Lecture 18: Pre-Main Sequence Evolution

APOD: T Tauri and Hind's Variable Nebula
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Review: Stars vs Cores

Dense cores and stars are both gravitationally bound, pressure supported spheres of gas.

Molecular cores are optically thin to millimeter and infrared radiation, they can cool efficiently and maintain a constant temperature. They can also be confined (in part) by external pressure.

Stars are optically thick. They are not isothermal and show strong temperature gradients that drive radiation transfer and convection. Stars must also be very close to hydrostatic equilibrium.

Question 1: How do we go between a collapsing core and a star?

Answer: increase in density. Whether the mass of a collapsing core is constant or whether its mass decreases with the Jeans mass, as a collapsing cloud shrinks its optical depth increases!

Optical Depth vs. Density

Imaging a sphere of mass M, constant density ρ, radius R, and absorption by mass of κ_λ. The the optical depth is

$$\tau = \kappa_\lambda \rho R = \kappa_\lambda \frac{3M}{4\pi R^3} R = \kappa_\lambda \frac{3M}{4\pi R^2} \tag{1}$$

Thus as the radius decreases and the density increases then the core eventually becomes optically thick. In reality, this will be the center of a collapsing cloud and the density will not be constant; however, the basic argument remains. As we pack more and more matter into a small space, the optical depth increases.

Opacity Limited Fragmentation

We see that as a cloud collapses, the optical depth increases. At the same time, the Jeans length and mass will decrease. This leads to two competing effects. On one hand, the density increases, leading to higher optical depths. At the same time, Fred Hoyle in the 1950s predicted that as a cloud collapsed, it would continue to fragment into smaller and smaller pieces as the density increased. The assumption was that the temperature was constant, a condition needed for Jeans fragmentation. Will this lead to an increase or decrease in the optical depth?

We remember from previous lectures that the Jeans mass is given by $M_J = \rho\lambda_J^3$, where λ_J is the Jeans length:

$$M_J = \left(\frac{\pi k}{G m_H}\right)^{3/2} T^{3/2} \rho^{-1/2} \tag{2}$$

or

$$M_J = \pi^{3/2} c_s^3 (G^3 \rho)^{-1/2} \tag{3}$$

To analyze the optical depth, let's consider the Jeans length,which is given by:

$$\lambda_J = \pi^{1/2} c_s (G\rho)^{-1/2} \tag{4}$$

Then the optical depth is:

$$\tau = \kappa_\lambda \rho \lambda_J = \chi_\lambda \pi^{1/2} c_s (\rho/G)^{1/2} \tag{5}$$

so that τ increases as the cloud collapses and ρ increases. Eventually, collapse and Hoyle fragmentation will produce optically thick fragments. At this point, the cores are no longer isothermal. Since Jeans fragmentation requires an isothermal gas, this will also halt the fragmentation of the collapsing gas into progressively smaller pieces. Low and Lynden-Bell (1976) calculated that this occurs at a few hundredths of a stellar mass. This suggests the initial pieces are much smaller than a stellar mass, they will need to grow by accreting gas from the surrounding cloud.

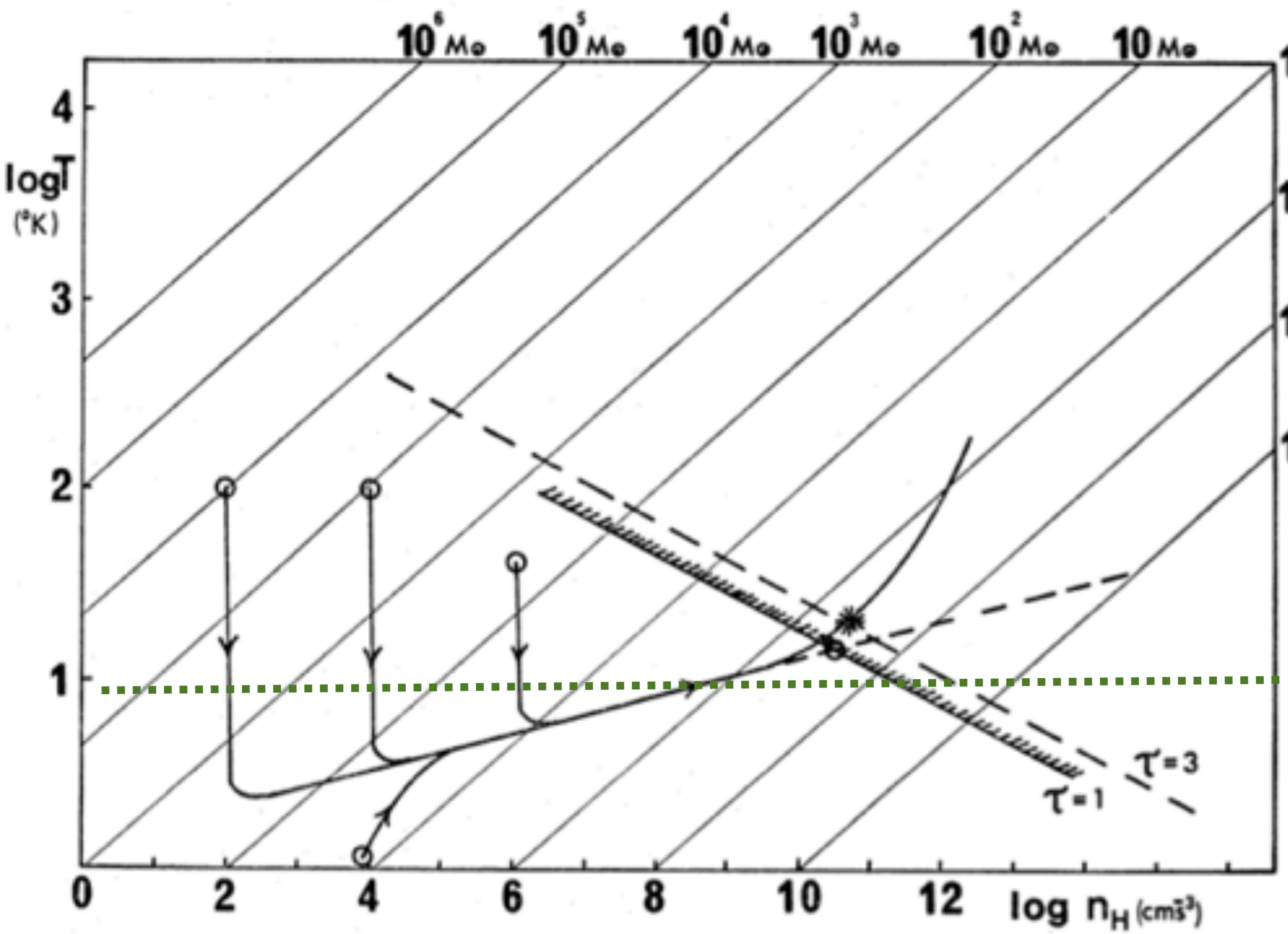


FIG. 1. *The log T – log ρ diagram. The lines of constant Jeans mass are indicated. To the right of the hatched line Jeans mass fragments are optically thick with $\tau \geqslant 1$. The other curve is the cooling curve for the idealized opacity law $\kappa = KT^2$, applicable to dust in equilibrium with gas. Notice the strong convergence of out-of-equilibrium solutions to the $C = 0$ solution (see equation (20)). The last fragmentation is at * where the trajectory bends away from the optically thin trajectory and becomes parallel to the lines of constant Jeans mass. This occurs at the optical depth $\tau \sim 3$. Notice that the point * corresponds very closely to the same mass as ○, the intersection of the $\tau = 1$ curve with the cooling curve calculated in the absence of opacity. It is the conditions at ○ that were calculated in the Introduction (equations (6)–(11)). $m = m_H$.*

Mass of Jeans Fragment

Line of constant temperature

Low & Lynden-Bell 1976

Question 2: How does an optical thick fragment evolve?

Today we look into pre-main sequence evolution: the contraction of optically thick, gravitationally supported objects before the onset of Hydrogen burning.

The presumption is that once a core becomes optically thick, it enters hydrostatic equilibrium - but this process is poorly understood.

Next lecture we investigate the protostellar phase, where mass accretion onto an optically thick fragment sets the initial conditions for pre-mains sequence contraction.

Pre-main sequence evolution: Following analysis of Hartmann

We start with the equation for an adiabatic gas:

$$P\rho^{-\gamma} = K_1 = P\rho^{-(1+1/n)} \tag{6}$$

where $\gamma = 5/3$ and n, the polytropic index is given by $n = 3/2$. This would be the case for a fully convective star, where convection maintains an adiabatic equation of state from the center of the star to the photosphere. As we did last semester, we can estimate the central pressure in the star by the equation of hydrostatic equilibrium:

$$\frac{dP}{dR} = \frac{P_c}{R} = \rho\frac{GM}{R^2} \tag{7}$$

which for a constant density star ($\rho = M_\star/4\pi R_\star^3$).

$$P_c = \rho\frac{GM}{R} = \frac{3GM_\star^2}{4\pi R_\star^4} \propto \frac{GM_\star^2}{R_\star^4} \tag{8}$$

Using ideal gas law, $P\rho^{-\gamma} = K$ and $P = \rho kT/\mu m_H$ we get

$$T_c \propto \frac{GM_\star}{R_\star} \tag{9}$$

To solve for the luminosity and effective temperature of the star, we must consider the stellar atmosphere where convection no longer is occuring. Here again we use hydrostatic equilibrium, approximating the atmosphere as a plane-parallel atmosphere with a gravitational constant $g = GM/R^2$.

The resulting equation is:

$$\frac{dP}{dz} = -\rho g \tag{10}$$

We integrate downward in the stellar atmosphere until $\tau = \int \kappa \rho dz = 2/3$. Accordingly:

$$P_{eff} = \frac{2}{3} \frac{g}{\kappa_{rm}} \tag{11}$$

where

$$\frac{1}{\kappa_{rm}} = \frac{\int_0^\infty \frac{1}{\kappa_\nu} \frac{dB_\nu(T)}{dT} d\nu}{\int_0^\infty \frac{dB_\nu(T)}{dT} d\nu} \tag{12}$$

Adopt an opacity law for the stellar atmosphere, $\kappa_{rm} = \kappa_0 \rho T_{eff}^a$. Due to H^- opacity, κ_{rm} is a strong function of temperature with $a \sim 10$. For a constant density star:

$$P_{eff} = \frac{2}{3} \frac{GM_\star}{R_\star^2 \kappa_0 \rho T_{eff}^a} = \frac{2}{3} \frac{G\rho 4\pi R_\star^3}{R_\star^2 3 \kappa_0 \rho T_{eff}^a} = \frac{8}{9} \frac{R_\star}{\kappa_0 T_{eff}^a} \tag{13}$$

For an adiabatic star (remember that $PT^{-5/2} = K_2$):

$$\frac{T_{eff}}{T_c} = \left(\frac{P_{eff}}{P_c}\right)^{-2/5} \tag{14}$$

By substituting the relationships for P_c, T_c, and P_{eff}, we get

$$T_{eff} \propto R^{2.5/(2.5+a)} M^{0.5/(2.5+a)} \tag{15}$$

and

$$L_{eff} \propto R^{(15+2a)/(2.5+a)} M^{2/(2.5+a)} \tag{16}$$

If a is large, then there T_{eff} is relatively constant while L_{eff} decreases as the radius decreases.

Convective (Hayashi) Tracks: Following Hartmann, Chapter 11

From the virial theorem we find that $2U + W = 0$ where U is total kinetic energy and W is total potential energy. Thus $U = W/2$ and the total energy $-W/2$.

For $\gamma = 5/3$ ($P \propto \rho^\gamma$):

$$L_\star = -\frac{d}{dt}\frac{3}{7}\frac{GM_\star^2}{R_\star} \tag{17}$$

with a high value of a, for a given mass

$$T_{eff} = \left(\frac{L_\star}{4\pi\sigma R_\star^2}\right)^{1/4} \approx constant \tag{18}$$

Using Teff as a constant and using $L_\star = 4\pi R_\star^2 T_{eff}^4$, we get:

$$R_{star}^2 = \frac{3GM_\star^2}{28\pi\sigma R_\star^2 T_{eff}^4}\frac{dR}{dt} \tag{19}$$

Integrating both sides (and starting with $t = 0$ and R being very large):

$$t = \frac{GM_{star}^2}{28\pi\sigma T_{eff}^4 R_\star^3} \tag{20}$$

$$4\pi R_\star^2 T_{eff}^4 = L_\star = (4\pi\sigma T_{eff}^4)^{1/3} \left(\frac{GM_\star^2}{7t}\right)^{2/3} \tag{21}$$

Defining the Kelvin-Helmoltz as

$$t_{KH} = \frac{3}{7}\frac{GM_\star^2}{R_0} \text{ and } L_0 = 4\pi\sigma T_{eff}^4 R_0^2 \tag{22}$$

where R_0 is the radius when $t = t_{KH}/3$, we get our final answer:

$$L_\star = 4\pi\sigma T_{eff}^4 R_0^2 \left(\frac{GM_\star^2}{7R_0 t}\right)^{2/3} = L_0 \left(\frac{3t}{t_{KH}}\right)^{-2/3} \tag{23}$$

Radiative (Henyey) Tracks: Following Hartmann, Chapter 11

Convection will when the radiative gradient is to high. This gradient is given by:

$$\nabla R = \frac{dlnT}{dlnP} = \frac{3\kappa_R P}{64 G\pi\sigma T^4}\frac{L}{M} \tag{24}$$

if $\nabla R < 0.4$ (remember that $R = 4$ for $\gamma = 5/3$ and $P \propto T^{5/2}$) then convection stop. This will occur as the star contracts and the luminosity drops (P_c/T_c^4 does not decrease with radius). At this point, radiation begins to be the main means for transporting energy, and we can use the radiative diffusion equation to estimate the luminosity:

$$L = \frac{64\pi\sigma r^2 T^3}{3\rho\kappa_r}\frac{dT}{dr} \tag{25}$$

using the basic equations for stellar structure and the Kramer's opacity

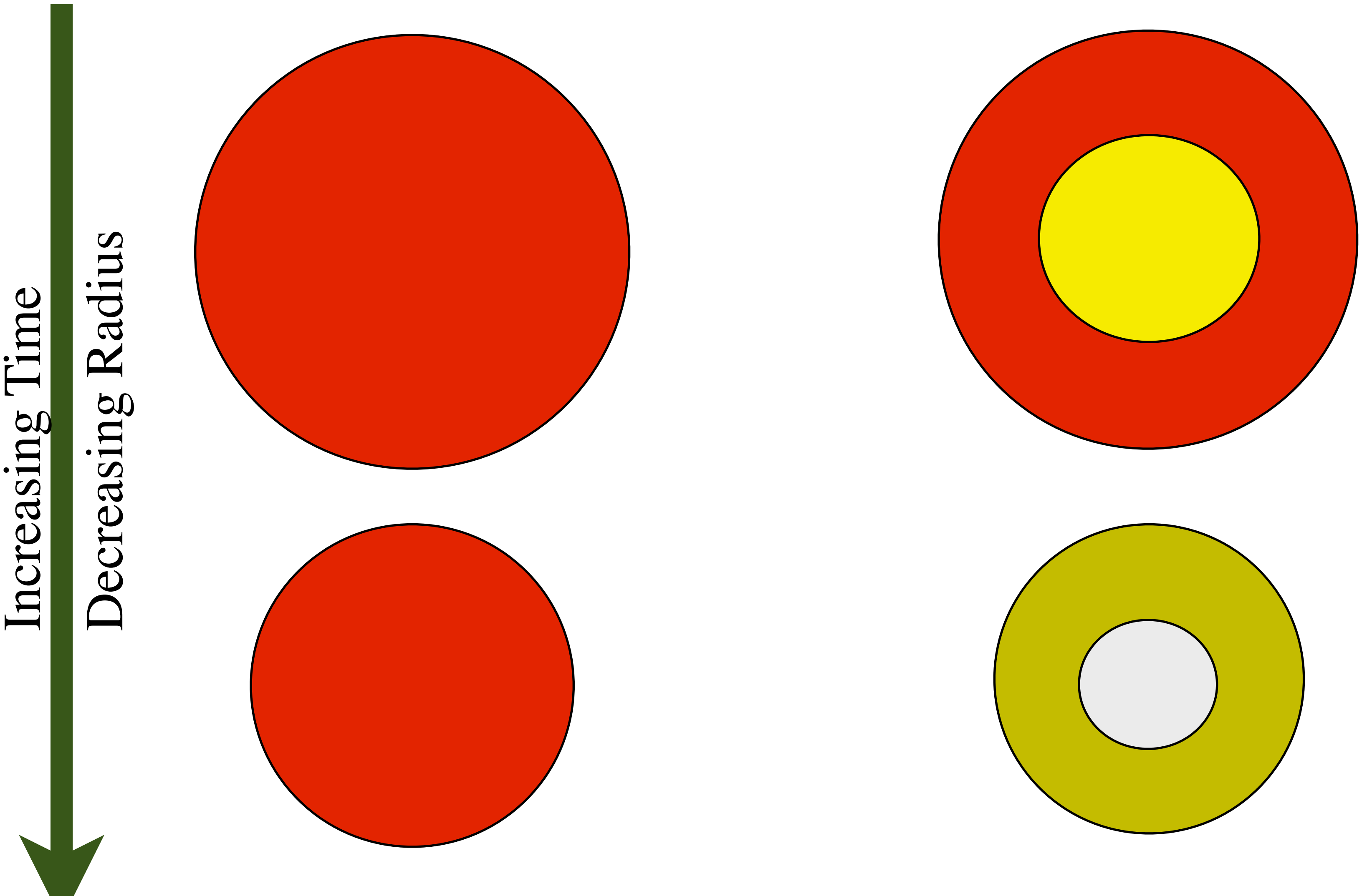
$$T \propto \frac{M_\star}{R_{star}}, \text{ and } \rho \propto M_\star/R_\star^3, \text{ and } \kappa \propto \rho T^{-3.5} \tag{26}$$

and by assuming that $dT/dR = T_c/R_\star$, we get:

$$L_\star \propto M_\star^{5.5} R_\star^{-0.5} \tag{27}$$

Convective (Hayashi) Track

Radiative (Henyey) Track



Constant Teff (Increasing Tc)
Decreasing Luminosity

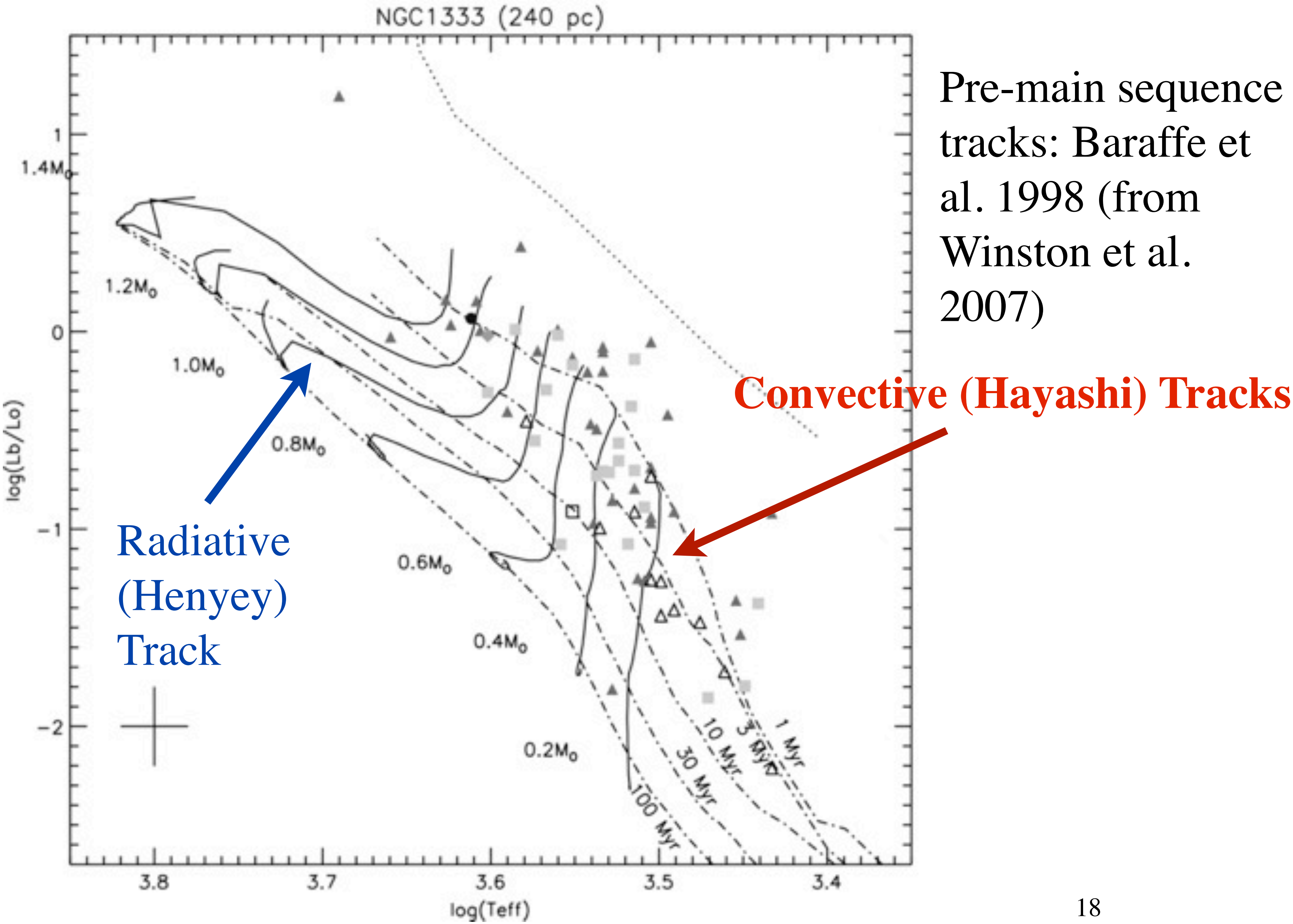
Increasing Teff (and Tc)
Increasing Luminosity

Core Temperatures

The core temperature scales at $T_c = k M / R$. Thus as a star contracts, 1/2 of its potential energy is converted into thermal energy, and the core temperature rises.

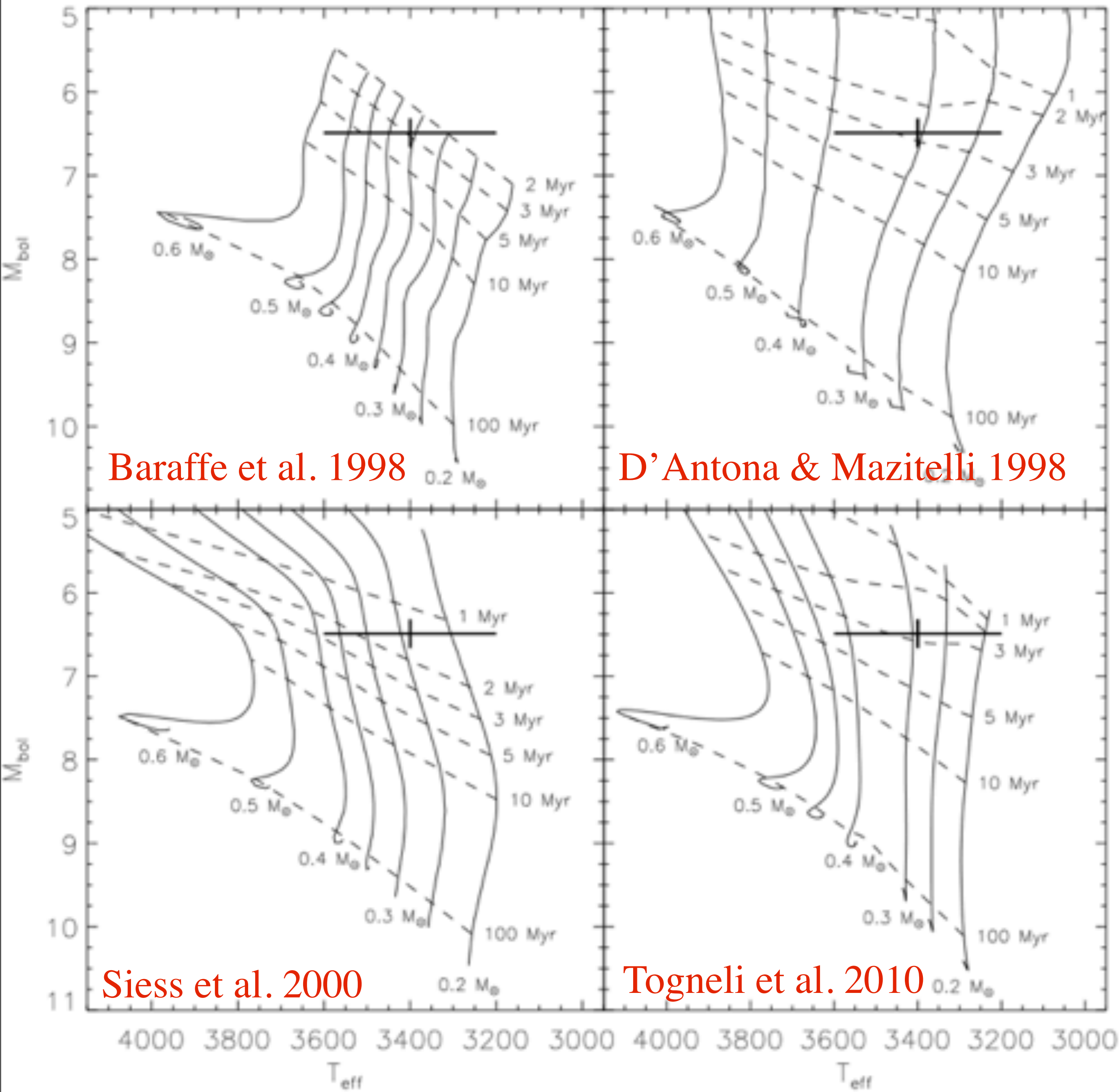
Pre-main sequence stars in this sense have negative heat capacities.

Initially core temperature are too low for nuclear fusion. However, as temperatures increase, nuclear fusion starts. We will discuss in future lectures



Pre-main sequence tracks: Baraffe et al. 1998 (from Winston et al. 2007)

Comparing pre-main sequence tracks



Pre-main sequence tracks require codes that take into account both stellar interiors and atmospheres.

Beware:
Different groups of modelers get different solutions for tracks.

Taken from Vacca et al. 2011

Summary

As molecular cores collapse, the density and the optical depth increase. The core eventually becomes optically thick.

This may eventually prevent Jeans fragmentation (which requires isothermal conditions). The exact mass when this happens is somewhere around 0.01 Msun.

The optically thick fragments then achieve hydrostatic equilibrium as core temperature rises (this is not well understood).

Once accretion ends, stars enter pre-main sequence contraction phase. The core temperature is to low to support nuclear fusion.

Low to intermediate mass stars are initially convective stars. The strong dependence of the atmospheric opacity on temperature ensures that the star maintains a relatively constant temperature. The luminosity decreases as the stars shrink. They follow Hayashi tracks.

Stars with masses > 0.5 msun develop radiative zones during their pre-main sequence contraction. These stars increase in temperature and luminosity as they shrink. They follow Henyey tracks.

Pre-main sequence stars have a negative heat capacity: the core temperature rises as the star contracts. Eventually the temperature is high enough for fusion to occur, and the star enters the main sequence.