Lecture 19: Protostars & the Stellar Birthline
The Road to the Main Sequence

Cloud: complicated balance between gravity, turbulent motions, and magnetic fields resulting into fragmentation. Isothermal and optically thin to IR and radio wavelengths.

Core: thermal pressure in rough balance with gravity (may not exist for massive stars). Isothermal and optically thin to IR and radio wavelengths.

Protostar: an optically thick object in hydrostatic equilibrium results from the collapse of unstable cores.
**Protostars:** Accretes mass throughout protostellar phase. Can undergo Deuterium fusion (for low mass stars) and Hydrogen fusion (high mass stars). Accretion onto star with a disk. Must overcome magnetic pressure, resulting in magnetospheric accretion for low mass stars. Must overcome photon pressure for high mass stars.

**Pre-Main Sequence Star:** only happens for low to intermediate stars. Star contracts and center heats up. Deuterium fusion possible.

**Main Sequence Stars:** hydrogen fusion. Long period of stable luminosity and radius.
Review: Convective Pre-Main Sequence Stars

Adopt an opacity law for the stellar atmosphere, \( \kappa_{rm} = \kappa_0 \rho T_{eff}^a \).

Where for cool stars, the H\(^-\) opacity gives \( a = 10 \)

By substituting the relationships for \( P_c, T_c, \) and \( P_{eff} \), we get

\[
T_{eff} \propto R^{2.5/(2.5+a)} M^{0.5/(2.5+a)}
\]

and

\[
L_{eff} \propto R^{(15+2a)/(2.5+a)} M^{2/(2.5+a)}
\]
Convective (Hayashi) Tracks: Following Hartmann

From the virial theorem we find that $2U + W = 0$ where $U$ is total kinetic energy and $W$ is total potential energy. Thus $U = W/2$ and the total energy $-W/2$.

For $\lambda = 5/3$.

$$L_* = -\frac{d}{dt} \frac{3 GM_*^2}{R_*}$$  \hspace{1cm} (19)

with a high value of $a$, for a given mass

$$T_{\text{eff}} = \left( \frac{L_*}{4\pi\sigma R_*^2} \right)^{1/4} \approx \text{constant}$$  \hspace{1cm} (20)

and

$$L_* = L_o \left( \frac{3t}{t_{kh}} \right)^{-2/3}$$  \hspace{1cm} (21)
In the centers of contracting pre-main sequence stars, as temperature increases, opacity drops so that radiation can effectively transport energy.

At this point, you begin to have radiative tracks where the luminosity *increases* as the radius of the star shrinks.

\[ L = \frac{64\pi\sigma r^2 T^3}{3\rho \kappa r} \frac{dT}{dr} \]  

using the basic equations for stellar structure and the Kramer’s opacity

\[ T \propto \frac{M_*}{R_{\text{star}}} \text{, and } \rho \propto \frac{M_*}{R_*^3} \text{, and } \kappa \propto \rho T^{-3.5} \]  

and by assuming that \(dT/dR = T_c/R_*\), we get:

\[ L_* \propto M_*^{5.5} R_*^{-0.5} \]
Convective (Hayashi) Track

Radiative (Henyey) Track

Increasing Time

Decreasing Radius

Constant Teff (Increasing Tc)
Decreasing Luminosity

Increasing Teff (and Tc)
Increasing Luminosity

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Pre-main sequence tracks: Baraffe et al. 1998 (from Winston et al. 2007)

Convective (Hayashi) Tracks

Radiative (Henyey) Track
Now go to protostellar phase
The Stellar Birthline

The Stellar Birthline is the initial point for pre-main sequence contraction. It comes from a calculation of the mass and radius of an accreting protostar with time. At some mass, the accretion is terminated, and pre-main sequence contraction begins.

Stahler 1983
Stahler 1988
Palla & Stahler 1990
The Stellar Birthline

Imagine an accreting a star. As the gas falls onto the central star, it lowers the potential energy onto the star by:

$$\Delta W = -\frac{GM_* \Delta m}{R} \quad (28)$$

At the same time, the gas may bring internal energy:

$$\Delta U = \epsilon \frac{GM_* \Delta m}{R} \quad (29)$$

If gas fell from infinity and then all the potential energy turned to internal energy, than $\epsilon = 1$ and there is no net energy gain. If the gas started at a bound orbit, the $\epsilon < 1$. If the gas is totally cold when it lands on the star, $\epsilon = 0$.

For a pre-main sequence star:

$$L_* = -\frac{3}{7} \frac{G(M)^2}{R} \left( \frac{\dot{R}}{R} \right) \quad (30)$$

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For a protostar we write this as a balance.

\[
-\frac{3}{7} \frac{G(M)^2}{R} \left( \frac{\dot{R}}{R} \right) = \frac{\dot{M}}{R} \frac{GM}{R} (\epsilon - 1) - \frac{dE_{surf}}{dt} + L_D
\]  

(31)

The term \(dE_{surf}/dt\) is the rate of radiative energy loss over the stellar surface. \(L_D\) is the luminosity from Deuterium burning. On the stellar surface, imagine that the accreting gas lands on a fraction of the surface given by the filling factor \(\delta\). Then:

\[
\frac{dE_{surf}}{dt} = 4\pi R^2 \left[ \delta F_{acc} + (1 - \delta)\sigma T_{eff}^4 \right]
\]  

(32)

where \(F_{acc}\) is the flux radiating from regions of accretion. The luminosity of the protostar is then:

\[
L_{phot} = 4\pi R^2 (1 - \delta)\sigma T_{eff}^4
\]  

(33)

which will go to the usual \(4\pi R^2 \sigma T_{eff}^4\) when \(\delta \to 0\). Let us define \(\alpha\), where
We define $\alpha$:

$$\dot{M} \frac{GM}{R} \alpha \equiv \dot{M} \frac{GM}{R} \varepsilon - 4\pi R^2 \delta F_{acc}$$

where $4\pi R^2 \delta F_{acc}$ is the energy radiated away by the accretion shock. Thus, if $\alpha$ is close to 1, the infalling gas keeps all of its energy. If $\alpha = 0$, it loses that energy - perhaps by radiating it back into space. Now, we can write the luminosity as:

$$L_* = -\frac{3}{7} \frac{GM_*^2}{R_*} \left[ \left( \frac{1}{3} - \frac{7\alpha}{3} \right) \frac{\dot{M}}{M_*} + \frac{\dot{R}_*}{R_*} \right] + L_D$$

(35)

Let us consider various limits of the equation. If $\dot{M} = 0$ (no accretion) and $L_D = 0$ (no Deuterium burning), then:

$$L_* = -\frac{3}{7} \frac{GM_*^2 \dot{R}_*}{R_*}$$

(36)
On the other hand, if \( L_* = 0 \), i.e. the luminosity is totally absorbed by the infalling gas, and \( \alpha = 0 \) (infalling gas is cold) and \( L_D = 0 \) (no Deuterium burning) then:

\[
\frac{-1}{3} \frac{\dot{M}}{M_*} = \frac{\dot{R}_*}{R_*}
\]  

(37)

or

\[
\frac{-1}{3} \frac{d\ln M_*}{dt} = \frac{d\ln R_*}{dt}
\]  

(38)

In this case \( R \propto M^{-1/3} \). On the other hand, imagine that \( \alpha = 1 \). In this limit:

\[
2 \frac{\dot{M}}{M_*} = \frac{\dot{R}_*}{R_*}
\]  

(39)

and \( R \propto M^2 \). So in the limit of no Deuterium burning, and the luminosity of the star is smothered by accretion, \( \alpha \) controls whether the star grows or shrinks. If \( \alpha > 1/7 \), the added gas is hot and the star grows, if \( \alpha < 1/7 \), the added gas is cold and the star shrinks.
What is $\alpha$? Consider the material that is accreting onto the central star has a temperature $T_{acc}$.

$$T_{acc}\Delta m = \frac{\Delta U}{C_V} = \frac{2}{3}\frac{\mu m_H}{k}\Delta U,$$

or

$$T_{acc} = \frac{2}{3}\frac{\mu m_H}{k}\frac{\Delta U}{\Delta m}$$  \hspace{1cm} (40)

The internal energy divided by $C_V = (3/2)(\mu m_H/k)$ for a $n = 3/2$ polytropic star (i.e. star convective star with $\gamma = 5/3$):

$$M < T > = \int T dM = \int \frac{u}{C_V} dM = \frac{3}{7}\frac{GM_*^2}{R_*} \left(\frac{2}{3}\frac{\mu m_H}{k}\right) = \frac{3}{7}\frac{\mu m_H}{k}\frac{GM_*^2}{R_*}$$  \hspace{1cm} (41)

where $u$ is the specific internal energy (internal energy per gram). Now we can write

$$\frac{\Delta U}{\Delta m} = \alpha \frac{GM}{R}$$  \hspace{1cm} (42)
Now we can write:

\[
\frac{\Delta U}{\Delta m} = \alpha \frac{GM}{R}
\]  

(42)

and solving for \( \alpha \):

\[
\alpha = \left( \frac{\Delta U}{\Delta m} \right) \frac{R}{GM_*}
\]  

(43)

we can further write:

\[
\alpha = \frac{9}{14} \frac{T_{acc}}{< T >}
\]  

(44)

Thus if \( T_{acc} = (2/9) < T > \) the \( \dot{R}/R \sim 0 \) if \( L_D \sim 0 \). For \( T_{acc} \) greater than this value, the star will grow, for \( T_{acc} \) less than this value, the star will shrink.
Hartmann et al 1997

Stellar Birthline for total luminosity $L=0$ and two different accretion rates.

Different values of $\alpha$ are used in the left panel.
Stellar birthline for total luminosity $L=0$
This shows the evolution of central temperature and $f$, the fraction of D/H (relative to the interstellar abundance.)

- Core temperature
- Fraction of Deuterium

Hartmann et al 1997

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The Effect of Surface Cooling

Hartmann et al 1997
Large variations in radius may occur to Deuterium burning. This may depend strongly on accretion rate.
Summary

The Stellar Birthline is the path a protostar takes in Mass vs Radius space as it creates.

The birthline sets the initial conditions of pre-main sequence contraction.

The birthline is strongly dependent on the amount of energy deposited by the accreting gas, the amount of the star covered by accreting gas, and energy generation by burning of Deuterium (next lecture).

The birthline may be dependent on accretion rate, and not the same for every star.