Lecture 20

Review: Types of Nuclear Burning

The energy generation per unit volume goes as the density squared. Thus, we can write the energy generation per mass, equivalent to the energy per volume divided by density, as

$$\epsilon \propto \rho T^n \tag{1}$$

$n = 4$ for the P-P chain, $n = 11.8$ for Deuterium burning, $n = 17$ for the CNO cycle.

Review: Pressure and the Internal Energy of a Gas

Last semester, we found that the Internal energy of a gas (per unit mass) is given by

$$P = (\gamma - 1)u\rho \tag{2}$$

where $\gamma = C_P/C_V$, the ratio of the specific heats. Thus, for an ideal gas, $\gamma = 5/3$, and:

$$u = \frac{3}{2}\frac{P}{\rho} \tag{3}$$

For radiation $\gamma = 4/3$ and

$$u_{rad} = 3P \tag{4}$$

where for a radiation of a temperature T

$$u_{rad} = 3P = \frac{4\sigma}{c}T^4 = aT^4 \tag{5}$$

The Virial Equilibrium of Stars: One More Time

Let's start with the Virial Theorem recast in the following form:

$$-3\int \frac{P}{\rho}dm = \Omega \tag{6}$$

where $dV = dm/\rho$. In the case of an ideal gas, we can write:

$$\frac{P}{\rho} = \frac{kT}{\mu m_H} \tag{7}$$

and

$$u_{gas} = \frac{3}{2}\frac{kT}{\mu m_H} \tag{8}$$

So are Virial equation does give $U = -1/2\Omega$, as expected. Let us try to generalize the equation to include radiation. Since we are interested in stars, and not clouds, the surface pressure term and magnetic field term are not important. Following an analysis in Prialnik:

$$\frac{P}{\rho} = \frac{P_{gas}}{\rho} + \frac{P_{rad}}{\rho} = \frac{kT}{\mu m_H} + \frac{aT^4}{3\rho} = \frac{2}{3}u_{gas} + \frac{1}{3}u_{rad} \tag{9}$$

where u_{gas} and u_{rad} are the specific internal energy (energy per mass). We can now express the virial equation as

$$U_{gas} = -\frac{1}{2}(\Omega + U_{rad}) \tag{10}$$

Note that radiation pressure effectively reduces the the gravitational attraction, with $\Omega + U_{rad}$ being an effective gravitational potential energy. The total energy is then:

$$E = \frac{1}{2}(\Omega + U_{rad}) = -U_{gas} \tag{11}$$

and the change in energy is given by:

$$\dot{E} = L_{nuc} - L \tag{12}$$

When $L_{nuc} = 0$, we have pre-main sequence evolution. In this case, the stars has a form of negative heat capacity. As E decreases, U_{gas} (and correspondingly the average internal temperature) increases!! Another way of thinking about this is that $T_C \propto M/R$. As the star contracts and the radius of the star shrinks, the central temperature must increase.

Consider a star near equilibrium such that $L_{nuc} = L$. Now perturb the star from that equilibrium. If $L_{nuc} < L$ then $\dot{E} < 0$. The internal gas energy will increase, the star will contract, and the internal temperature will increase. This increases the rate of nuclear fusion, which is a strong power of T. In the opposite case, if $L_{nuc} > L$ then $\dot{E} > 0$: the star then expands, the internal gas energy will decrease, and the internal temperature decreases. The rate of fusion will then also decrease. Hence, the equilibrium achieved when L_{nuc} is stable.