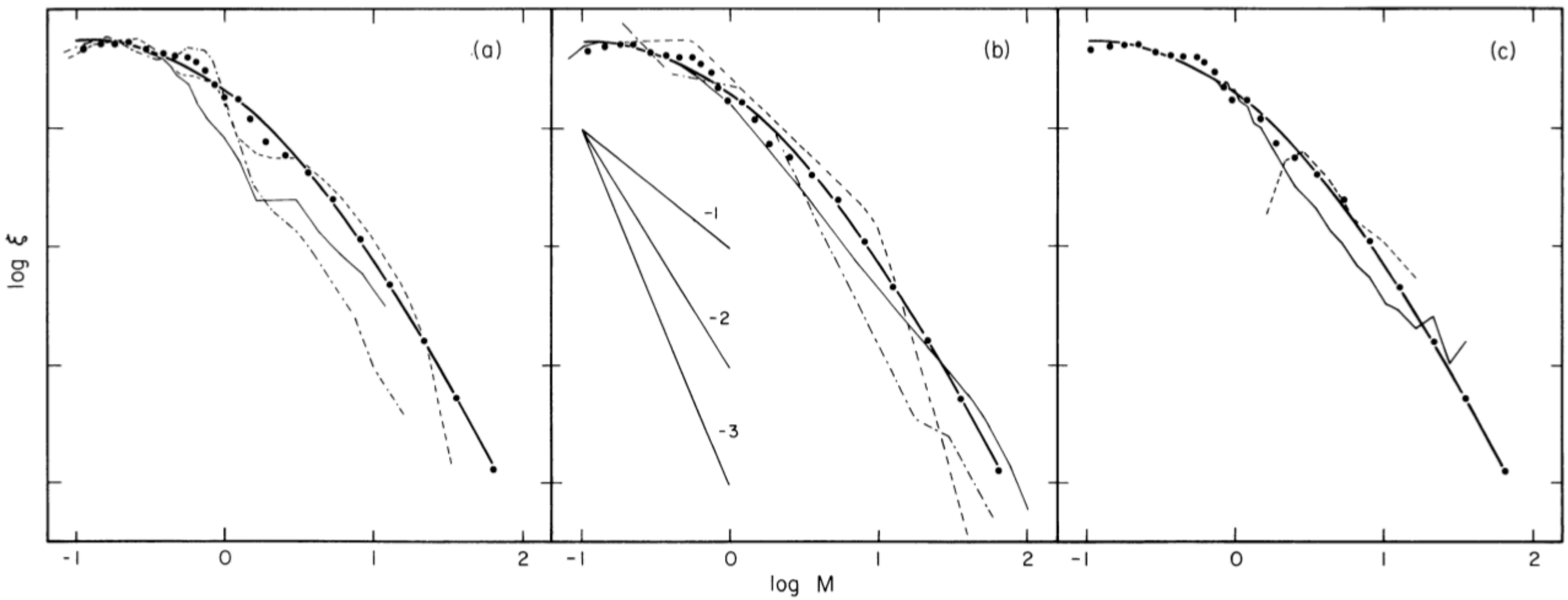


FIG. 11.—Comparison of published empirical IMFs. The filled circles and thick line are the IMF of the present paper for a constant birthrate with $T_0 = 12 \times 10^9$ yr. (*a*) *Thin solid line*, Salpeter (1955); *dashed line*, Sandage (1957) as given in Warner (1961); *dot-dashed line*, Schmidt (1959) constant ($n = 0$) birthrate. (*b*) *Thin solid line*, Limber, (1960); *dashed line*, Hartmann (1970); *dot-dashed line*, Audouze and Tinsley (1976). (*c*) *Thin solid line*, mean cluster IMF of Taff (1974); *dashed line*, IMF of Ori OB1 from data in Warren and Hesser (1978). The IMFs are arbitrarily normalized to agree at the low-mass end except for that of Ori OB1, where the luminosity function is incomplete for $M \leq 2.5\ M_{\odot}$. The ticks on the ordinate show one unit in log ξ. Approximate slopes of any section of these IMFs may be estimated by a comparison with the three straight lines of indicated slope in Fig. 11*b*.

Analytic Forms of the IMF

A quadratic fit:

$$log\xi(logM) = A_0 + A_1 logM + A_2(logM)^2 \tag{10}$$

A log-normal IMF:

$$\xi(logM) = C_1 e^{C_2(logM - C_3)} = ke^{-\frac{(logM - logM_c)^2}{2\sigma^2}} \tag{11}$$

or a multi-power-law form where over a given range of masses:

$$\xi(logM) = D_0 M^{D_1} \tag{12}$$

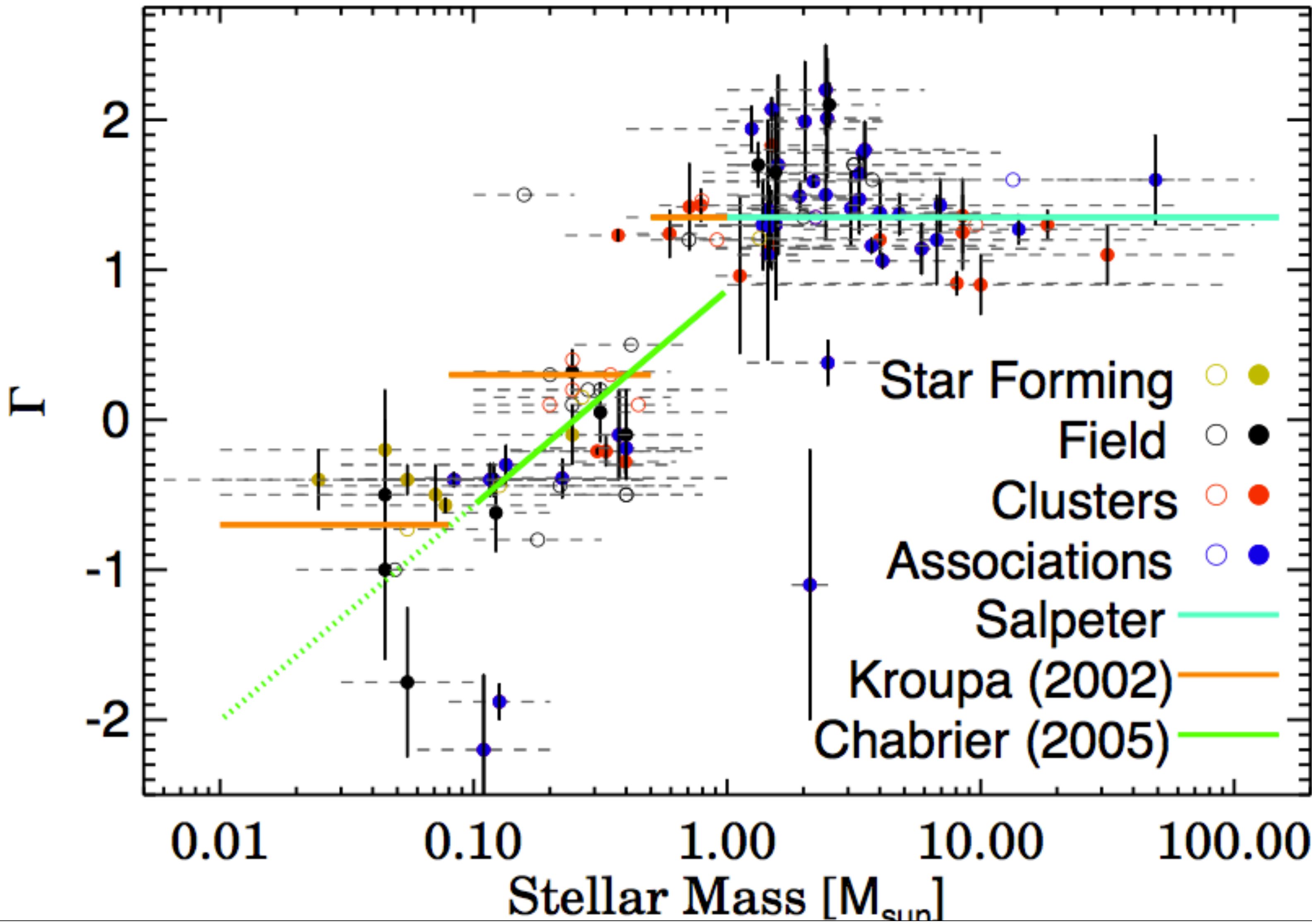
where values given are by Miller & Scalo (1979) and have been updated by numerous more recent works.

TABLE 7

PARAMETERS FOR ANALYTIC FITS TO THE IMF

BIRTHRATE	MI	CONSTANT			MD
T_0 (10^9 yr)	9, 12, 15	9	12	15	9, 12, 15
Quadratic Fit to log ξ(log M)					
A_0	1.43	1.47	1.53	1.59	1.96
A_1	−0.88	−1.02	−0.96	−0.87	−0.92
A_2	−0.50	−0.44	−0.47	−0.50	−0.49
Half-Gaussian Fit to ξ(log M)					
C_0	66.2	113.3	106.0	93.2	242.4
C_1	1.15	1.02	1.09	1.14	1.14
C_2	−0.88	−1.15	−1.02	−0.88	−0.93
Three-Segment Power-Law Fit to ξ(log M)					
$0.1 \leq M/M_\odot \leq 1$:					
D_0	32	36	42	42	100
D_1	−0.4	−0.4	−0.4	−0.4	−0.4
$1 \leq M/M_\odot \leq 10$:					
D_0	32	36	42	42	100
D_1	−1.5	−1.5	−1.5	−1.5	−1.5
$10 \leq M/M_\odot$:					
D_0	180	200	240	240	560
D_1	−2.3	−2.3	−2.3	−2.3	−2.3

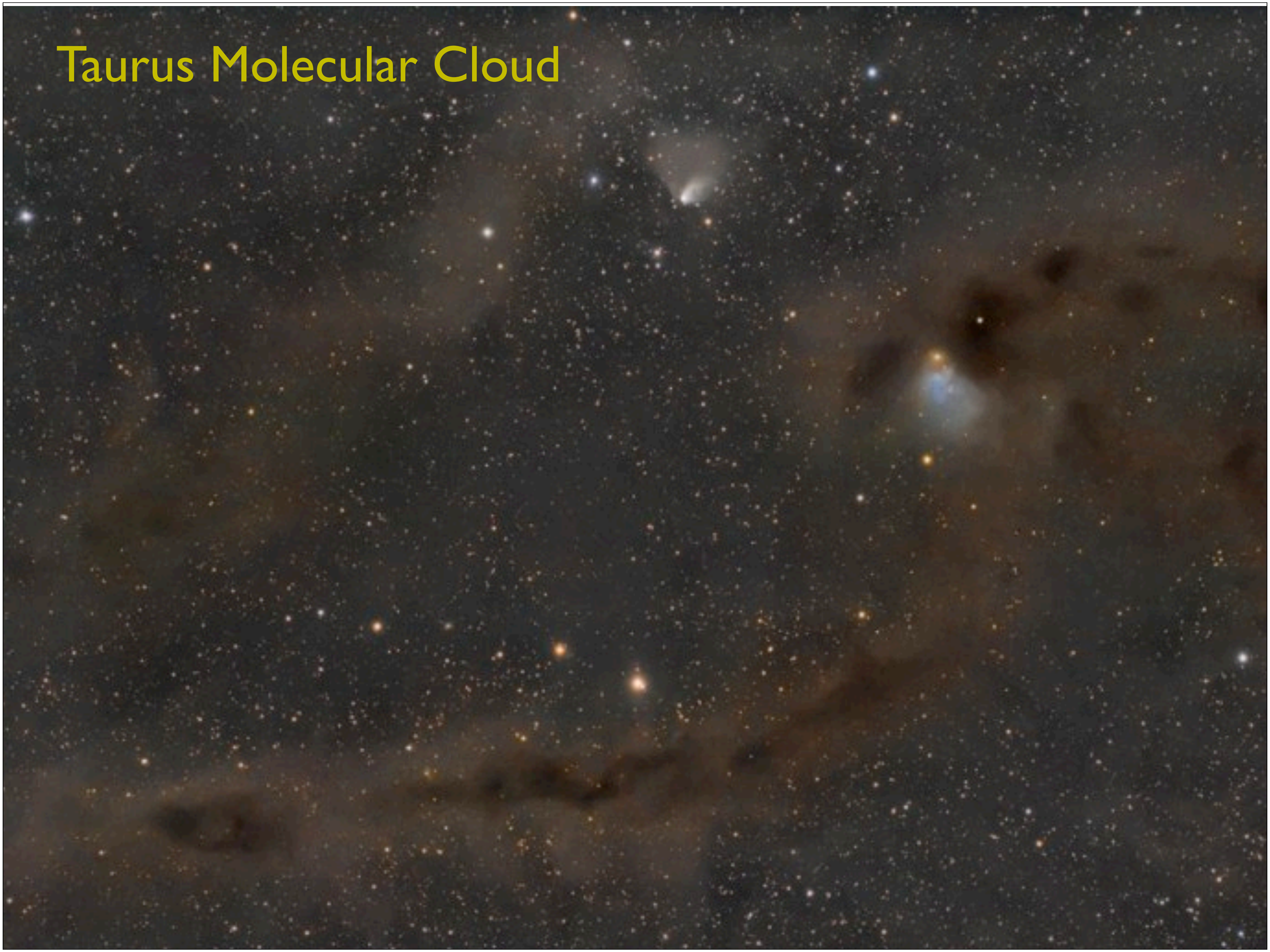
Γ vs Mass

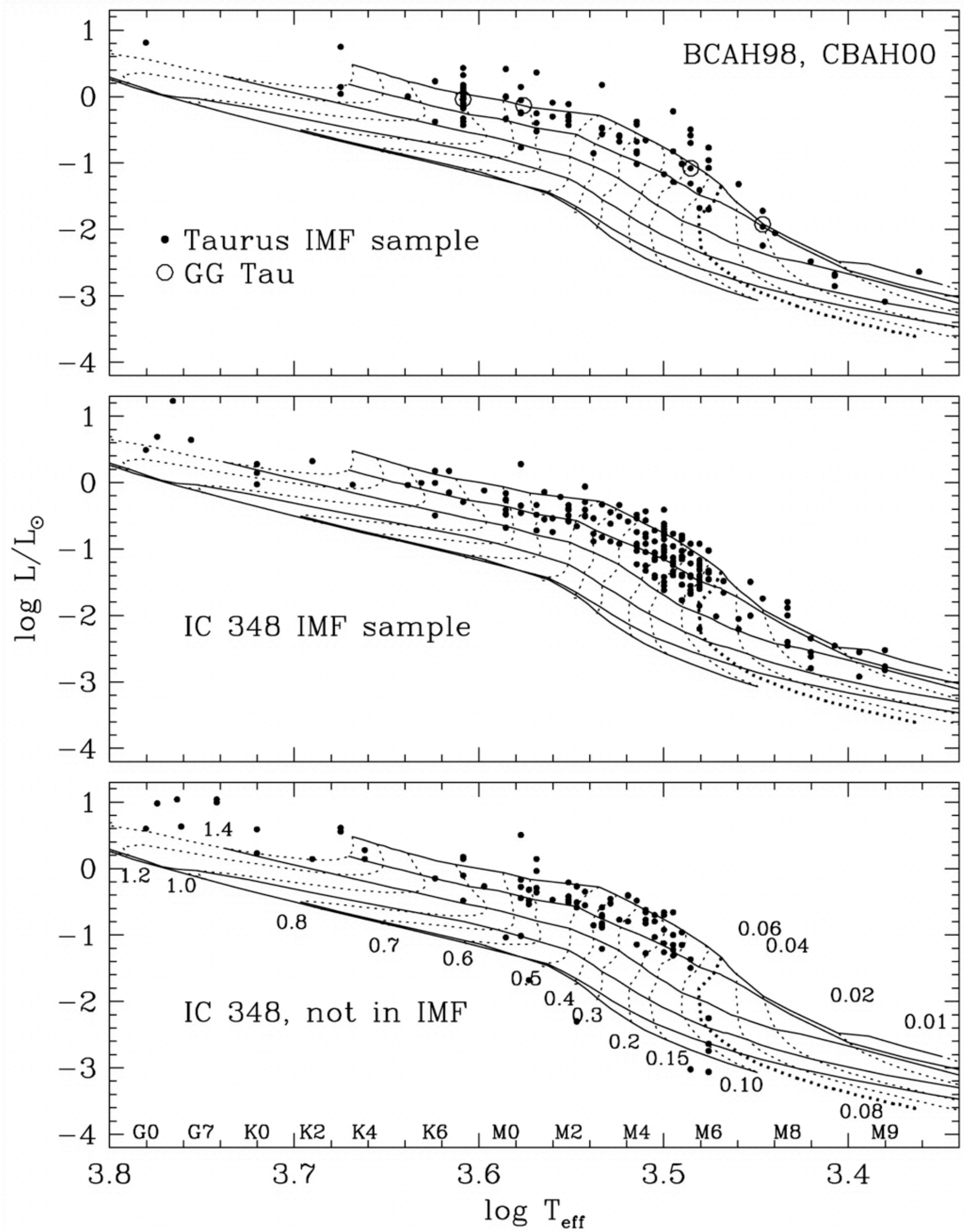


Star Forming
Field
Clusters
Associations
Salpeter
Kroupa (2002)
Chabrier (2005)
Γ
2
1
0
-1
-2
0.01
0.10
1.00
10.00
100.00
Stellar Mass [M_sun]

Using Young Clusters to Determine the IMF


IC 348 in Perseus

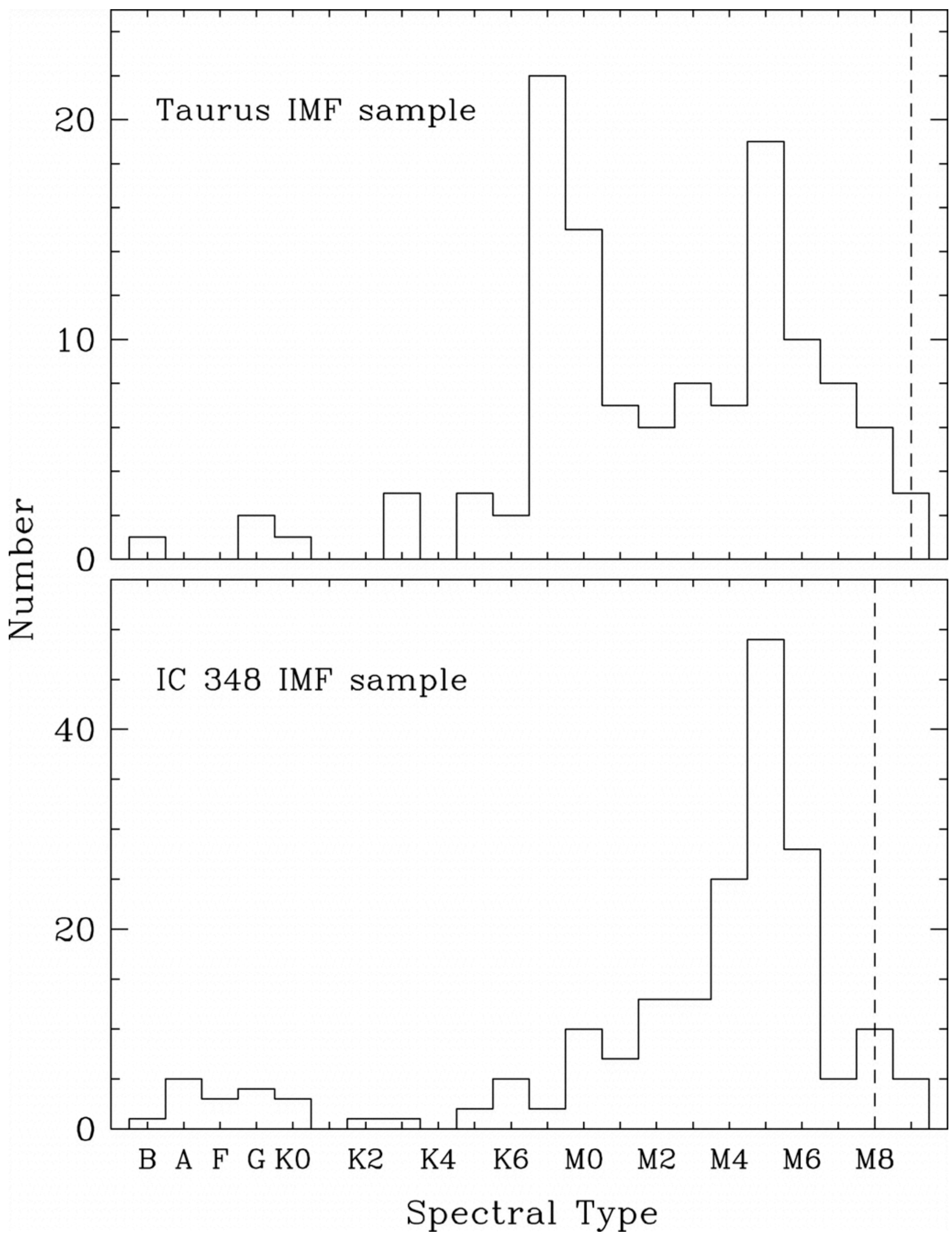

Taurus Molecular Cloud



Converting Photometry and Spectroscopy of young stars into masses and ages:

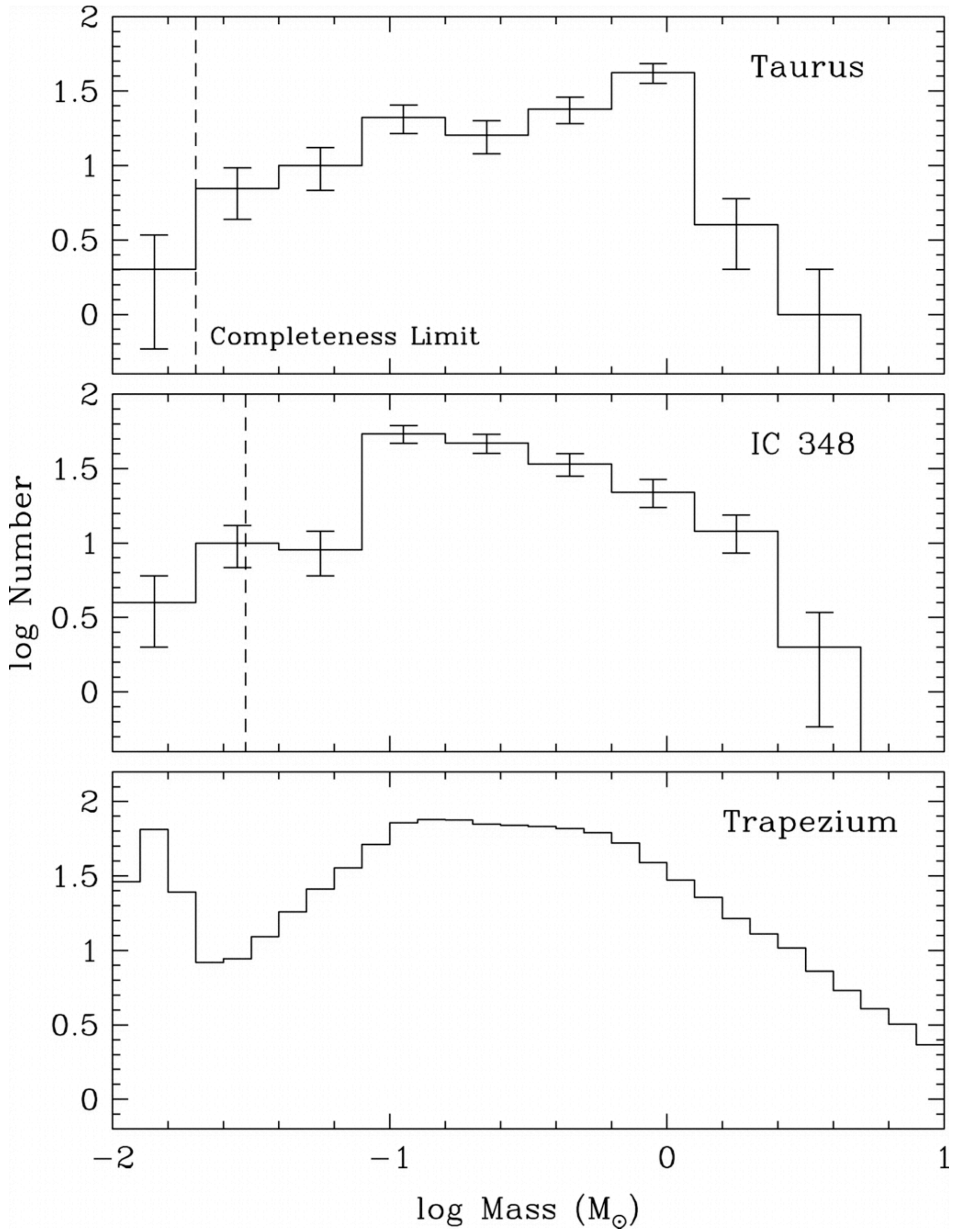
Pre-Main Sequence Tracks

Luhman et al. 2003



Distribution of spectral types in Taurus and IC348

Luhman et al. 2004

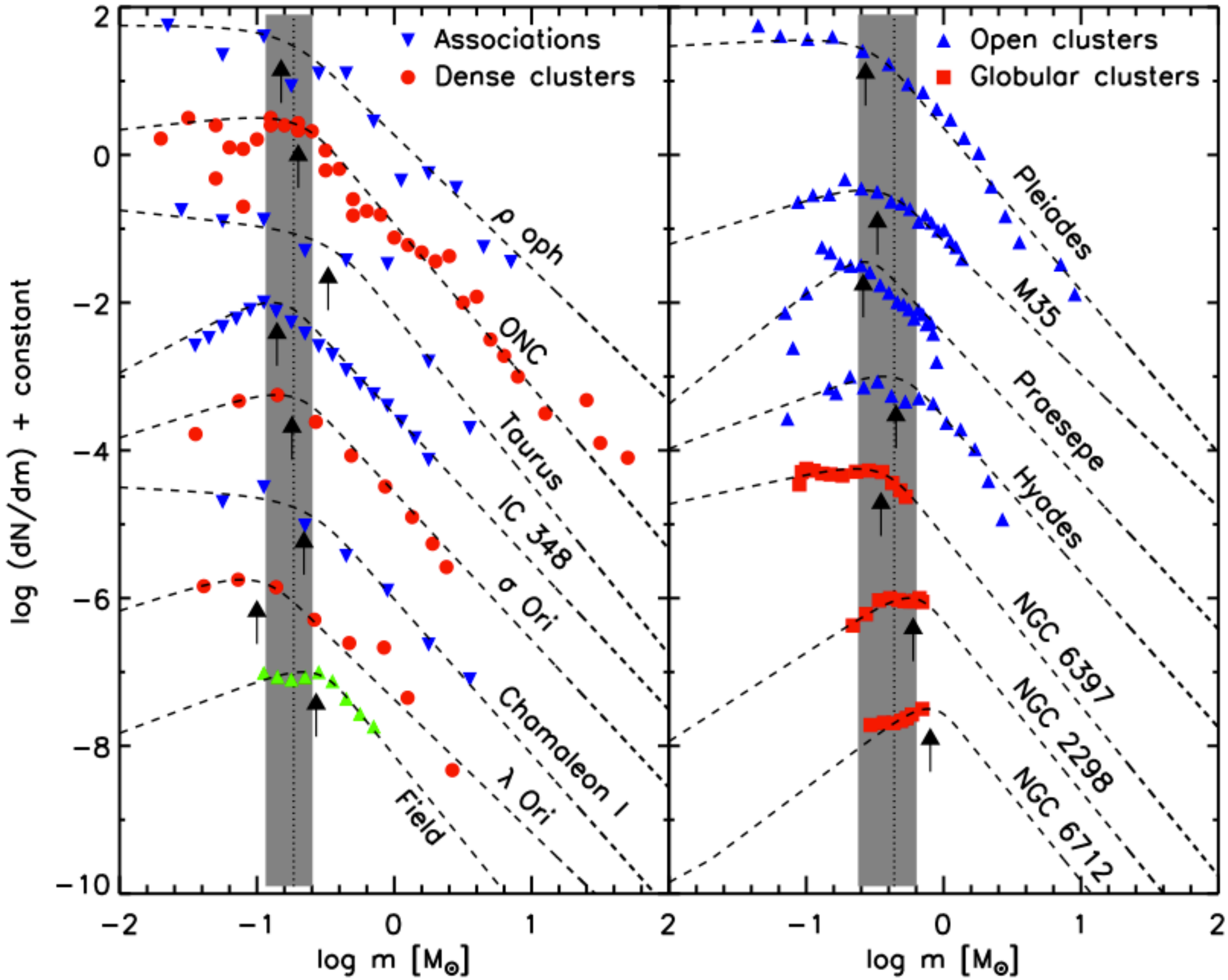


IMFs from star forming regions

Taken from Luhman et al. 2004

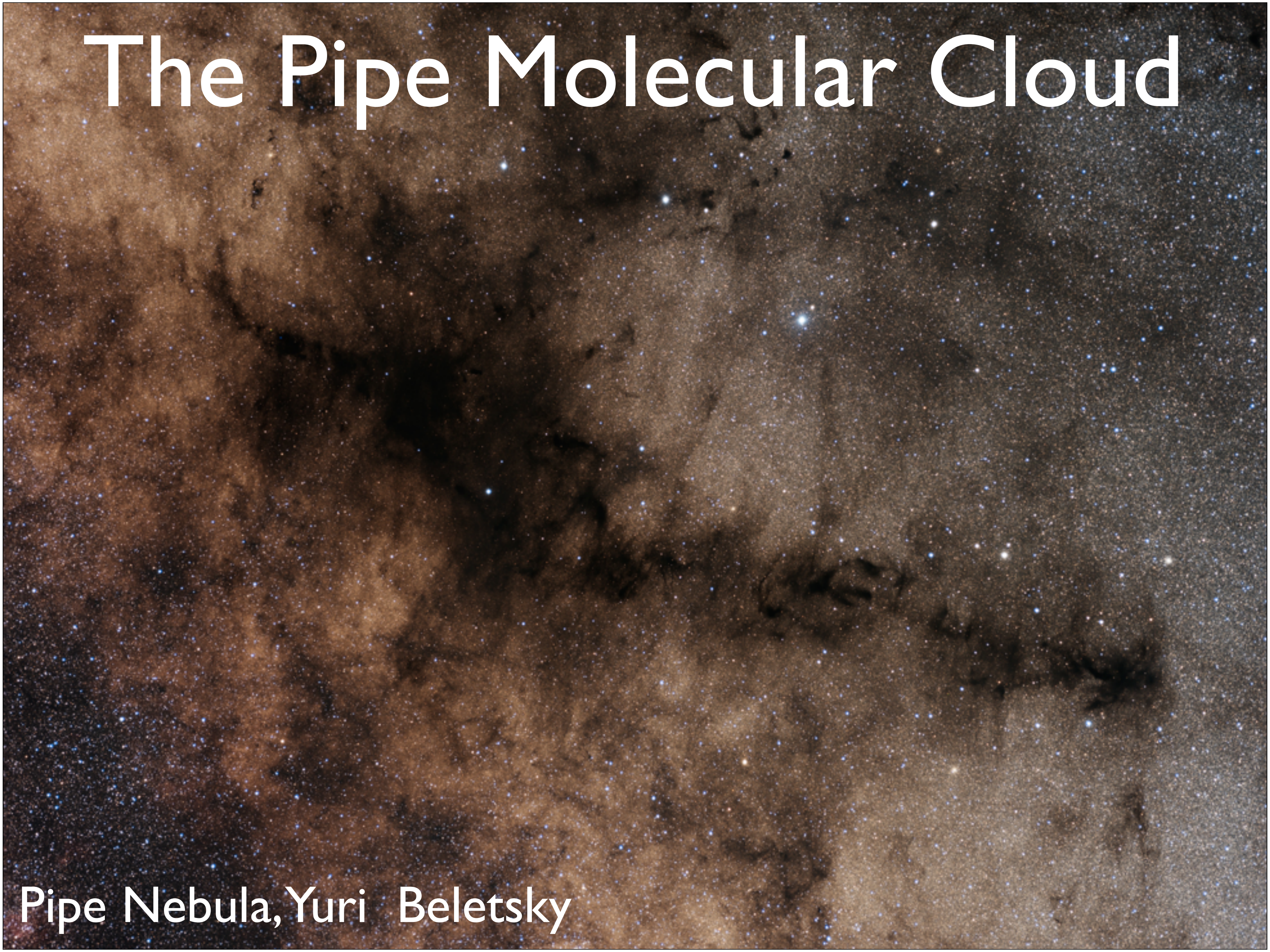
Trapezium IMF originally from Muench et al. 2002

Variations in the IMF?

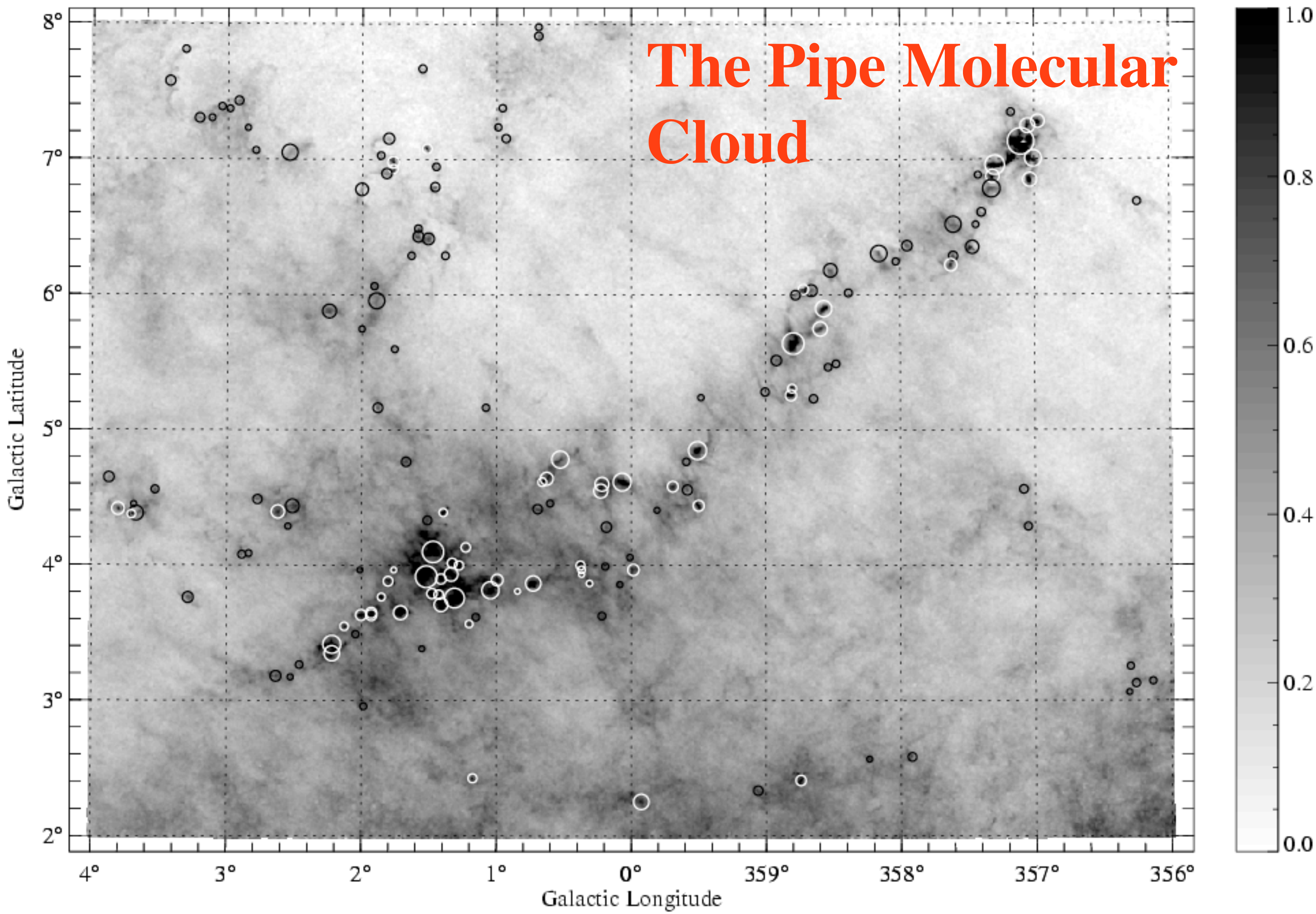


The Origin of the IMF

The Pipe Molecular Cloud

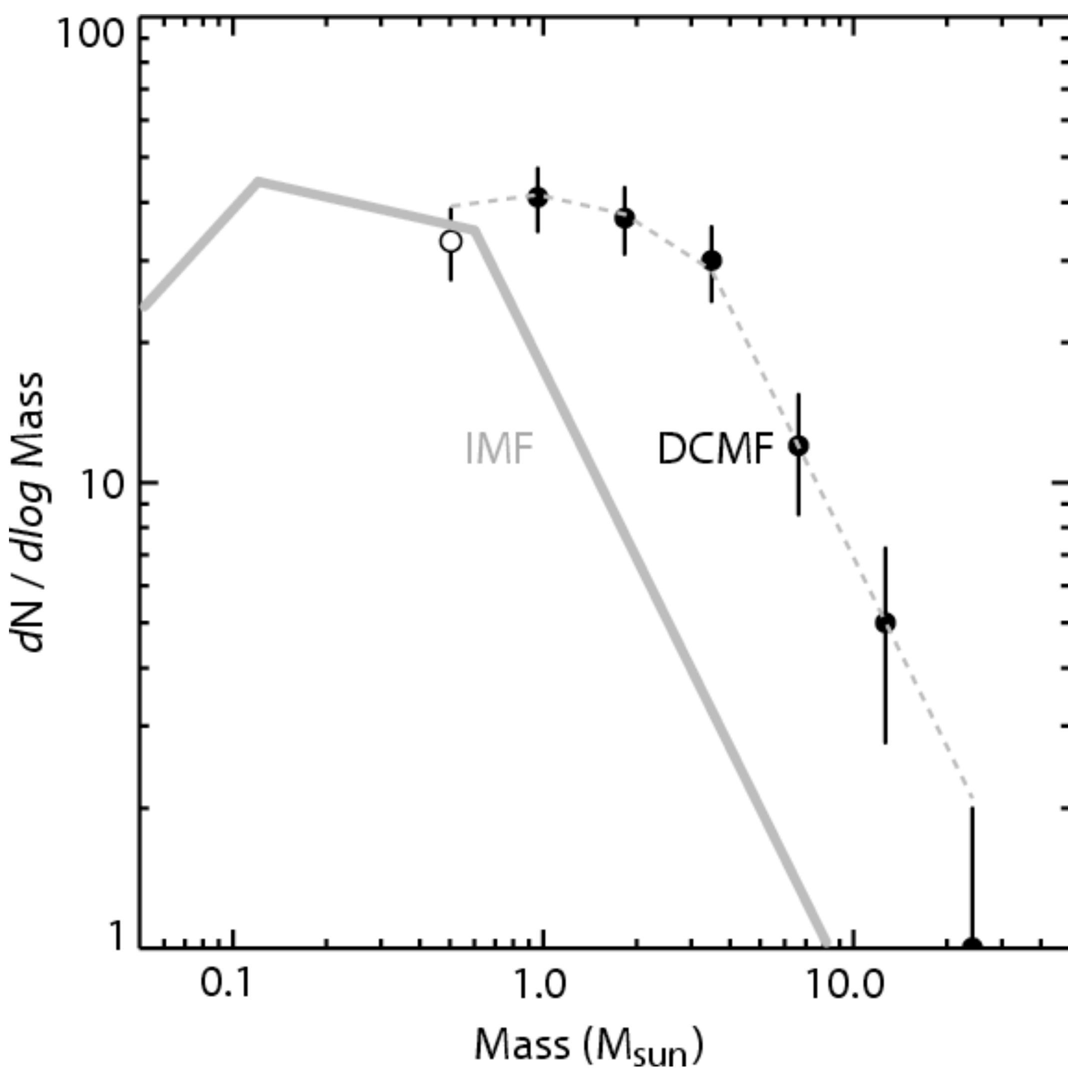


Pipe Nebula, Yuri Beletsky



The Pipe Molecular Cloud
Galactic Latitude
Galactic Longitude
8°
7°
6°
5°
4°
3°
2°
4°
3°
2°
1°
0°
359°
358°
357°
356°
1.0
0.8
0.6
0.4
0.2
0.0

The Core Mass Function



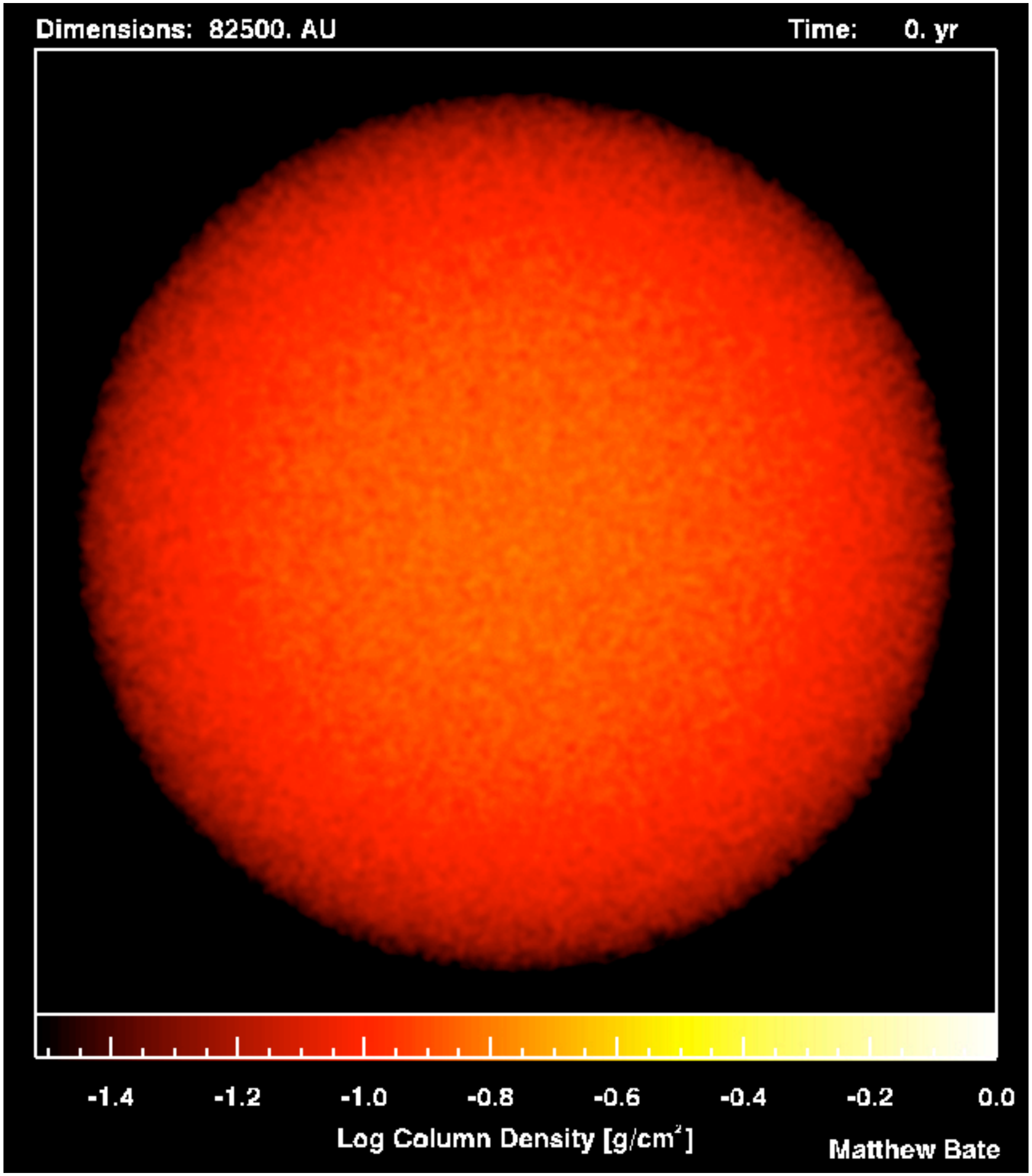
Alves et al. 2007

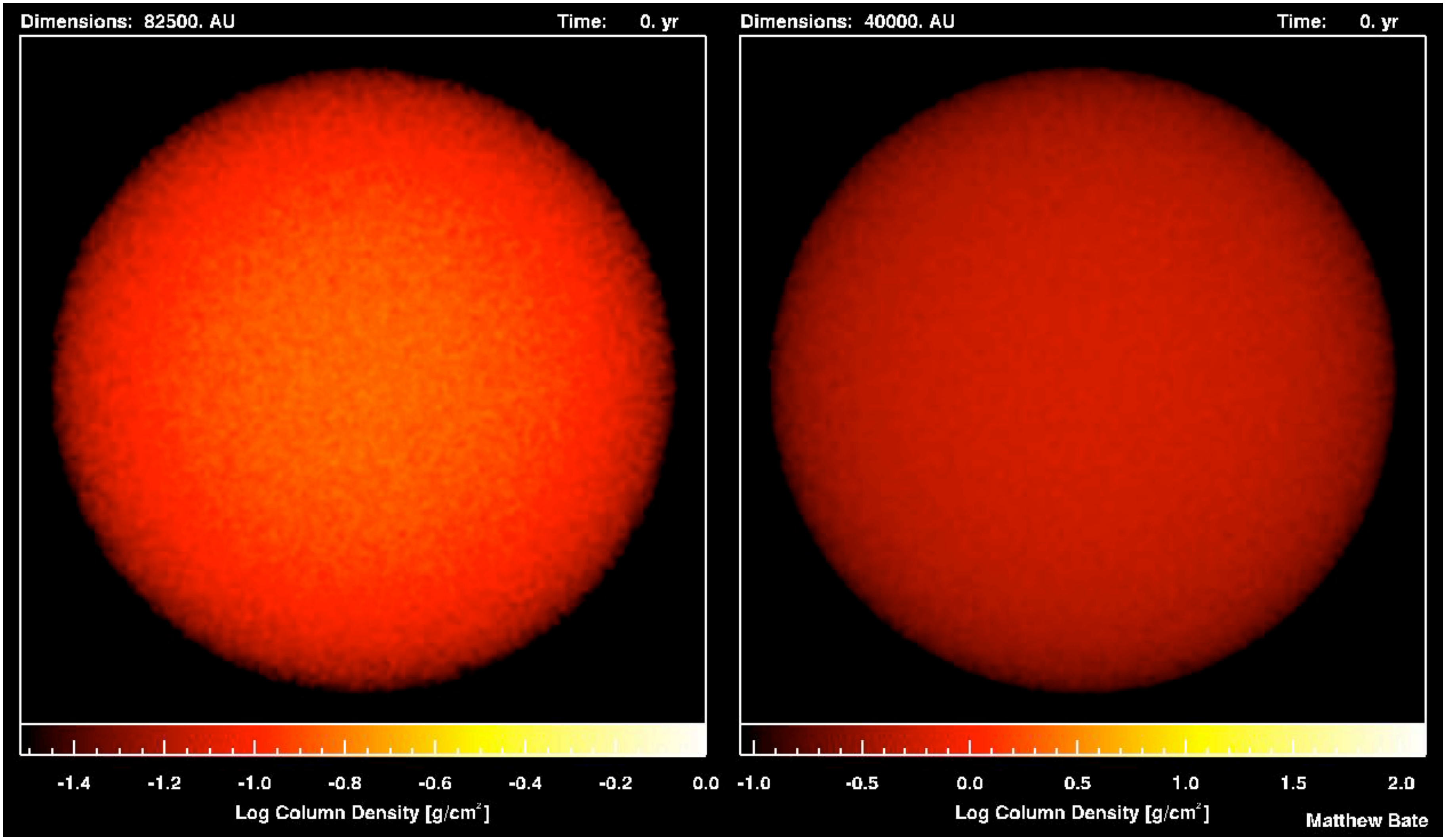
Some explanations of the IMF

- Outflow stops accretion. Log-Normal shapedue to central limit theorem.
- Originates from core mass function
- Competitive Accretion
- A bit of everything

Competitive Accretion (i.e. Bondi-Hoyle Accretion)

$$\dot{M} = 4\pi\rho\frac{(GM_\star)^2}{(v^2 + c_s^2)^{3/2}}$$

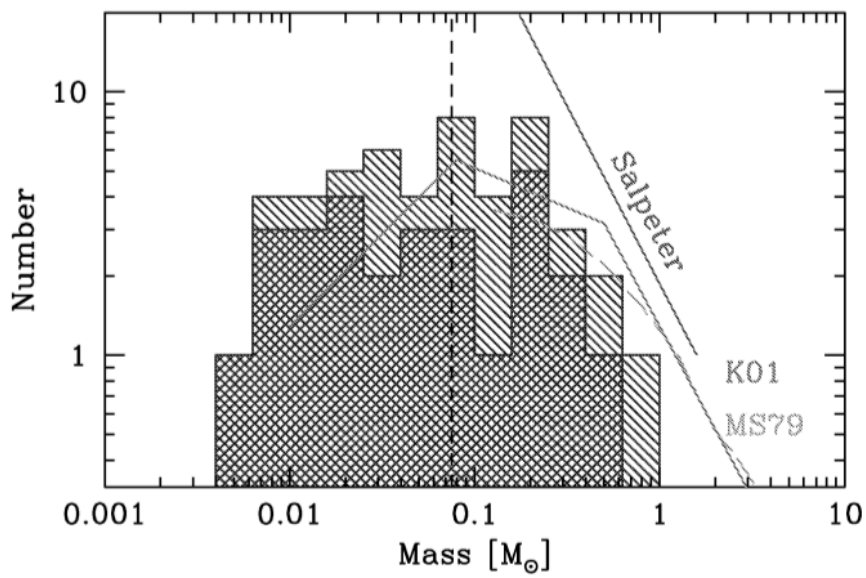

Dimensions: 82500. AU
Time: 0. yr
-1.4
-1.2
-1.0
-0.8
-0.6
-0.4
-0.2
0.0
Log Column Density [g/cm²]
Matthew Bate



Dimensions: 82500. AU
Time: 0. yr
-1.4 -1.2 -1.0 -0.8 -0.6 -0.4 -0.2 0.0
Log Column Density [g/cm^2]
Dimensions: 40000. AU
Time: 0. yr
-1.0 -0.5 0.0 0.5 1.0 1.5 2.0
Log Column Density [g/cm^2]
Matthew Bate

2005MNRAS.356.1201B

 2005



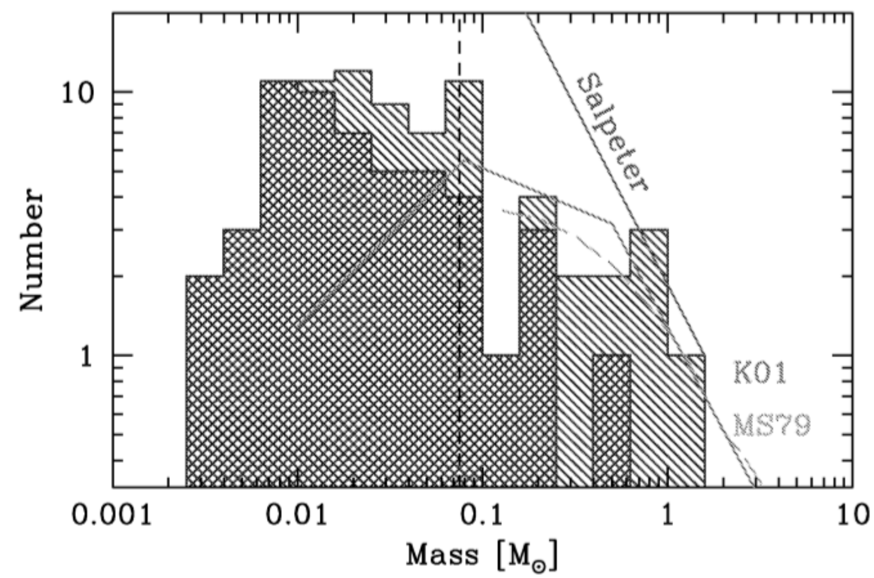


Figure 6. The initial mass functions produced by the two calculations. Calculation 2 (right-hand panel) had the lower initial mean thermal Jeans mass and produced a much higher fraction of brown dwarfs. The single shaded regions show all of the objects; the double shaded regions show only those objects that have finished accreting. The mass resolution of the simulations is 0.0011 $M_\odot$ (i.e. 1.1 M_J), but no objects have masses lower than 2.9 M_J due to the opacity limit for fragmentation. We also plot fits to the observed IMF from Miller & Scalo (1979) (dashed line) and Kroupa (2001) (solid broken line). The Salpeter (1955) slope (solid straight line) is equal to that of Kroupa (2001) for $M > 0.5\ M_\odot$. The vertical dashed line marks the star/brown dwarf boundary.

M_J = 1 solar mass

M_J = 0.33 solar mass

A Bit of Everything