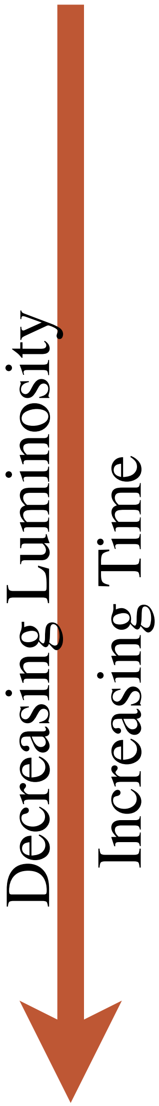
Lecture 21: The Main Sequence and Leaving the Main Sequence

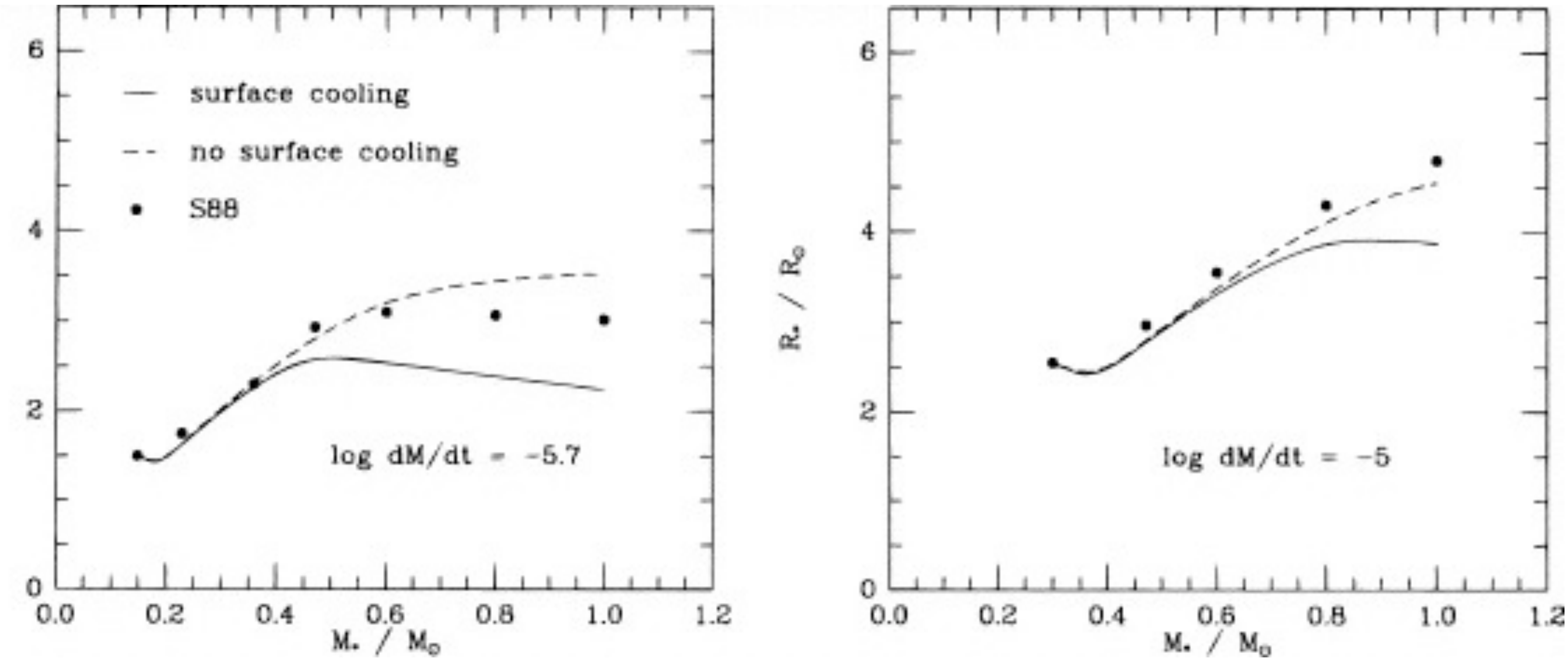


Protostar: Accretes mass throughout protostellar phase. Can undergo Deuterium fusion (for low mass stars) and Hydrogen fusion (high mass stars). Accretion onto star with a disk. Must overcome magnetic pressure, resulting in magnetospheric accretion for low mass stars. Must overcome photon pressure for high mass stars.

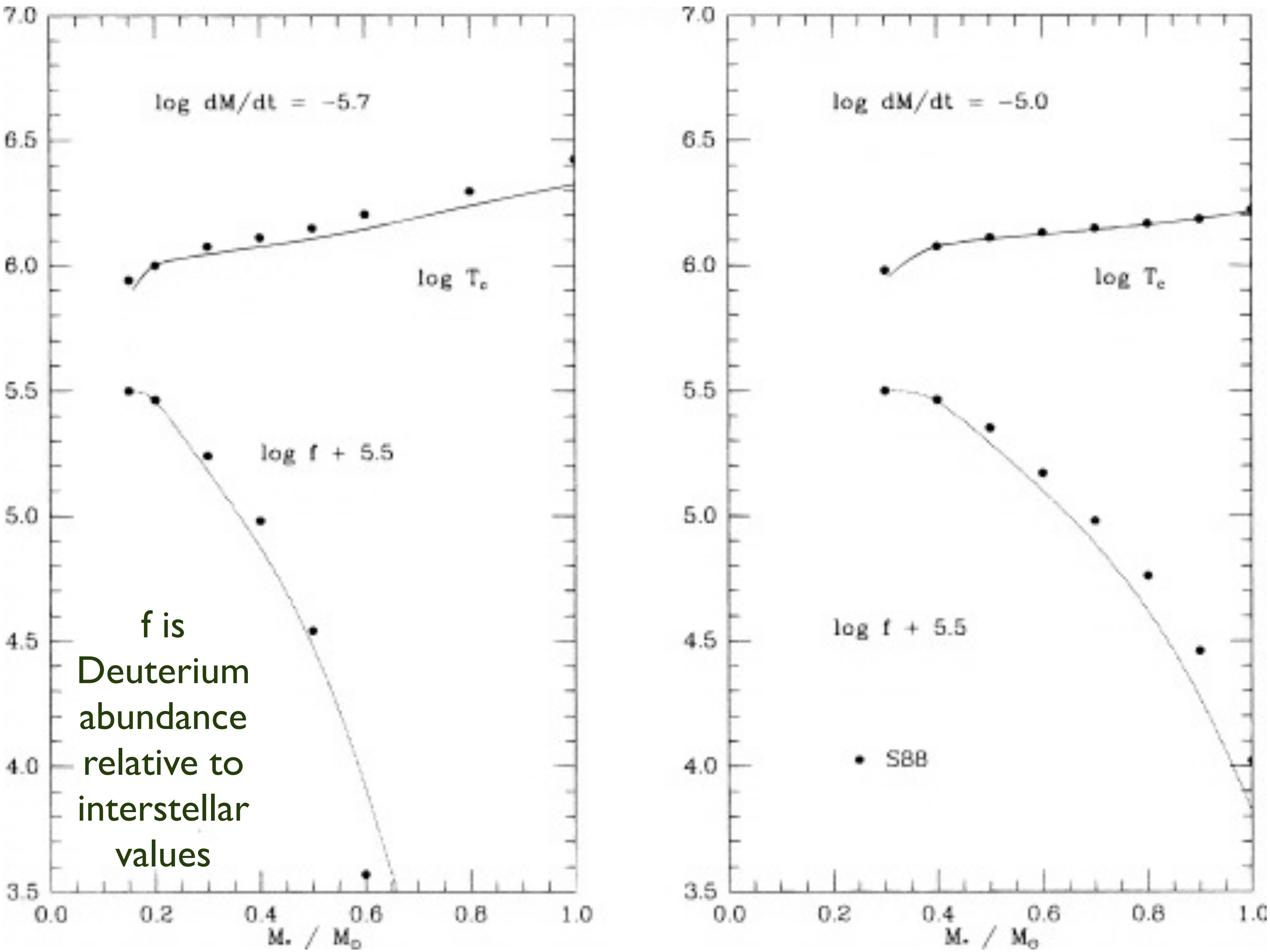
Pre-Main Sequence Star: only happens for low to intermediate stars. Star contracts and center heats up. Deuterium fusion possible.

Main Sequence Star: hydrogen fusion. Long period of stable luminosity and radius.

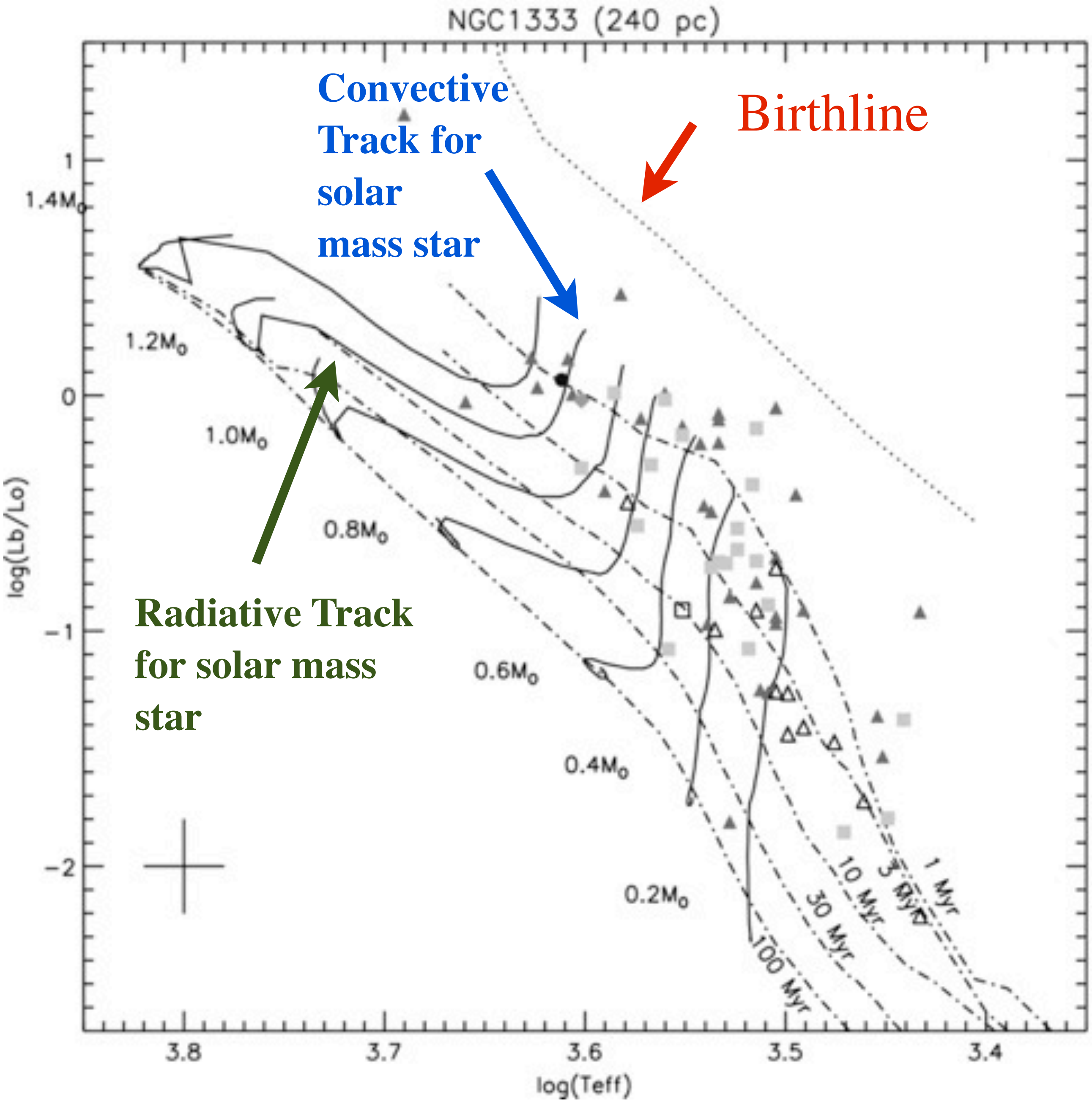
Protostellar Birthline



These show models of how the radius of a protostar changes as it grows in mass and Deuterium burning occurs.

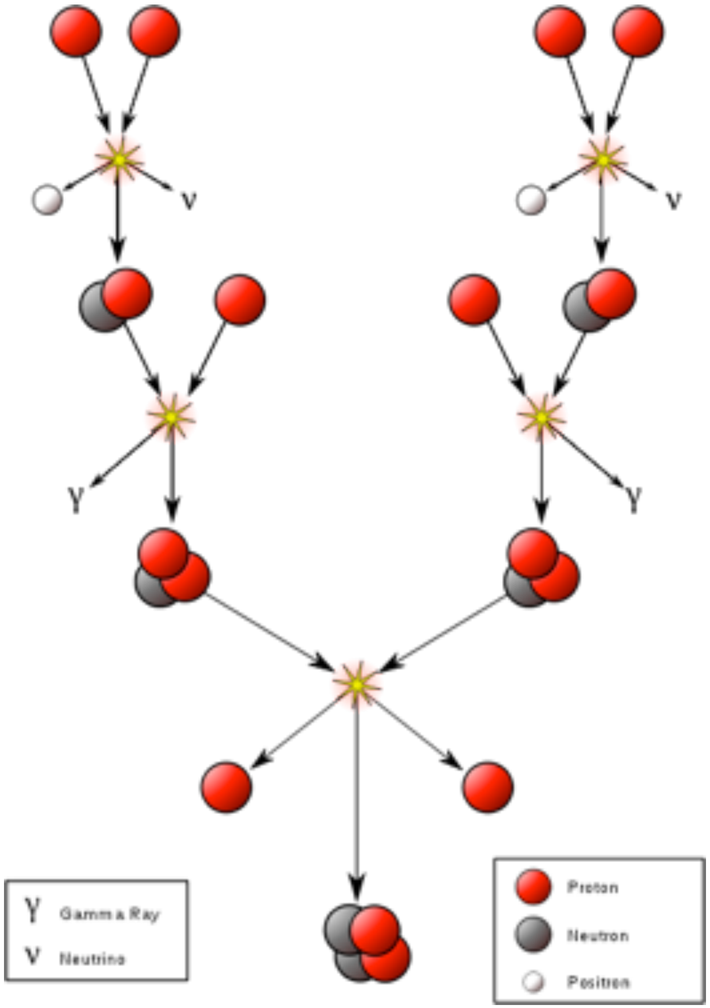


These show models of how the radius of a protostar changes as it grows in mass and Deuterium burning occurs.



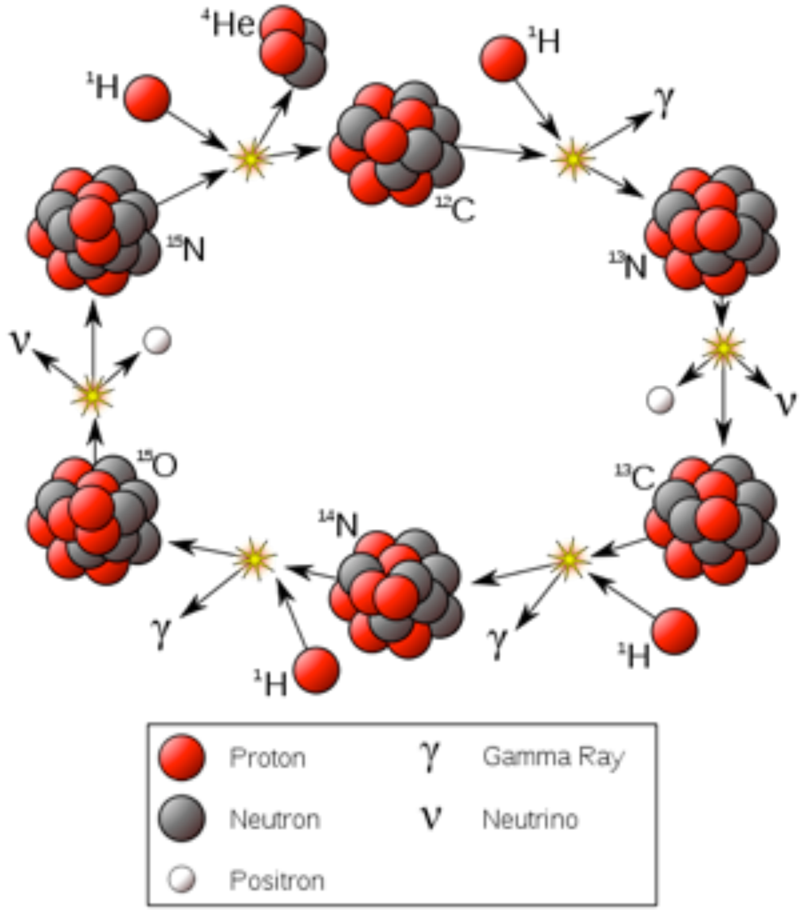
Pre-main sequence tracks: Baraffe et al. 1998 (from Winston et al. 2007)

The P-P Chain pp 1 branch



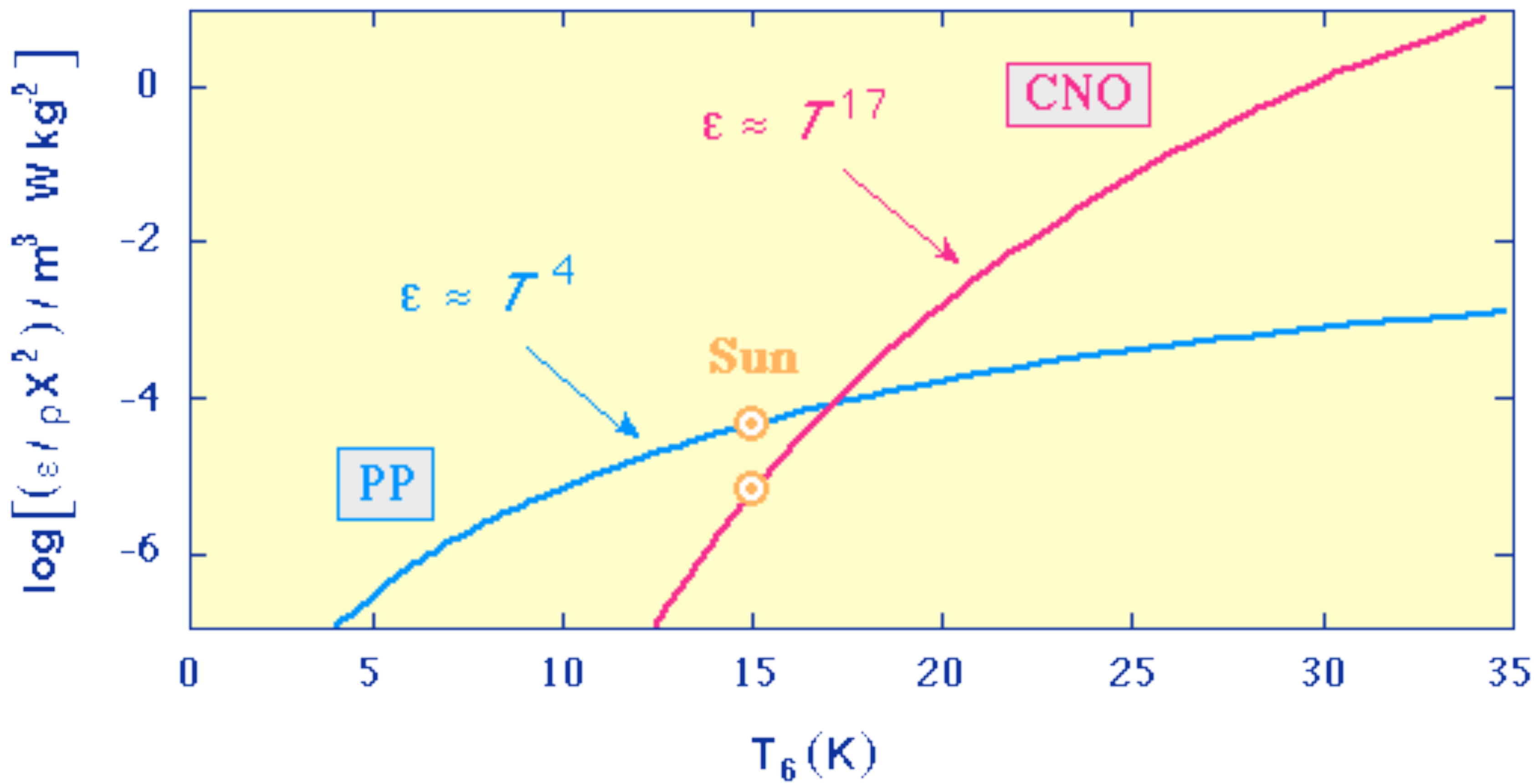
$$\varepsilon = \varepsilon_0 \varrho T^4$$

The CNO Cycle



$$\varepsilon = \varepsilon_0 \varrho \, T^{17}$$

P-P Chain vs CNO Cycle



http://csep10.phys.utk.edu/astr162/lect/energy/cno-pp.html

For a Virialized Star on Main Sequence

$$E = \frac{1}{2}(\Omega + U_{rad}) = -U_{gas} \quad (11)$$

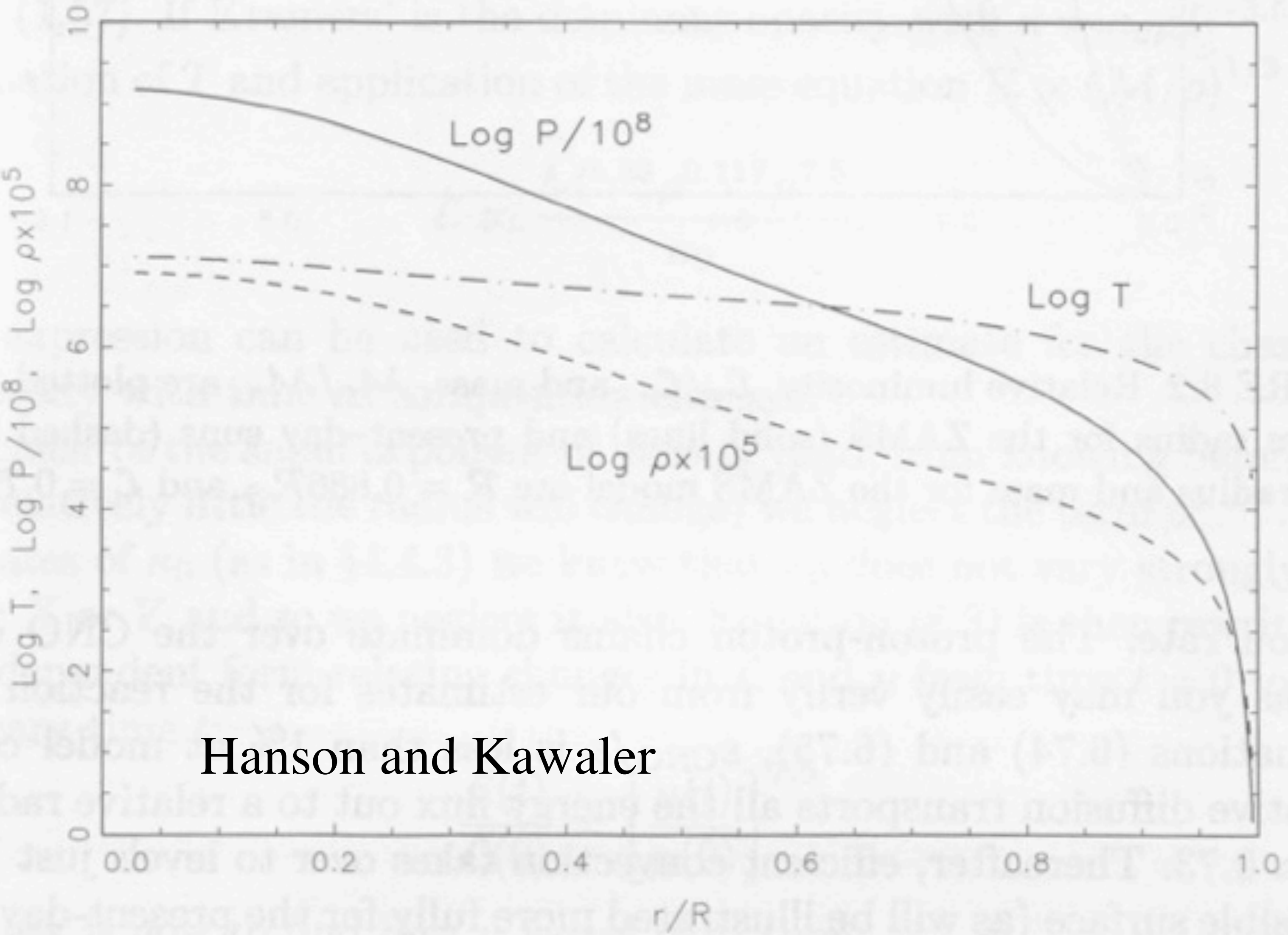
and the change in energy is given by:

$$\dot{E} = L_{nuc} - L \quad (12)$$

The Stellar Thermostat

If L_{nuc} goes up, energy goes up, star expands and L_{nuc} goes back down.

If L_{nuc} goes down, energy goes down, star contracts and L_{nuc} goes back up.



Hanson and Kawaler

FIGURE 8.1. Shown are the runs of pressure, temperature, and density for a model of the zero-age sun. Note that the pressure has been multiplied by 10^{-8} and the density by 10^5. The abscissa is the relative radius $r/\mathcal{R}$ where $\mathcal{R} = 0.886\mathcal{R}_\odot$.

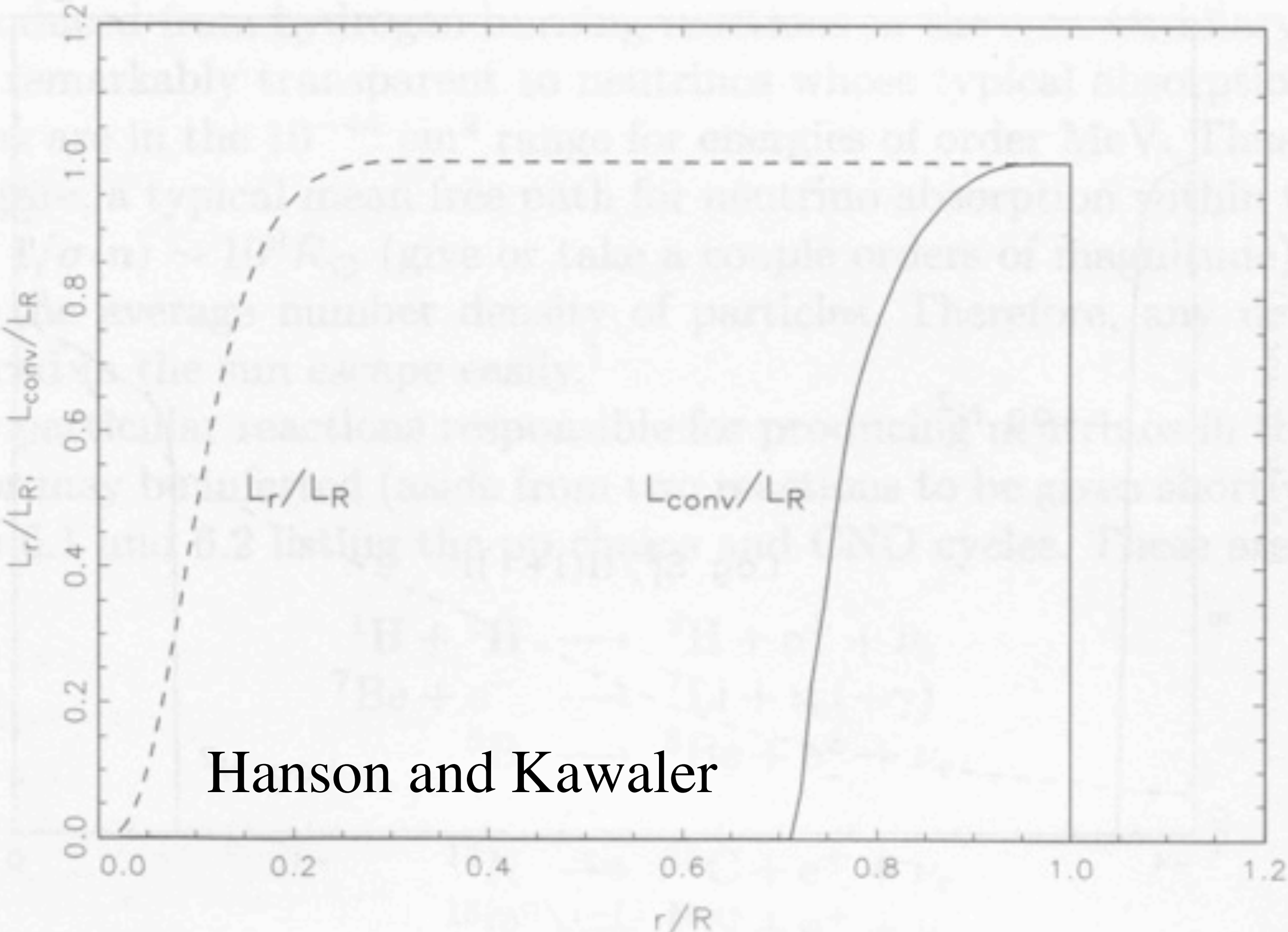


FIGURE 8.5. The ratios $\mathcal{L}_{tot}/\mathcal{L}_R$ (dashed line) and $\mathcal{L}_{conv}/\mathcal{L}_R$ (solid line) are shown as a function of relative radius in the present-day sun. $\mathcal{L}_R$ is equal to the luminosity at the surface. The convective luminosity drops to zero just as the photosphere is reached at $r = R$.

Monday, May 2, 2011

Review: calculation of mean atomic weight of an ionized gas (μ)

Given a mass fraction X_i (or abundance) for an ionic (or atomic) species with atomic weight A_i, we can can calculate μ by:

$$\frac{1}{\mu(ion)} = \sum \frac{X_i}{A_i} \tag{1}$$

For the electrons:

$$\frac{1}{\mu(e^-)} = \sum \frac{Z_i X_i}{A_i} \tag{2}$$

where Z_i is the charge of the ion. Finally, the total value of μ is:

$$\frac{1}{\mu} = \frac{1}{\mu(e^-)} + \frac{1}{\mu(ion)} = \sum \frac{Z_i X_i}{A_i} + \frac{X_i}{A_i} \tag{3}$$

Multiplying $1/\mu$ by ρ/m_H gives the number of particles needed for the ideal gas law. We use X for the mass fraction (abundance) of Hydrogen, Y for Helium, Z for everything else. We will consider an envelope, with normal "solar" abundances, and a core where Hydrogen has been depleted and the abundance is dominated by Helium. For ions:

$$\frac{1}{\mu(ion)} = X + \frac{1}{4}Y + \frac{1-X-Y}{<A>} \approx \frac{1}{4}(1+3X) \tag{4}$$

We now approximate $Y = 1-X$ and $Z = 0$. Given $X = 0.707$ and $Y = 0.274$ and $<A> \sim 20$ then $\mu(ion) = 1.29$. On the other hand, if $Y \sim 1$ then $\mu(ion) = 4$. For electrons:

$$\frac{1}{\mu(e^-)} = X + \frac{1}{2}Y + (1-X-Y)\frac{<Z>}{<A>} = X + \frac{1}{2}Y = \frac{1}{2}(1+X) \tag{5}$$

For a normal (envelope) abundance, $\mu(e^-) = 1.17$, for $X = 0$ (in the Hydrogen depleted core), $\mu(e^-) = 2$. The total value of μ is given by.

$$\frac{1}{\mu} = \frac{1}{\mu(e^-)} + \frac{1}{\mu(ion)} \approx \frac{3+5x}{4} \tag{6}$$

For an ionized gas with normal interstellar abundances, $\mu_{env} = 0.6$. If we convert the Hydrogen into Helium, then $\mu_{core} = 1.33$, *doubling the mean atomic weight per particle.*

Hanson and Kawaler

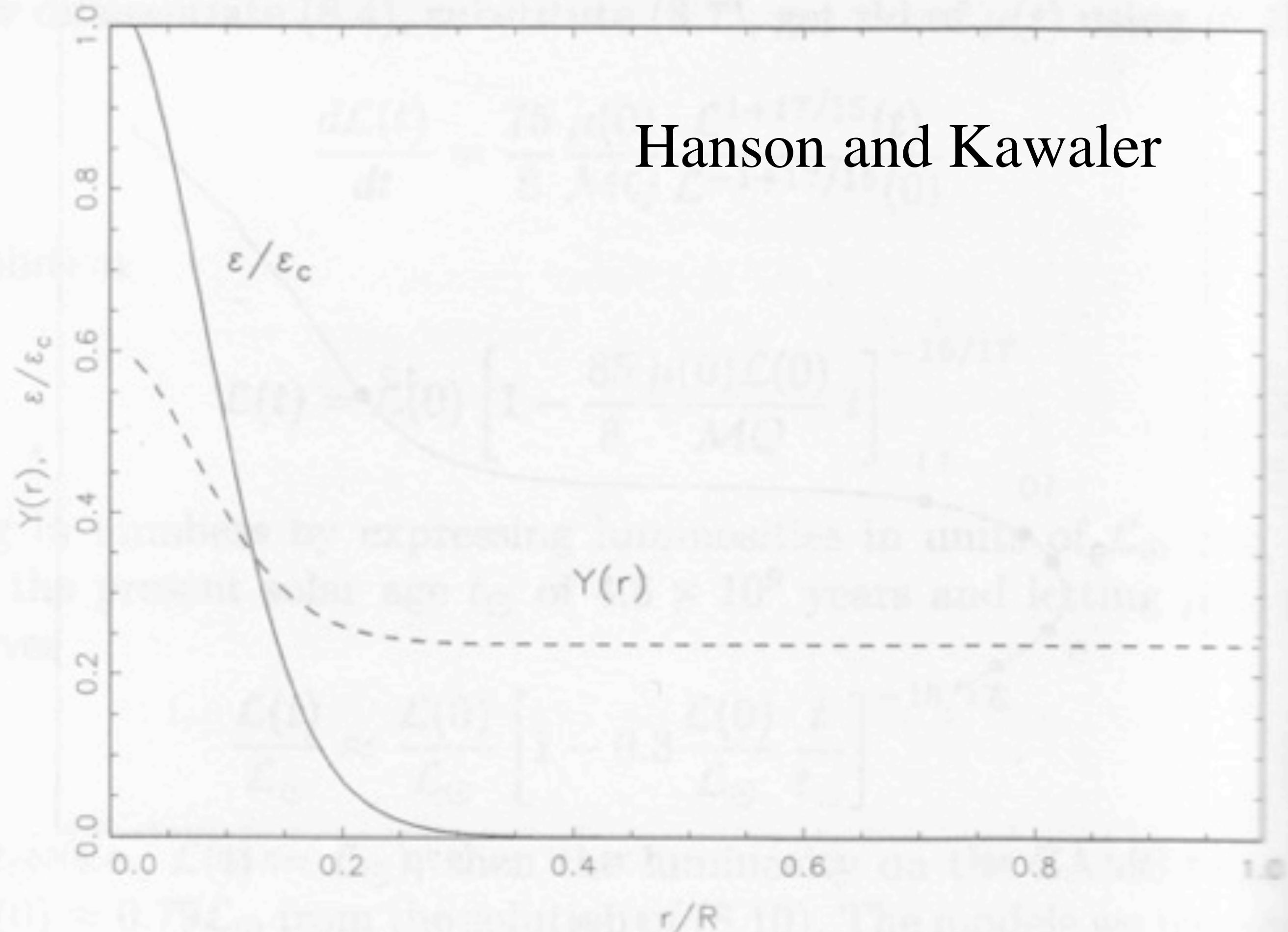


FIGURE 8.4. Because of hydrogen-burning in the radiative solar core, the mass fraction, Y, of helium in the present-day sun has increased while the mixture becomes less hydrogen-rich. Also shown is the energy generation rate $\varepsilon/\varepsilon_c$ compared to its central value ε_c.

The Effect of Hydrogen Burning

In the evolution of stars, the changing of the mean atomic weight of the gas particles by nuclear fusion can have a dramatic effect on the pressure, hence driving stellar evolution. Consider the ideal gas law:

$$P = \frac{\rho}{\mu m_H} kT \tag{7}$$

When we convert hydrogen into He, μ doubles, from 0.6 to 1.3. Thus, for the same temperature, the pressure drops by a factor of two.

The Main Sequence Phase

Is the main sequence phase characterized by constant energy? We can rewrite the Virial theorem, ignoring U_{rad}.

$$P_c = C\frac{M^2}{R^4} \tag{8}$$

The ideal gas law implies that

$$T_c = C\frac{\mu m_H}{k\rho}\frac{M^2}{R^4} \tag{9}$$

If we let $R = (3M/4\pi\rho)^{1/3}$ then:

$$T_c = C\left(\frac{4\pi}{3}\right)^{4/3}\frac{\mu m_H}{k}M^{2/3}\rho^{1/3} \tag{10}$$

From radiative diffusion

$$L = -\frac{64\pi\sigma r^2}{\kappa\rho} T^3 \frac{dT}{dr} \tag{11}$$

Approximating $dT/dr \sim T/R$, then:

$$L \propto \frac{RT^4}{\kappa\rho} \tag{12}$$

If we adopt the Kramers opacity $\kappa = \kappa_0 \rho T^{-3.5}$

$$L \propto \frac{M^{5.53} \rho^{0.166} \mu^{7.5}}{\kappa_0} \tag{13}$$

Since M is constant, and there is a low dependence on ρ (which is probably also close to constant):

$$\frac{L(t)}{L(0)} = \left[\frac{\mu(t)}{\mu(0)}\right]^{7.5} \tag{14}$$

The Faint Sun Paradox

The result is that as Hydrogen is converted into Helium, the Sun becomes more luminous. Early on, it was 25% less luminous. The faint young Sun paradox is that the early Earth was too cold for liquid water to exist. The most common explanation is that the greenhouse effect kept the surface warmer (the Earth's atmosphere was rich in CO_2 early on). As the same ages, it will get warmer. In about 1 billion years, predictions show the Oceans evaporating in a runaway greenhouse effect.

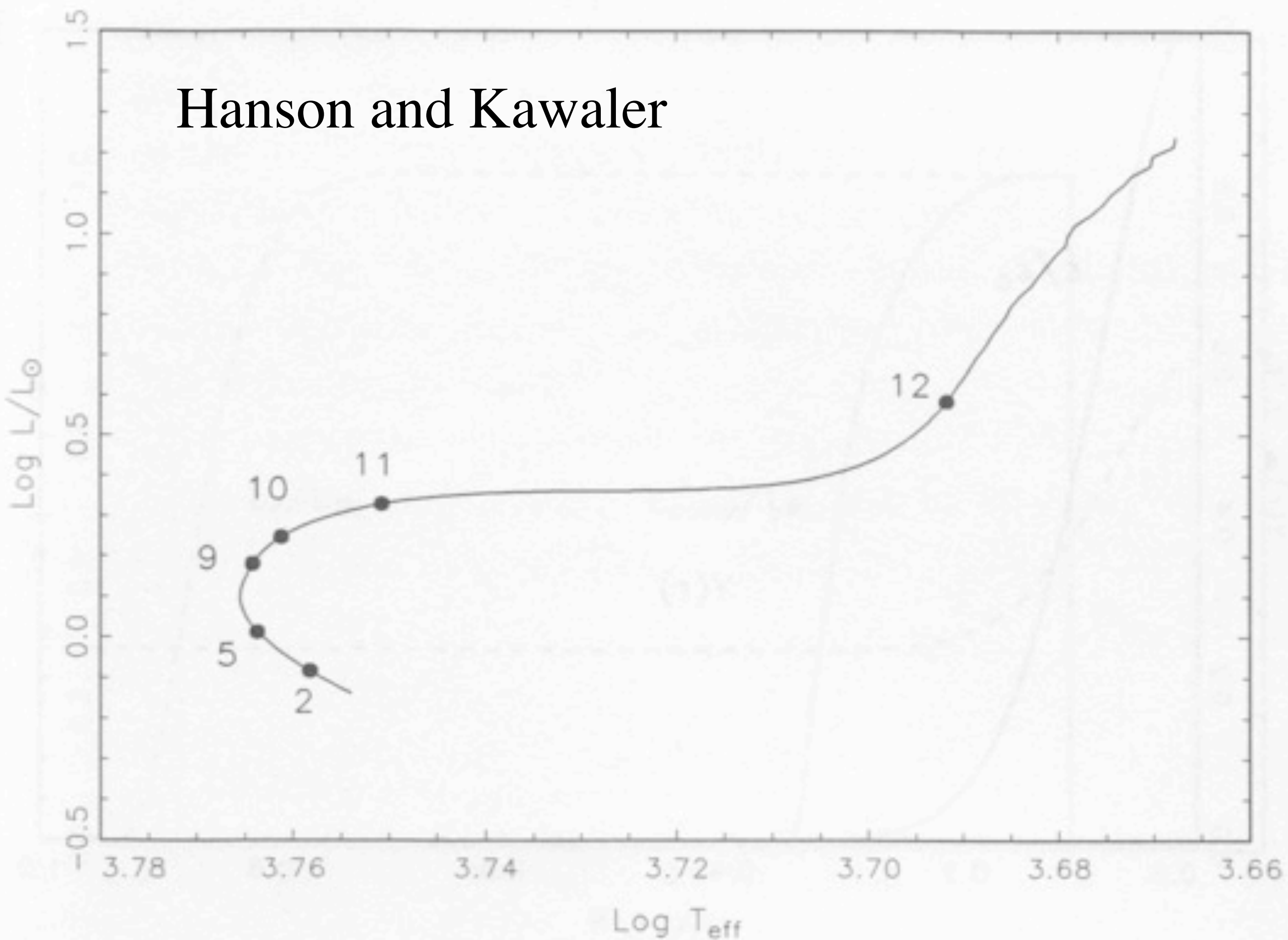


FIGURE 8.3. A solar model evolution track in the Hertzsprung–Russell diagram from the ZAMS to the red giant stage. Elapsed evolutionary time from the ZAMS is indicated by the filled circles where the units are Gyr.

The Red Giant Phase

Now imagine that the central core of the star is depleted in Hydrogen, and that fusion is taking place primarily in a shell surrounding the central core (either by the P-P chain or the CNO cycle). What happens to the central core?

Let us consider the virial theorem for a shell. Now, we have a virial theorem similar to that which we considered for clouds and cores with an internal and external pressure (see lecture 4 equations, section 2).

$$\int_0^{V_c} P dV = P_s V_c + \frac{1}{3}\alpha \frac{GM_c^2}{R_c} \tag{15}$$

Now assume an isothermal gas where $P = \rho kT/\mu m_H$. Dividing by the volume, we get

$$P_s = \frac{3}{4\pi}\frac{kT}{\mu m_H}\frac{M_c}{R_c^3} - \frac{\alpha G}{4\pi}\frac{M_c^2}{R_c^4} \tag{16}$$

If we set $P_S = 0$, we get a minimum radius:

$$R_0 = \alpha \frac{\mu m_H}{3k} \frac{M_c \mu_c}{T_c} \tag{17}$$

Below this radius, there is no stable configuration. However, if there is external pressure On the other hand, we can find a maximum value of P_s, by finding out where $dP_s/dR_c = 0$. At this point:

$$0 = -\frac{9}{4\pi} \frac{kT}{\mu m_H} \frac{M_C}{R_c^4} + \frac{\alpha G}{\pi} \frac{M_c^2}{R_c^5} \tag{18}$$

Giving the solution

$$R_1 = \frac{4\alpha G}{9R_\star} \frac{M_c \mu_c}{T_c} \tag{19}$$

Thus, if $R < R_0$, the core is unstable if the pressure was 0. If there is an external pressure, the core is unstable if $R < R_1$. It is stable for $R > R_1$. At R_1, there is a maximum in the pressure where if the pressure is exceeded, the core will collapse. By substituting this back into the equation for P_S, we get

$$P_{s,max}(M_c) = C_1 \frac{T_c^4}{M_c^4 \mu_c^4} \tag{20}$$

The Minimum Pressure from the Envelope

The surrounding pressure is due to the weight of the envelope. We can estimate in by using the equation of hydrostatic equilibrium:

$$\frac{dP}{dm} = \int_0^{M_\star} \frac{Gm}{4\pi r^4} dm \qquad (21)$$

Since $r < R$, where R is the radius of the star, we get the inequality:

$$P_{env} > \int_0^{M_\star} \frac{Gm}{4\pi R^4} dm = \frac{GM_\star^2}{8\pi R_\star^4} \qquad (22)$$

Thus, the core is unstable if then

$$P_{s,max}(M_c) = C_1 \frac{T_c^4}{M_c^2} \mu_c^4 \geq \frac{GM_\star^2}{8\pi R_\star^4} \qquad (23)$$

Now, we assume can use the equation

$$T_c = C_2 \frac{\mu_{env} m_H}{k} \frac{GM_\star}{R_\star} \tag{24}$$

giving the final relationship

$$\frac{M_c}{M_\star} \leq C_3 \left(\frac{\mu_{env}}{\mu_c} \right)^2 \tag{25}$$

Schönberg and Chandrasekhar derived that $C_3 = 0.37$.

What are μ_{env} and μ_c? Let us assume that the envelope is dominated by Hydrogen, while the core is dominated by Helium. The total value of μ is given by:

$$\frac{1}{\mu} = \frac{1}{\mu(e^-)} + \frac{1}{\mu(ion)} \approx \frac{3+5x}{4} \tag{26}$$

This gives us $\mu_{env} = 0.6$ and $\mu_{core} = 1.33$

The critical ratio is when the mass of the Helium approaches the size:

$$\frac{M_c}{M_\star} \leq 0.37 \left(\frac{0.6}{1.33} \right)^2 \sim 0.1 \tag{27}$$

Thus, when 10% of the Hydrogen is converted into Helium, the star becomes unstable. At this point, it enters the Red Giant phase.

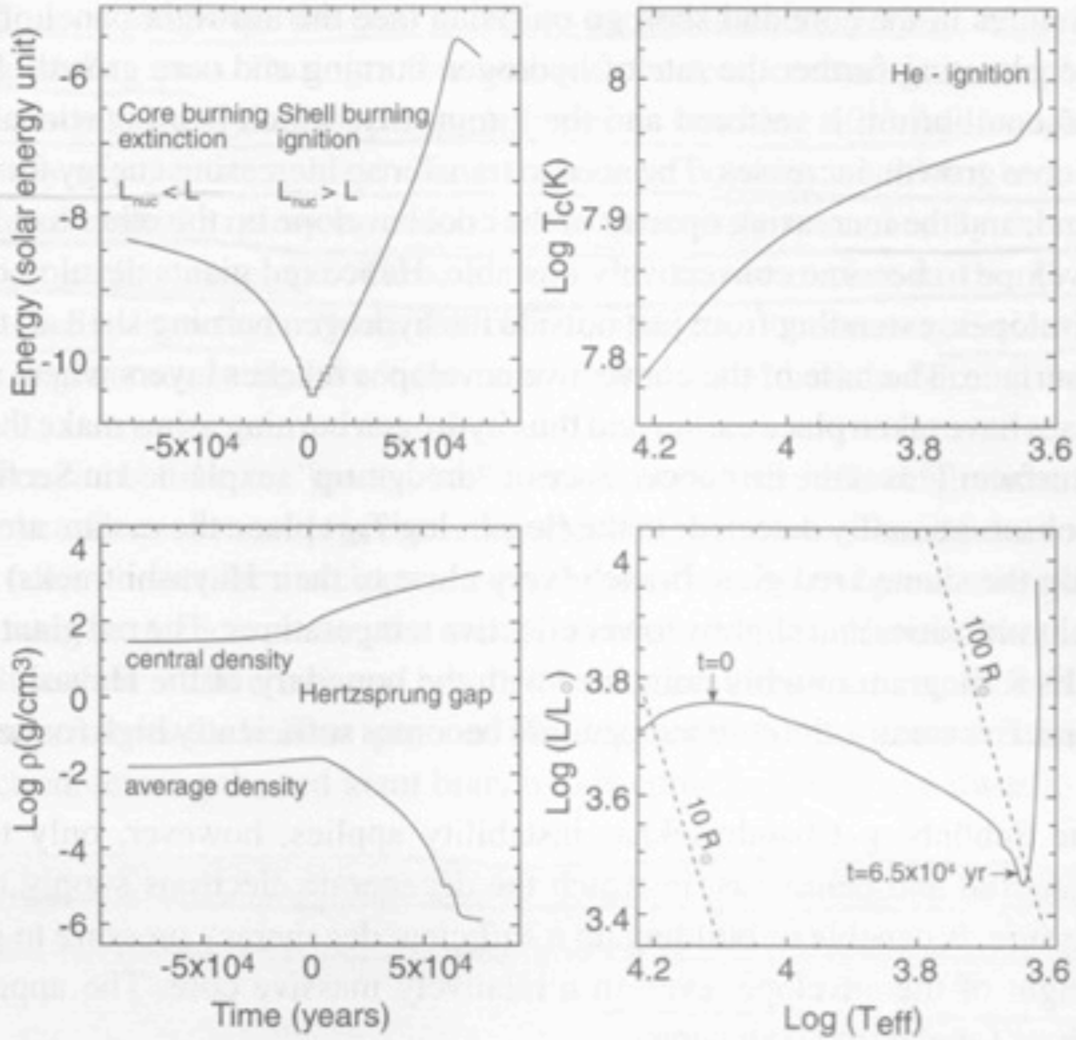


Figure 8.6 Evolution of an intermediate mass star ($7M_\odot$) during the crossing of the *Hertzsprung gap*: *top left*: total energy as a function of time (the time is arbitrarily set to zero at the onset of core contraction); *bottom left*: central density and average density ($3M/4\pi R^3$) as a function of time; *bottom right*: evolutionary track in the H–R diagram (where lines of equal radius are marked); *top right*: changing of central temperature with effective temperature.

Prialnik

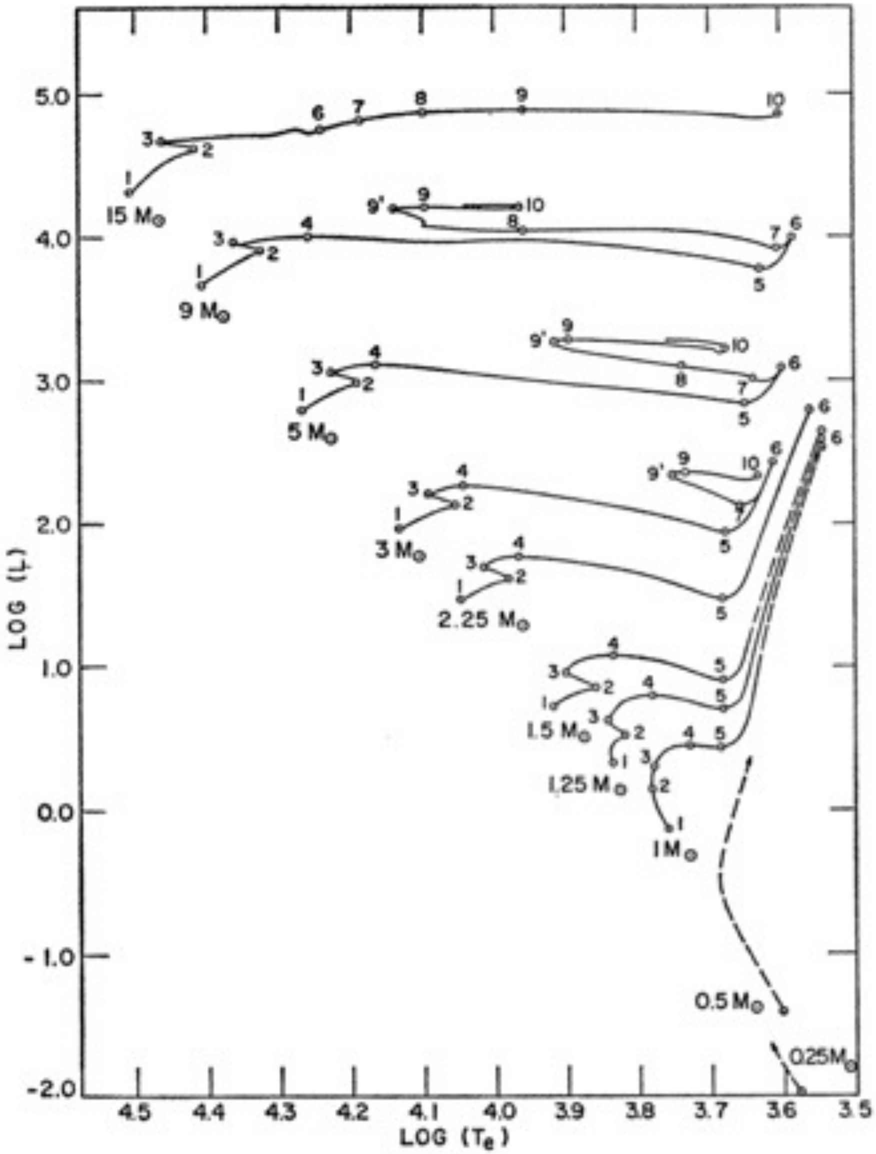


FIG. 3. Paths in the H-R diagram for metal-rich stars of mass $(M/M_\odot) = 15$, 9, 5, 3, 2.25, 1.5, 1.25, 1, 0.5, 0.25. Units of luminosity and surface temperature are the same as in Figure 1. Traversal times between labeled points are given in Tables III and IV. Dashed portions of evolutionary paths are estimates.

Iben 1967
Annual Review of
Astronomy and
Astrophysics

Summary

- We traced the evolution of a 1 solar mass star from birth to leaving the main sequence.
- During the main sequence, stars slowly grow more luminous as the mean atomic weight of the gas increases (and the number of particles per gram decreases).
- A growing central core of Helium becomes unstable as it grows in mass and gravity begins to dominate over pressure, this occurs when about 10% of the total mass of a star is converted into Helium.