1. Viscosity: from Shu's *Physics of Astrophysics* Volume II

Consider a gas with sheer $\frac{\partial u_x}{\partial y}$ where u_x is the velocity in the fluid in the $\hat{x}$ direction. Now consider a layer of gas, in that shearing fluid. It is going to receive momentum carried by particles escaping from the faster moving layers, and it is going to lose momentum going into the slower moving layers. Those particles are going to carry momentum

$$\frac{dp_x}{dt} = l_{mfp} v_{th} \rho \frac{\partial u_x}{\partial y} A \tag{1}$$

where l_{mfp} is the mean free path, which is equal to $1/(n\sigma)$ and v_{th} is the mean velocity from thermal motions.

$$l_{mfp} = \frac{1}{n\sigma} = \frac{\mu m_H}{\rho\sigma}, v_{th} = c_s = \sqrt{\frac{kT}{\mu m_H}} \tag{2}$$

giving

$$\frac{dp_x}{dt} = \frac{\mu m_H c_s}{\sigma} \frac{\partial u_x}{\partial y} A \tag{3}$$

Now in a constant velocity sheer, each layer receives as much momentum from the upper layer as it loses to the lower layers. Show a change in velocity, or a

$$\frac{dp_x}{dt} = \frac{\partial}{\partial y} \left(\frac{\mu m_H c_s}{\sigma} \frac{\partial u_x}{\partial y} \right) A \Delta y \tag{4}$$

we can divide out $A\Delta y$ to determine the force unit per volume

$$f_x = \frac{\partial}{\partial y} \left(\frac{\mu m_H c_s}{\sigma} \frac{\partial u_x}{\partial y} \right) \tag{5}$$

this can be written as

$$f_x = \frac{\partial \pi_{xy}}{\partial y}, \ where \ \pi_{xy} = \rho\nu \frac{\partial u_x}{\partial y} \tag{6}$$

$$\nu = \frac{\mu m_H c_s}{\rho\sigma} = l_{mfp} c_s \tag{7}$$