The Kelvin-Helmholtz Time

The Kelvin-Helmhotz time, or t_{KH}, is simply the cooling time for a pressure supported (i.e. in hydrostatic equilibrium), optically thick object. In other words, a pre-main sequence star. This is given by the potential energy over the luminosity.

$$t_{KH} = \frac{GM^2}{RL} \tag{1}$$

Since luminosity is a strong function of mass, t_{KH} declines with increasing mass. For a high mass object, let's take $M = 10\ M_\odot$, $R = 3$ R$_\odot$ and $L = 10^4\ L_\odot$, then t_{KH} = 10,000 years. On the other hand, for a low mass object, $M = 1\ M_\odot$, $R = 4$ R$_\odot$ and $L = 15$ L$_\odot$, then t_{KH} = 1.5 million years

Pressure Supported Core for a Massive Star

Consider a dense core supported by pressure. This core must satisfy the equation:

$$\frac{GM}{R} = c_s^2 \tag{2}$$

For $T = 20$ K, $c_s = 0.24$ km s^{-1}. In this case, for $M = 1$ M$_\odot$, we get an $R = 0.07$ pc: this is the size of observed dense cores. On the other hand, if $M = 10$ M$_\odot$, R = 0.76 pc. These objects don't seem to exist. There are large, parsec size structures, but they are turbulent, more than 10 M$_\odot$ im mass, and usally filled with a cluster of low mass stars. On suggestion is that we use the turbulent linewidth. In this case we replace c_s with $\sigma_{turb} = v_{NT}/2\sqrt{2ln(2)}$. If we set σ_{turb} to 1 km s^{-1}, then $R = 0.05\ pc$, which is much more acceptable. However, it is not clear that turbulence can act as a 3D, isotropic pressure and stabilize the core. Furthermore, the turbulent energy is quickly dissipated in shocks. Thus, any such turbulent cores are probably not stable.

The Eddington Luminosity

The Eddington Luminosity is the luminosity at which the radiation pressure exceeds the force of gravity. It is typically calculated for ionized gas where the primary opacity is Thompson scattering. Remember that the momentum carried by a photon is $h\nu/c$. For a given flux, F, the radiation pressure is then F/c. (*Here we assume that the radiation is coming from a single direction and the surface is perpendicular to that direction.* If the radiation is isotropic, and the flux is the radiation passing through the surface in one direction only since the net flux would be zero, then the pressure is given by $4F/3c$). Accordingly, the radiation pressure on a parcel of gas by a source of luminosity L and radius R is given by the equation:

$$P_{rad} = \frac{\chi\rho}{c}\frac{L}{4\pi R^2}dr \tag{3}$$

where χ is the sum of the absorption cross section and scattering cross section per mass and dr is the thickness of the gas layer. Note that the force absorbed goes up with the thickness of the absorbing slab, this says that the pressure goes up with the optical depth of the slab ($\chi\rho dr$). If we assume the gas is purely ionized Hydrogen, then the main source of opacity is Thomson scattering by electrons. Now, consider a parcel of gas in a stellar atmosphere with an area A and thickness dr. The force by photons per area is given by:

$$\frac{dP_{rad}}{dr} = -\frac{\chi\rho}{c}\frac{L}{4\pi R^2} = -\frac{\sigma_T\rho}{m_H c}\frac{L}{4\pi R^2} \tag{4}$$

Here we have divided by dr to give dP_{rad}/dr. Since the force is outward, dP/dr is negative. If there is a balance between gravity and radiation pressure, then you essentially get the equation for Hydrostatic equilibrium. The luminosity that gives you this balance is the Eddington luminosity.

$$\frac{dP_{rad}}{dr} = -\frac{\sigma_T\rho}{m_H c}\frac{L}{4\pi R^2} = -\frac{GM\rho}{R^2} \tag{5}$$

The Eddington luminosity can then be written as:

$$L_{edd} = \frac{4\pi GMm_H c}{\sigma_T} \tag{6}$$

This can also be stated as a luminosity to mass ratio:

$$\frac{L_{edd}}{M} = \frac{4\pi G m_H c}{\sigma_T} = 3.2 \times 10^4 \frac{L_\odot}{M_\odot} \tag{7}$$

When this ratio is exceeded, the radiation pressure exceeds gravity. Note that it is independent of radius, since both the gravity and the photon flux decrease by $1/R^2$.

"Eddington Luminosity" Calculation for Infall onto a Massive Star

Now consider an infalling envelope for a massive star. In this case, the primary source of opacity is the absorption and scattering of photons by dust grains. Assume that the photon momentum transferred to the dust grains by the absorption or scattering of a photon are subsequently transferred to the gas. Also, assume the luminosity is the combination of intrinsic luminosity of the central protostars (which may be on the main sequence) plus accretion luminosity. For infall to occur, gravity has to exceed radiation pressure:

$$\frac{L}{M} = \frac{L_\star + L_{acc}}{M} \leq \frac{4\pi Gc}{\chi_{eff}} \tag{8}$$

There is a significant difference here from the standard Eddington luminosity discussed in the previous section. The Thomson scattering opacity is independent of the frequency of light, but the opacity of the grains depends on the wavelength of the radiation field. Thus, as an opacity we need to use is weighted by the radiation field:

$$\chi_{eff} = \frac{\int \chi_\nu F_\nu d\nu}{\int F_\nu d\nu} = \frac{\int \chi_\nu B_\nu(T_{rad}) d\nu}{\int B_\nu(T_{rad}) d\nu} \tag{9}$$

We can assume the radiation field is described by the Planck equation. What is the temperature of the radiation field? Consider an infalling envelope. As the gas and dust falls inward, the temperature increases until, at a temperature between 1000 and 2000 K, the grains sublimate. This is called the dust destruction radius. At this point, the opacity of the gas drops. Thus, the light of star travels freely until it reaches the dust destruction radius and is radiated. We assume all the light is absorbed and re-emitted at the dust destruction radius. That temperature is approximately the dust sublimation temperature. Thus $T_{rad} \sim 2000$ K, much lower than that of the stellar photosphere. This reduces the effect of the radiation pressure significantly. As shown in the lecture, this may allow infall to occur, depending on the assumed dust opacities.

The inner boundary: Ram Pressure vs. Radiation Pressure

We can increase the L/M ratio at which infall can occur by absorbing the photons at the dust sublimation radius and re-emitting the luminosity at longer wavelengths. To do this, we need to bring gas and dust down to the sublimation radius. Remember that the dust at the sublimation radius is directly exposed to the radiation from the star, and thus the photon pressure will exceed gravity for this dust. Fortunately, that dusty gas has the additional pressure of all the material falling in from larger radius. Thus, at the dust sublimation radius, we must consider the balance of ram pressure of a gas with number density n, bulk velocity v and average particle mass μm_H. Consider the momentum absorbed by a surface, per unit area. The rate of particles hitting the surface is (assuming the velocity is perpendicular to the surface):

$$R = nv \tag{10}$$

If each particle has an average momentum $\mu m_H v$, then the rate of momentum being absorbed by the surface per area is:

$$P_{ram} = \mu m_H v n v = \rho v_{ff}^2 \tag{11}$$

where v_{ff} is the free fall velocity. For the infalling gas to move inward to the dust destruction radius, the ram pressure must exceed the radiation pressure up to the dust destruction radius (at which point the gas is no longer opaque):

$$\rho v_{ff}^2 > \frac{L}{4\pi R^2 c} \tag{12}$$

We can write the infall rate as $\dot{M} = \rho v 4\pi R^2$, hence

$$\dot{M} v > \frac{L}{c} = \frac{L_\star + GM\dot{M}/R}{c} \tag{13}$$

Quenched HII Regions

The young hot stars will also produce a strong UV field capable of ionizing hydrogen. Why don't we see an HII region? Perhaps the density is high enough that the Stromgren sphere is constrained to be near the star. Imagine that the star is producing a certain number of Lyman-continuum photons per seconds N_{lyc}. We use the equation for a Stromgren sphere (modified into a Stromgren shell):

$$N_{lyc} = 4\pi r^2 dr n^2 \beta \tag{14}$$

where n is the density of electrons or ions (the both are approximately equally) and $n^2\beta$ is the recombination rate per unit volume. We can relate $\dot{M}$ to $n(r)$ through (Walmsley RMxAC 1995):

$$\dot{M} = 4\pi m_H n(r) v_{ff} r^2 \tag{15}$$

If we solve for density we get:

$$n = \sqrt{N_{lyc}/\beta 4\pi r^3} \tag{16}$$

where we have set $dr \approx r$. Plugging this value of n into the equation for $\dot{M}$ and setting $v_{ff} = \sqrt{GM/r}$, we get:

$$\dot{M} = (4\pi m_H^2 N_{lyc} G M_\star m_H^2 \beta^{-1})^{0.5} \tag{17}$$

Accretion Through HII Regions

If escape velocity exceeds the sound speed in ionized gas, we can get a gravitationally bound HII region. In such a region, infall can occur through the HII region. The sound speed of ionized gas, which has a $T_K \sim 10^4$ K, is 10 km s^{-1}. For this occur, the radius of the ionized region should be less than the HII radius

$$R_{HII} = \frac{2GM}{c_{II}^2} \tag{18}$$

which for a 20 solar mass star is 400 AU.