Lecture 22: Why Stars Become Red Giants

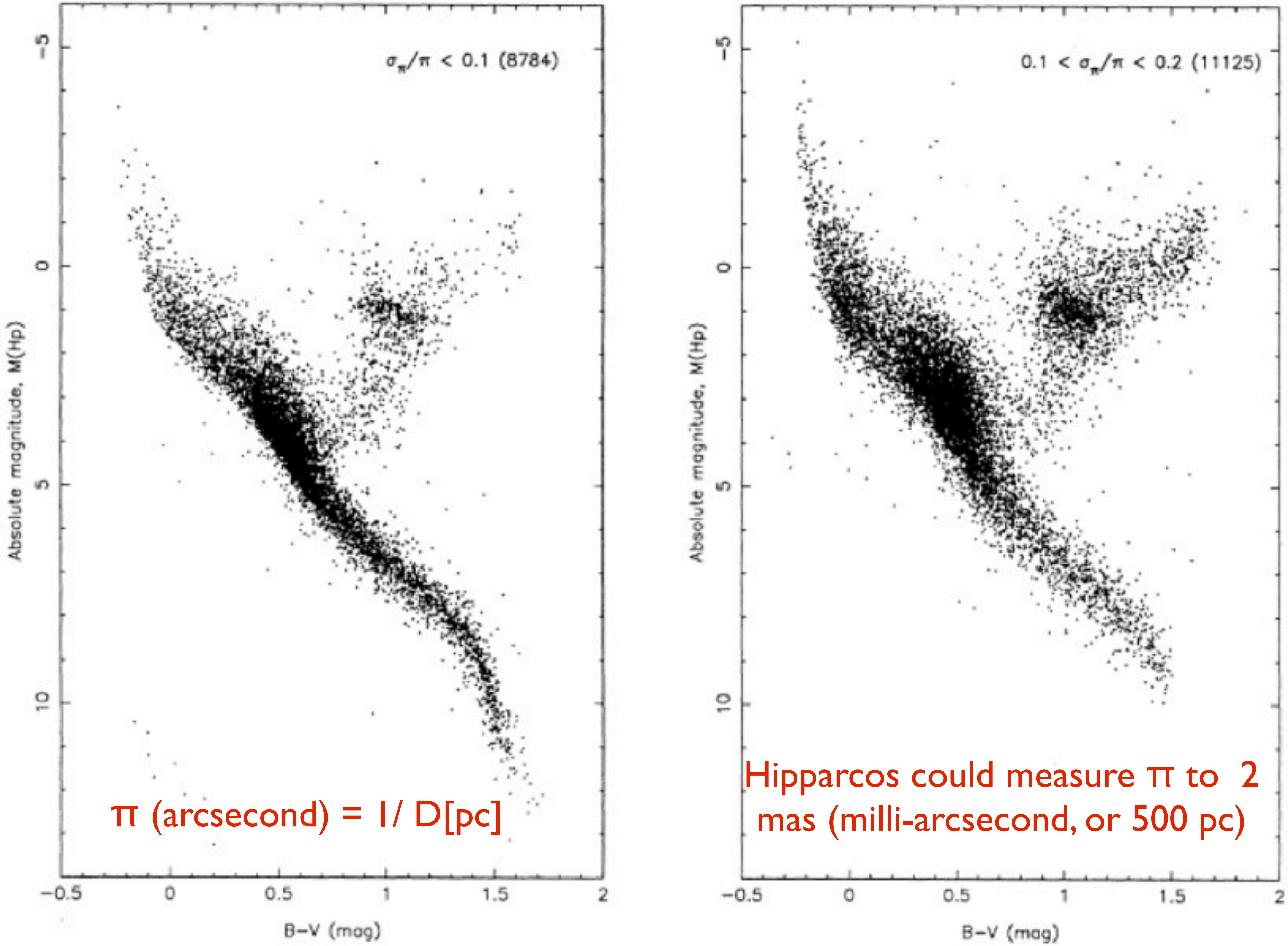
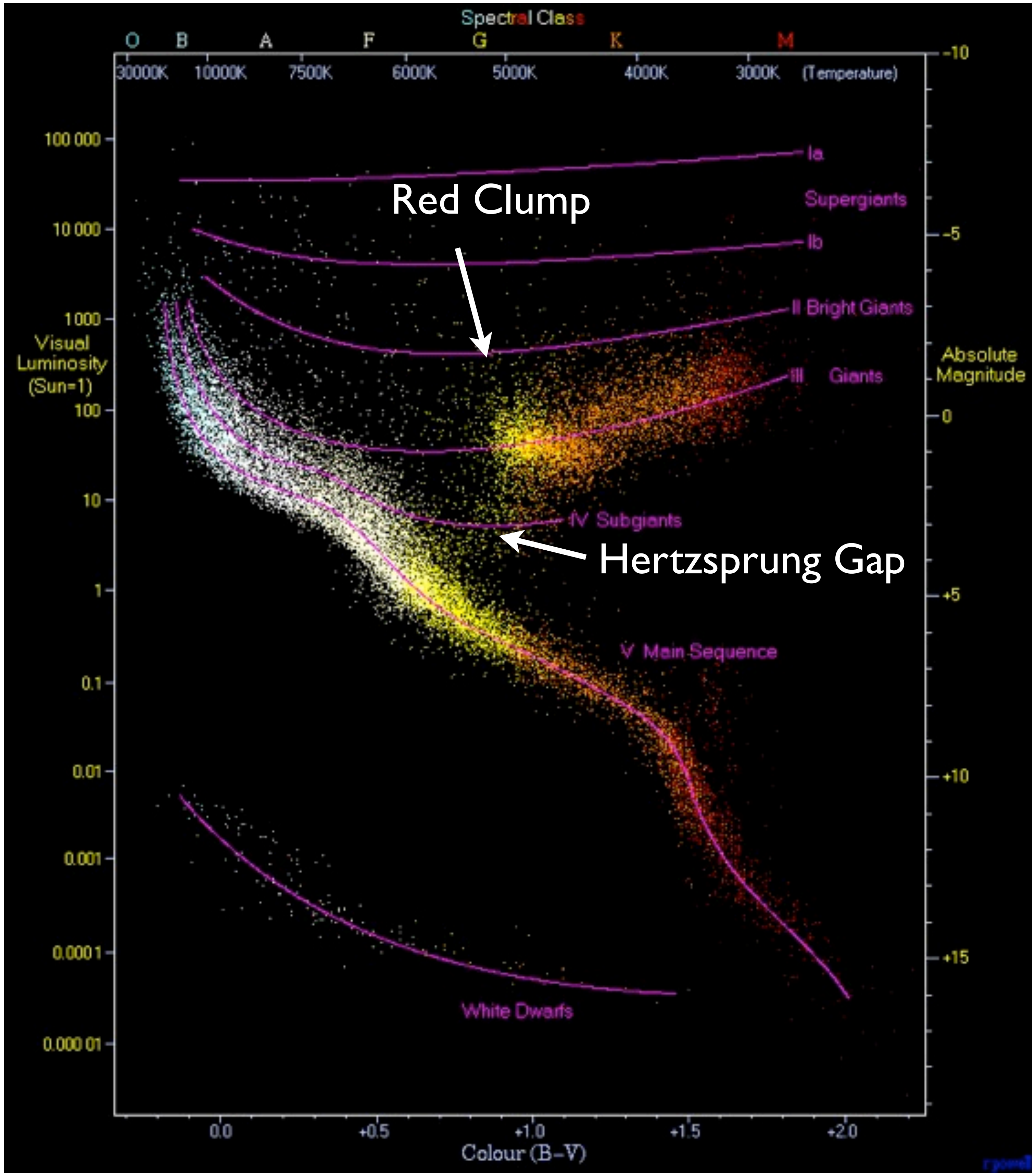
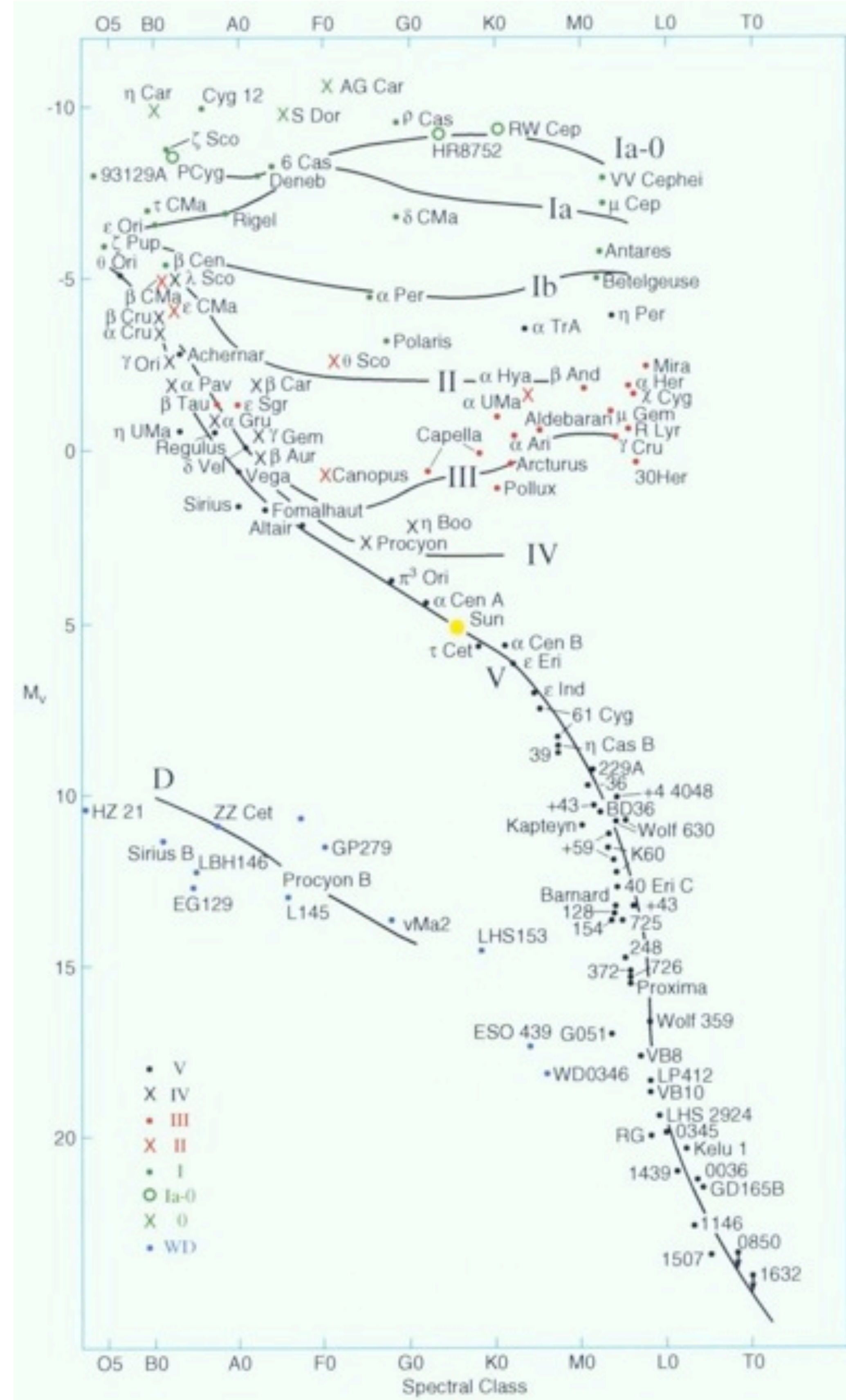


Fig. 6. a The observational HR diagram constructed from the preliminary Hipparcos catalogue H30, for the 8784 stars with $\sigma_\pi/\pi < 0.1$ and $\sigma_{B-V} < 0.025$ mag, and supplemented by six white dwarfs as described in the text. The ordinate gives the absolute magnitude, M_{Hp}, derived from the satellite-determined parallaxes and the median satellite-derived Hp magnitudes. The abscissa gives the colour index $(B - V)$, derived from the ground-based observations compiled in the Hipparcos Input Catalogue. **b** as for **a**, but based on the 11 125 stars from H30 satisfying $0.1 \leq \sigma_\pi/\pi < 0.2$ and $\sigma_{B-V} < 0.025$ mag.

Perrymann et al. 1995 A&A 304, 69

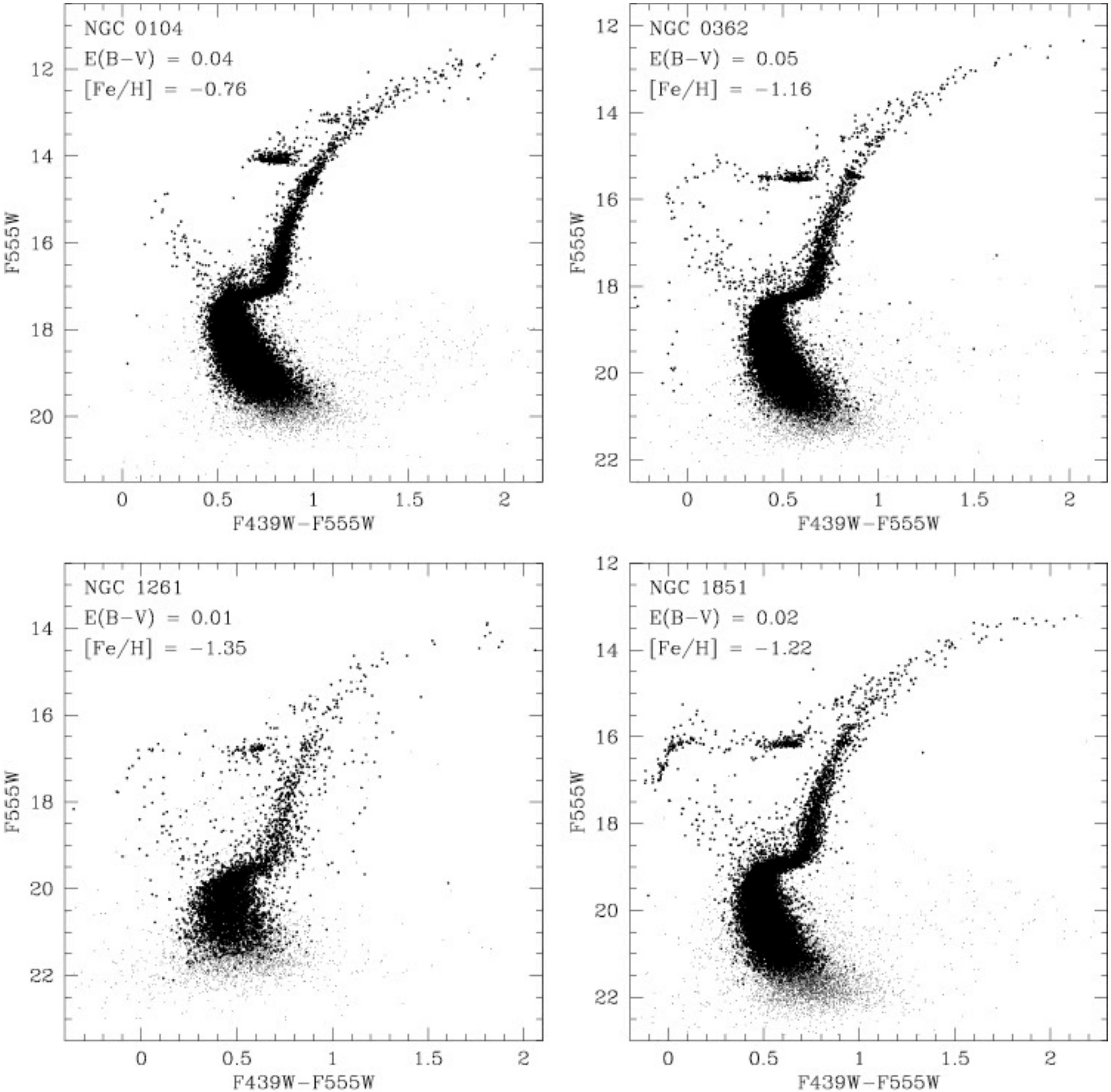


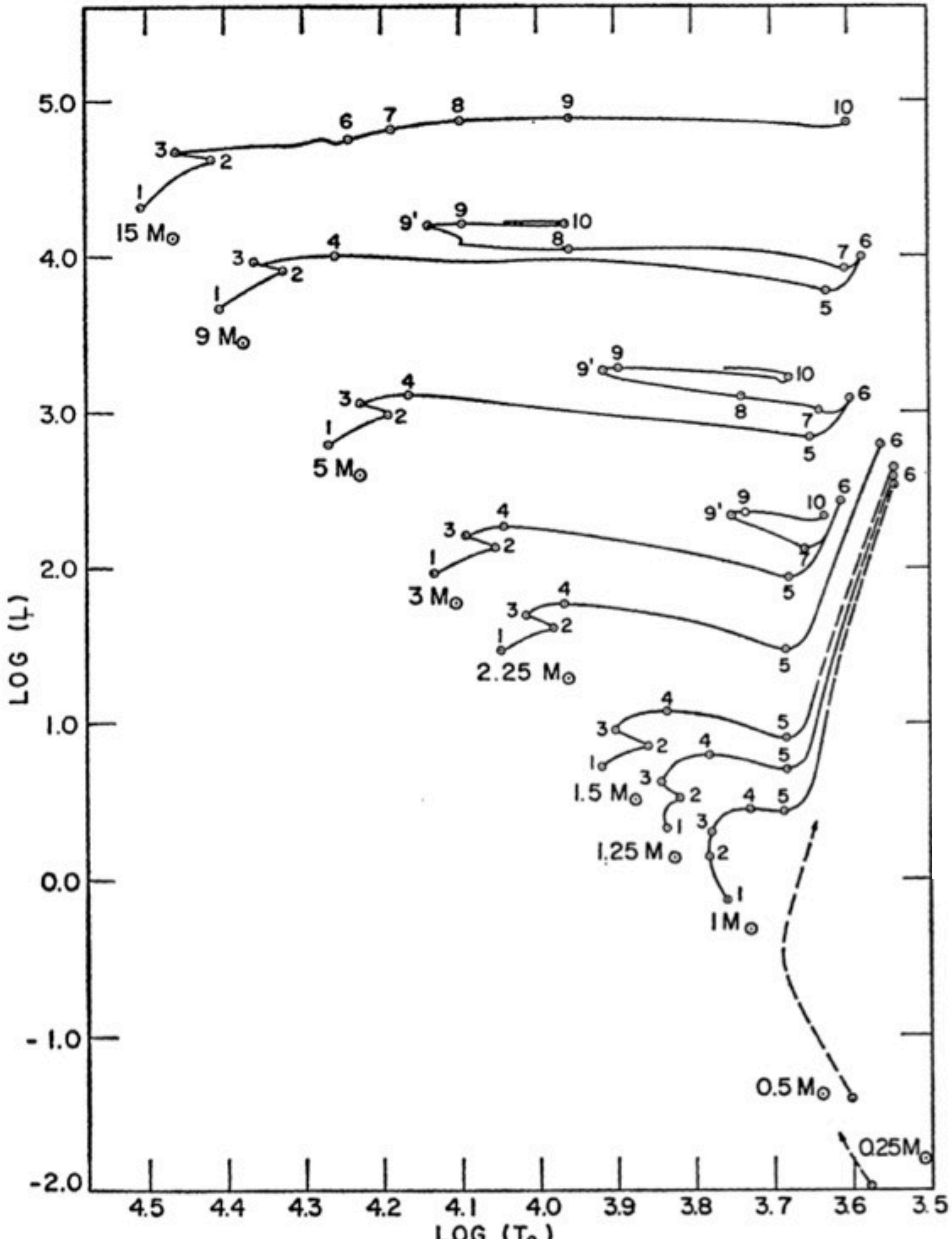


http://www.atlasoftheuniverse.com/hr.html

http://stars.astro.illinois.edu/sow/hrd.html

Piotto et al. 2002





Iben 1967

TABLE III

STELLAR LIFETIMES (yr)[a]

Mass ($M_\odot$) \ Interval ($i-j$)	(1–2)	(2–3)	(3–4)	(4–5)	(5–6)
15	1.010 (7)	2.270 (5)		7.55 (4)	
9	2.144 (7)	6.053 (5)	9.113 (4)	1.477 (5)	6.552 (4)
5	6.547 (7)	2.173 (6)	1.372 (6)	7.532 (5)	4.857 (5)
3	2.212 (8)	1.042 (7)	1.033 (7)	4.505 (6)	4.238 (6)
2.25	4.802 (8)	1.647 (7)	3.696 (7)	1.310 (7)	3.829 (7)
1.5	1.553 (9)	8.10 (7)	3.490 (8)	1.049 (8)	≥2 (8)
1.25	2.803 (9)	1.824 (8)	1.045 (9)	1.463 (8)	≥4 (8)
1.0	7 (9)	2 (9)	1.20 (9)	1.57 (8)	≥1 (9)

[a] Numbers in parentheses beside each entry give the power of ten to which that entry is to be raised.

TABLE IV

STELLAR LIFETIMES (yr)[a]

Mass ($M_\odot$) \ Interval ($i-j$)	(6–7)	(7–8)	(8–9)	(9–10)
15	7.17 (5)	6.20 (5)	1.9 (5)	3.5 (4)
9	4.90 (5)	9.50 (4)	3.28 (6)	1.55 (5)
5	6.05 (6)	1.02 (6)	9.00 (6)	9.30 (5)
3	2.51 (7)	4.08 (7)		6.00 (6)

[a] Numbers in parentheses beside each entry give the power of ten to which that entry is to be raised.

The Red Giant Phase

Now imagine that the central core of the star is depleted in Hydrogen, and that fusion is taking place primarily in a shell surrounding the central core (either by the P-P chain or the CNO cycle). What happens to the central core?

Let us consider the virial theorem for a shell. Now, we have a virial theorem similar to that which we considered for clouds and cores with an internal and external pressure (see lecture 4 equations, section 2).

$$\int_0^{V_c} PdV = P_s V_c + \frac{1}{3}\alpha\frac{GM_c^2}{R_c} \tag{15}$$

Now assume an isothermal gas where $P = \rho kT/\mu m_H$. Dividing by the volume, we get

$$P_s = \frac{3}{4\pi}\frac{kT}{\mu m_H}\frac{M_c}{R_c^3} - \frac{\alpha G}{4\pi}\frac{M_c^2}{R_c^4} \tag{16}$$

The Minimum Pressure from the Envelope

The surrounding pressure is due to the weight of the envelope. We can estimate in by using the equation of hydrostatic equilibrium:

$$\frac{dP}{dm} = \int_0^{M_\star} \frac{Gm}{4\pi r^4} dm \tag{21}$$

Since $r < R$, where R is the radius of the star, we get the inequality:

$$P_{env} > \int_0^{M_\star} \frac{Gm}{4\pi R^4} dm = \frac{GM_\star^2}{8\pi R_\star^4} \tag{22}$$

Thus, the core is unstable if then

$$P_{s,max}(M_c) = C_1 \frac{T_c^4}{M_c^2} \mu_c^4 \geq \frac{GM_\star^2}{8\pi R_\star^4} \tag{23}$$

The critical ratio is when the mass of the Helium approaches the size:

$$\frac{M_c}{M_\star} \leq 0.37 \left(\frac{0.6}{1.33}\right)^2 \sim 0.1 \quad (27)$$

Thus, when 10% of the Hydrogen is converted into Helium, the star becomes unstable. At this point, it enters the Red Giant phase.

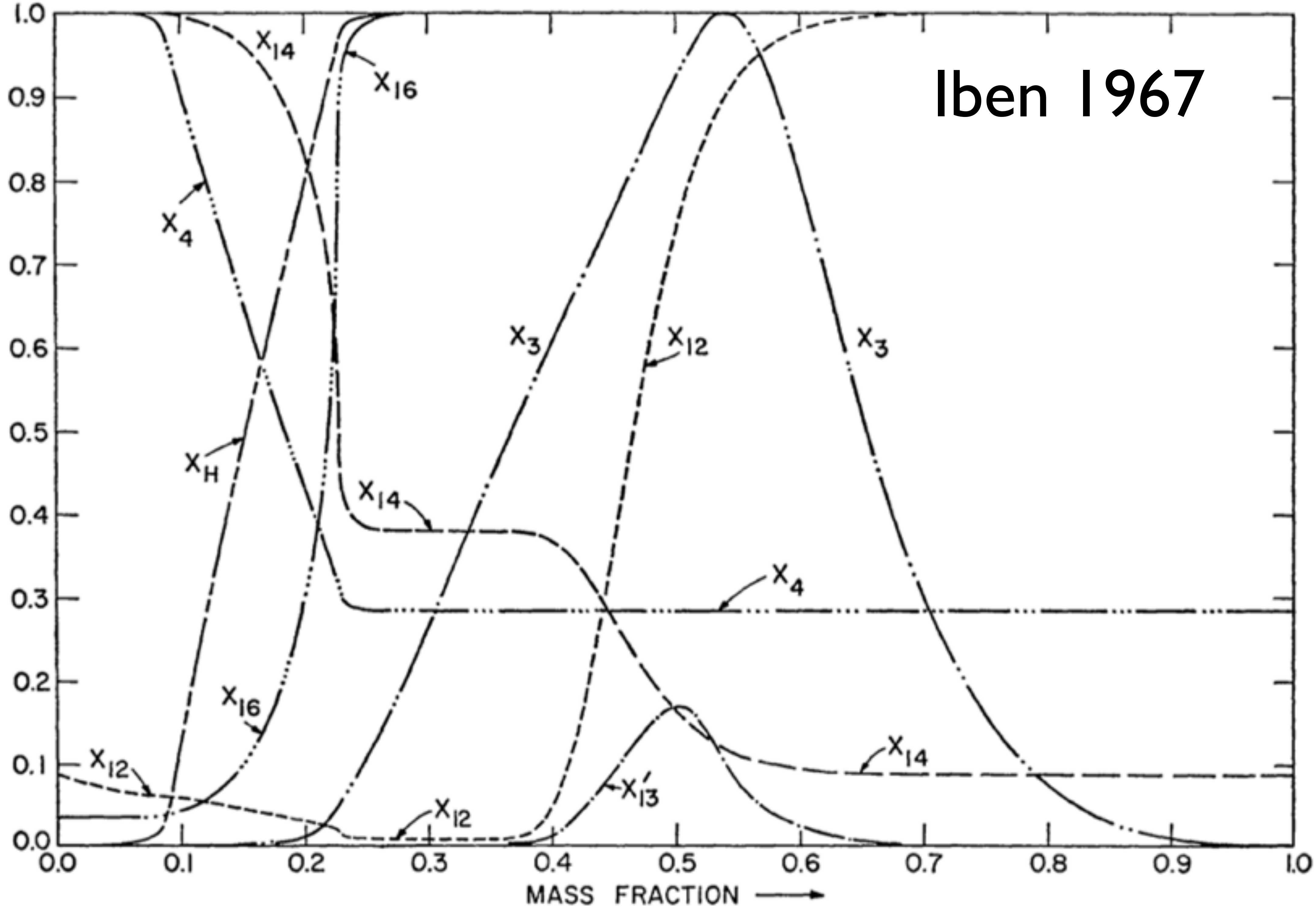


FIG. 2. The variation of composition with mass fraction in the $5M_\odot$ star immediately following the phase of overall contraction (point 3 in Figure 1). The X_i are abundances by mass of $H^1(X_H)$, $He^3(X_3)$, $He^4(X_4)$, $C^{12}(X_{12})$, $C^{13}(X'_{13})$, $N^{14}(X_{14})$, and $O^{16}(X_{16})$. Scale limits correspond to $0.0 \leq X_H < 0.708$, $0.0 \leq X_3 \leq 1.30 \times 10^{-4}$, $0.0 \leq X_4 \leq 0.976$, $0.0 \leq X_{12}$, $X'_{13} \leq 3.61 \times 10^{-3}$, $0.0 \leq X_{14} \leq 1.45 \times 10^{-2}$, and $0.0 \leq X_{16} \leq 1.08 \times 10^{-2}$.

The Subgiants

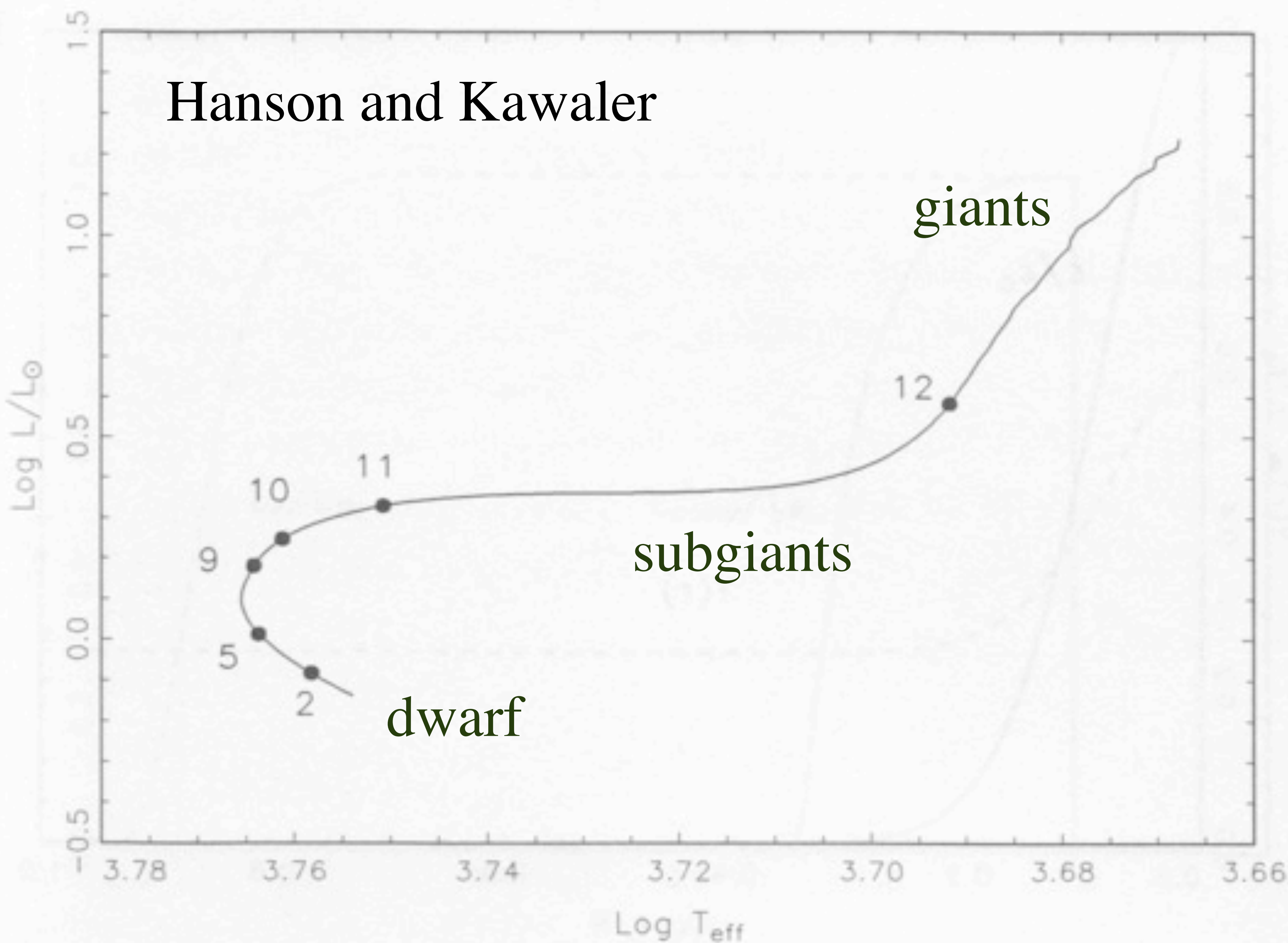


FIGURE 8.3. A solar model evolution track in the Hertzsprung–Russell diagram from the ZAMS to the red giant stage. Elapsed evolutionary time from the ZAMS is indicated by the filled circles where the units are Gyr.

In outer atmosphere (following Applegate 1988):

$$\frac{\frac{dP}{dr} = -\frac{GM\rho}{r^2}}{\frac{dT}{dr} = -\frac{L}{4\pi r^2 K}}$$

and

$$P = \frac{\rho k T}{\mu m_H}$$

$$\frac{dlnP}{dlnT} = n + 1 = 4\pi \left(\frac{\mu m_H}{k}\right) \frac{GMK}{L}$$

$P \propto T^{n+1}$

Where $K = \frac{4acT^3}{3\kappa\rho}$ and $\kappa = \kappa_0 \rho T^{-7/2}$ (Kramers Opacity)

Setting $M = constant$ and substituting in for $K = K_0 T^{13/2} \rho^{-2}$, $n = 3.25$

This means derivation only good in outer atmosphere

$$\rho = \left[\frac{16\pi}{17} \frac{GMK_0}{L} \left(\frac{\mu m_H}{k}\right)\right]^{1/2} T^{13/4}$$

n = 3.25

$$K(\rho, T) = \frac{17}{16\pi} \left(\frac{k}{\mu m_H}\right) \frac{L}{GM}$$

Inside star, assume limiting power-law solution

Let's first assume $M_e >> M_c$, that $R_c << R_e$ and all the luminosity comes from a central point. Now consider a power law solution:

$$P = P_c \left(\frac{r}{R_c}\right)^{-a}, \; T = T_c \left(\frac{r}{R_c}\right)^{-b} \tag{19}$$

$$\rho = \rho_c \left(\frac{r}{R_c}\right)^{-c}, \; M = M_c \left(\frac{r}{R_c}\right)^{-d} \tag{20}$$

For $r > R_c$ and $M > M_c$

$a = 42/11, b = 10/11, c = 32/11, d = 1/11$ and $n = c/b = 32/10 = 3.2$.

$$\rho_c = \frac{1}{44\pi} \frac{M_c}{R_c^3} \tag{21}$$

$$T_c = \frac{11}{42} \left(\frac{GM_c}{R_c}\right) \left(\frac{\mu m_H}{k}\right) \tag{22}$$

$n+1 = a/b \;\; \Rightarrow n = 3.2$

Applegate 1988

n+1 = a/b => n = 3.2

$$\frac{dlnP}{dlnT} = n + 1 = 4\pi \left(\frac{\mu m_H}{k}\right) \frac{GMK}{L}$$

$$\rho_c = \frac{1}{44\pi} \frac{M_c}{R_c^3}$$

$$K = K_0 T^{13/2} \rho^{-2}$$

$$T_c = \frac{11}{42} \left(\frac{GM_c}{R_c}\right) \left(\frac{\mu m_H}{k}\right)$$

This would result in a luminosity of:

$$L_{PL} = 9.443 K_0 \left(\frac{G\mu m_H}{k}\right)^{15/2} \frac{M_c^{11/2}}{R_c^{1/2}}$$

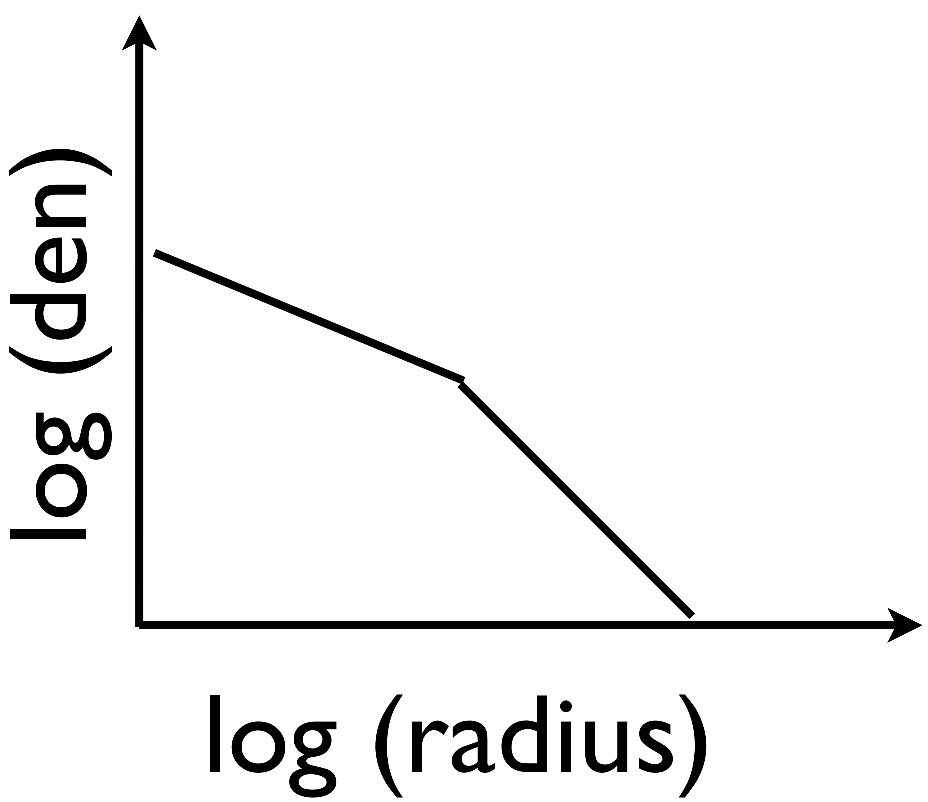
$$L_{pl} = 2490 \left(\frac{M_c}{M_\odot}\right)^{11/2} \left(\frac{R_\odot}{R_c}\right)^{1/2}$$

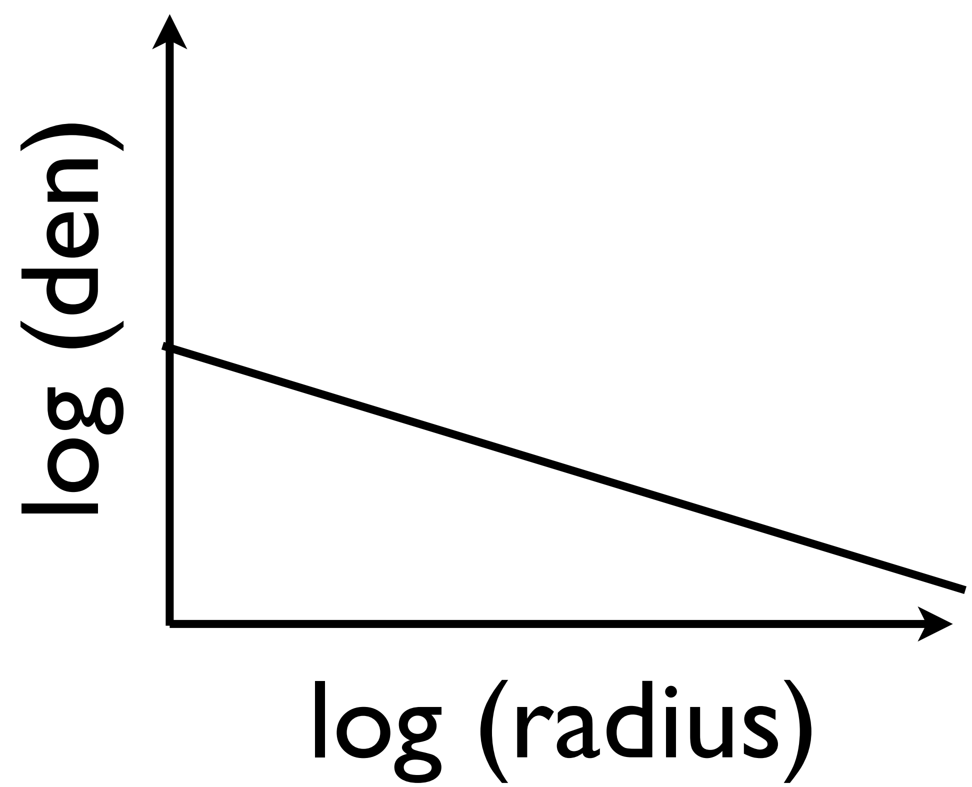
Applegate 1988

This is a maximum luminosity:

Upper envelope of Author's calculated models:

Power-law most extended star (literally infinite):





More extended stars means weaker gravity => lower overall density => lower overall opacity

Applegate 1988

TABLE 2

DETAILS OF THE $M_c = 0.3\ M_\odot$ MAIN-SEQUENCE MODEL

log R	M	log K	log T	log ρ	n
10.842......	1.000	15.118	3.748	−8.856	3.250
10.733.....	0.998	15.118	5.982	−1.594	3.248
10.660......	0.988	15.121	6.241	−0.756	3.236
10.551......	0.935	15.140	6.499	0.074	3.180
10.442......	0.816	15.186	6.685	0.657	3.054
10.332......	0.641	15.268	6.828	1.078	2.847
10.223......	0.455	15.384	6.927	1.374	2.566
10.114......	0.300	15.523	7.020	1.576	2.244

NOTE.—All quantities in cgs except M in $M_\odot$.

TABLE 3

DETAILS OF THE $M_c = 0.3\ M_\odot$ MAXIMUM LUMINOSITY MODEL

log R	M	log K	log T	log ρ	n
24.000......	1.000	15.981	−6.634	−43.030	3.250
23.306......	0.999	15.982	−5.940	−40.774	3.250
21.917......	0.995	15.983	−4.553	−36.266	3.249
20.181......	0.982	15.989	−2.822	−30.642	3.247
18.089......	0.938	16.008	−0.754	−23.933	3.242
16.016......	0.827	16.061	1.284	−17.334	3.230
14.627......	0.705	16.130	2.614	−13.048	3.218
13.586......	0.598	16.200	3.592	−9.904	3.210
12.544......	0.493	16.283	4.556	−6.813	3.204
11.503......	0.400	16.373	5.510	−3.757	3.201
10.461......	0.323	16.467	6.459	−0.718	3.200
10.114......	0.300	16.499	6.775	0.293	3.200

NOTE.—All quantities in cgs except M in $M_\odot$.

Applegate 1988

TABLE 1

PARAMETERS OF ENVELOPE MODELS

$M_c/M_\odot$	$R_c/10^{10}$ cm	$L_{max}/L_\odot$	L_{max}/L_*
0.2	1.0	0.9	2370
0.3	1.3	7.3	2370
0.4	1.5	33	2370
0.5	1.7	114	2550
0.6	2.0	328	2920
0.7	2.3	940	3850
0.8	2.6	3130	6530
0.9	3.2	16500	20000
0.99	4.8	1.8×10^6	1.58×10^6

NOTE.—$L_* = 1(M_c/M_\odot)^{5.5}(R_\odot/R_c)^{0.5}\ L_\odot$.

There is a large pressure drop outside the core:

Assume most of the mass in the core, and $R_c << R_e$

$$\frac{dT}{dr} = -\frac{L}{4\pi r^2 K} \qquad K(\rho, T) = \frac{17}{16\pi}\left(\frac{k}{\mu m_H}\right)\frac{L}{GM}$$

imply:

$$T(r) = \frac{4}{17}\frac{GM\mu m_H}{k}\left(\frac{1}{r} - \frac{1}{R_e}\right)$$

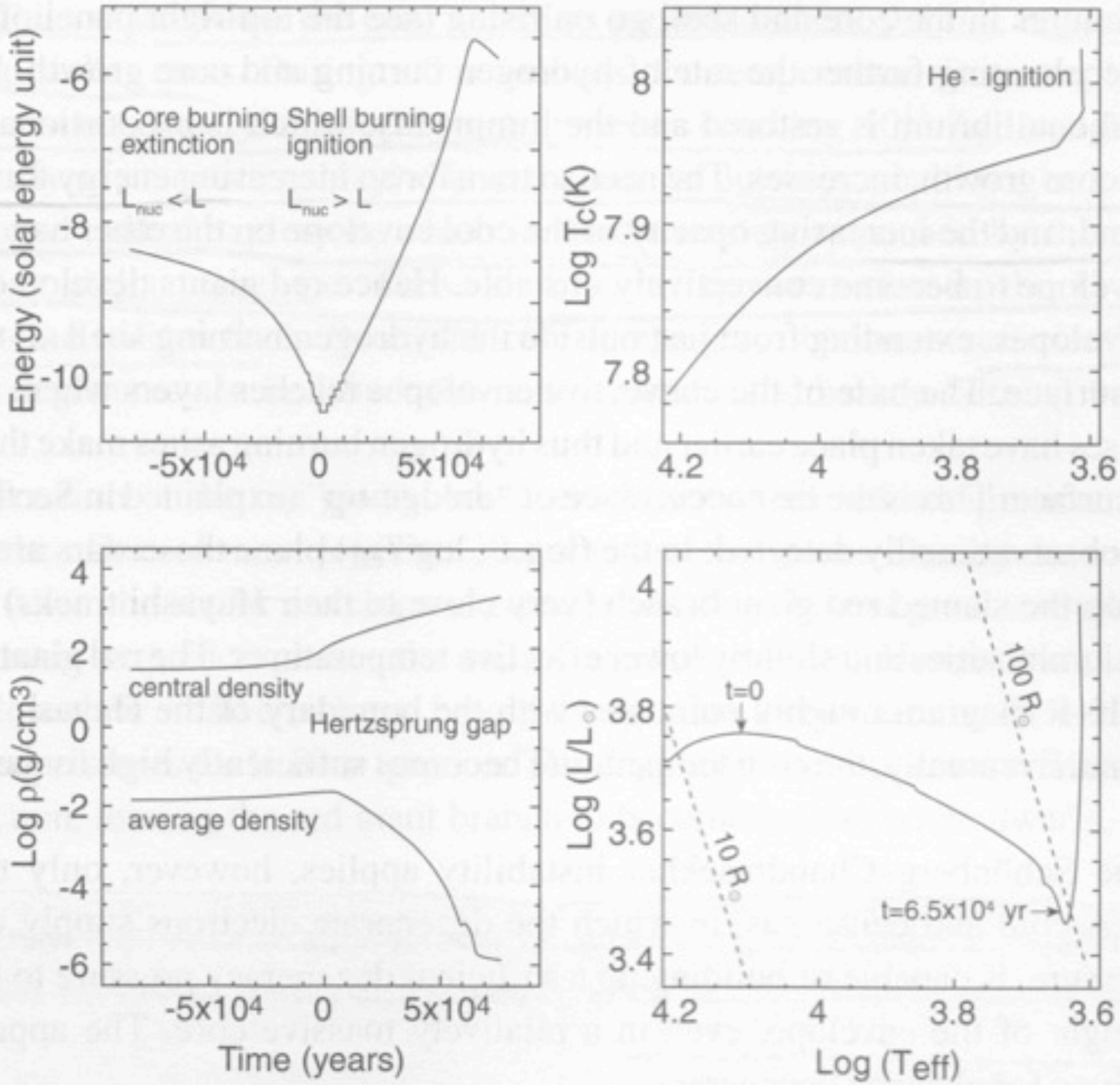
Note: T(r) only dependent on Mass (mainly core mass)

$$\rho = \left[\frac{16\pi}{17}\frac{GMK_0}{L}\left(\frac{\mu m_H}{k}\right)\right]^{1/2} T^{13/4}$$ then implies

$$\rho(r) = 0.0156\left(\frac{K_0}{L}\right)^{1/2}\left(\frac{GM\mu m_H}{k}\right)^{15/4}\left(\frac{1}{r} - \frac{1}{R_e}\right)^{13/4}$$

As luminosity grows, density drops

Applegate 1988



Convective Track

Figure 8.6 Evolution of an intermediate mass star ($7M_\odot$) during the crossing of the *Hertzsprung gap*: *top left*: total energy as a function of time (the time is arbitrarily set to zero at the onset of core contraction); *bottom left*: central density and average density ($3M/4\pi R^3$) as a function of time; *bottom right*: evolutionary track in the H–R diagram (where lines of equal radius are marked); *top right*: changing of central temperature with effective temperature.

Prialnik

Sunday, May 1, 2011

Onset of Convection:

Convection occurs when the temperature gradient required for radiative transport becomes steeper than that for an adiabatic equation of state:

For adiabatic gas: $P = K\rho^{\gamma_a}$ γ_a= 5/3 for monotonic gas

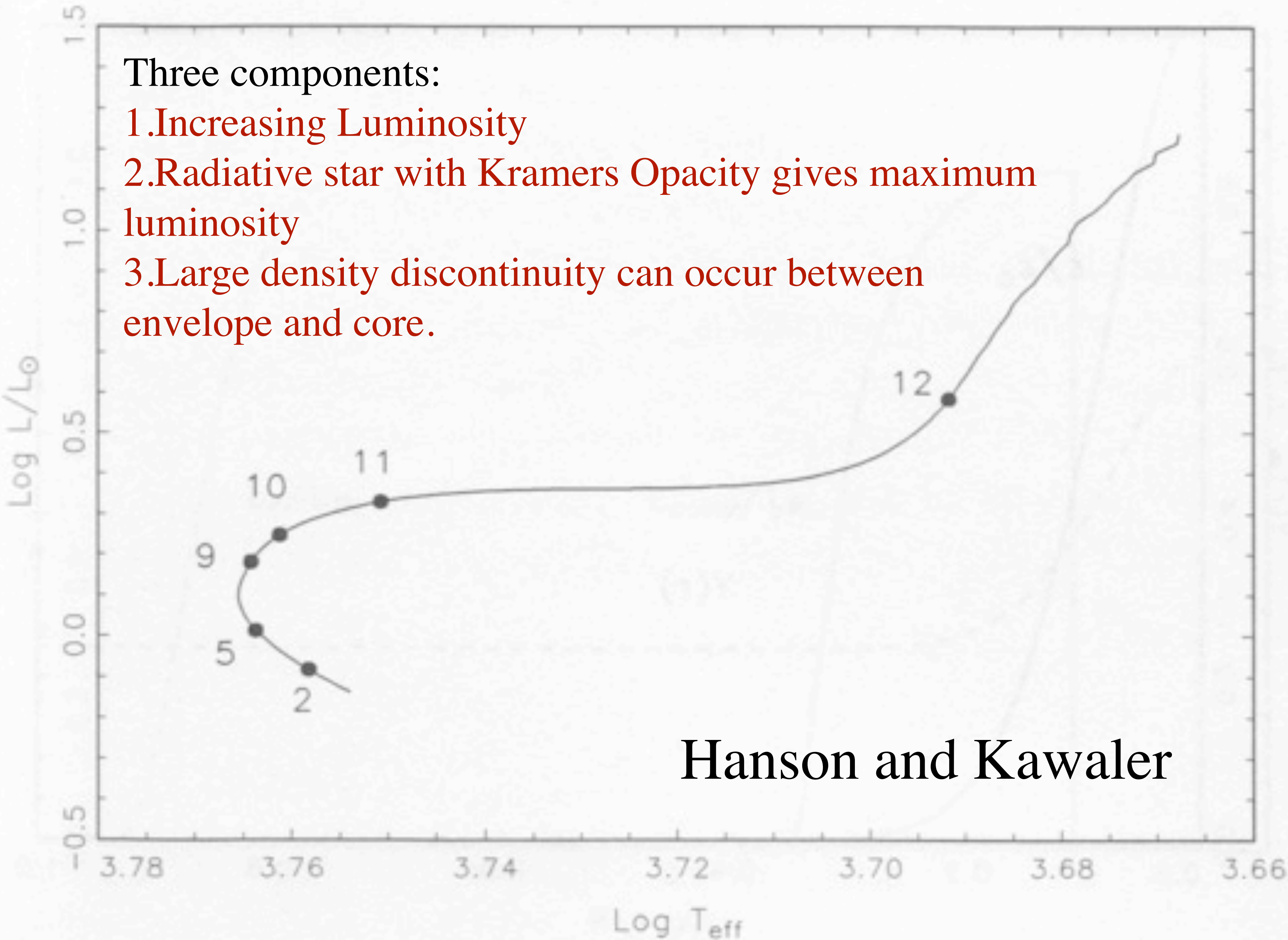
Convective Stability Requires:

$$\gamma \equiv \frac{\rho}{P}\left(\frac{dP}{d\rho}\right)_{star} < \gamma_a \qquad \frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T}$$

$$\frac{P}{T}\left(\frac{dT}{dP}\right)_{star} < \frac{\gamma_a - 1}{\gamma_a} \qquad \boxed{\left|\frac{dT}{dr}\right| < \left(\frac{\gamma_a - 1}{\gamma_a}\right)\frac{T}{P}\left|\frac{dP}{dr}\right|_{star}}$$

Three components:

1. Increasing Luminosity
2. Radiative star with Kramers Opacity gives maximum luminosity
3. Large density discontinuity can occur between envelope and core.



Hanson and Kawaler

FIGURE 8.3. A solar model evolution track in the Hertzsprung–Russell diagram from the ZAMS to the red giant stage. Elapsed evolutionary time from the ZAMS is indicated by the filled circles where the units are Gyr.

On the Red Giant Branch (same as for pre-ms stars)

For a convective atmosphere, we start by setting boundary condition at the photosphere.

$$\frac{dP}{dz} = -\rho g \qquad\qquad P_{eff} = \frac{2}{3}\frac{g}{\kappa_{rm}}$$

where $\kappa_{rm} = \kappa_0 \rho T_{eff}^a$

$$P_{eff} = \frac{2}{3}\frac{GM_\star}{R_\star^2 \kappa_0 \rho T_{eff}^a} = \frac{2}{3}\frac{G\rho 4\pi R_\star^3}{R_\star^2 3\kappa_0 \rho T_{eff}^a} = \frac{8}{9}\frac{R_\star}{\kappa_0 T_{eff}^a}$$

For a cool stellar atmosphere, the opacity decreases strongly with temperature due to H^- opacity:

a ~ 10, implies a small change in T_{eff} is a large change in P_{eff}: this is why the temperature doesn't change much.

Hartmann

For an adiabatic star (remember that $PT^{-5/2} = K_2$):

$$\frac{T_{eff}}{T_c} = \left(\frac{P_{eff}}{P_c}\right)^{-2/5}$$

By substituting the relationships for P_c, T_c, and P_{eff}, we get

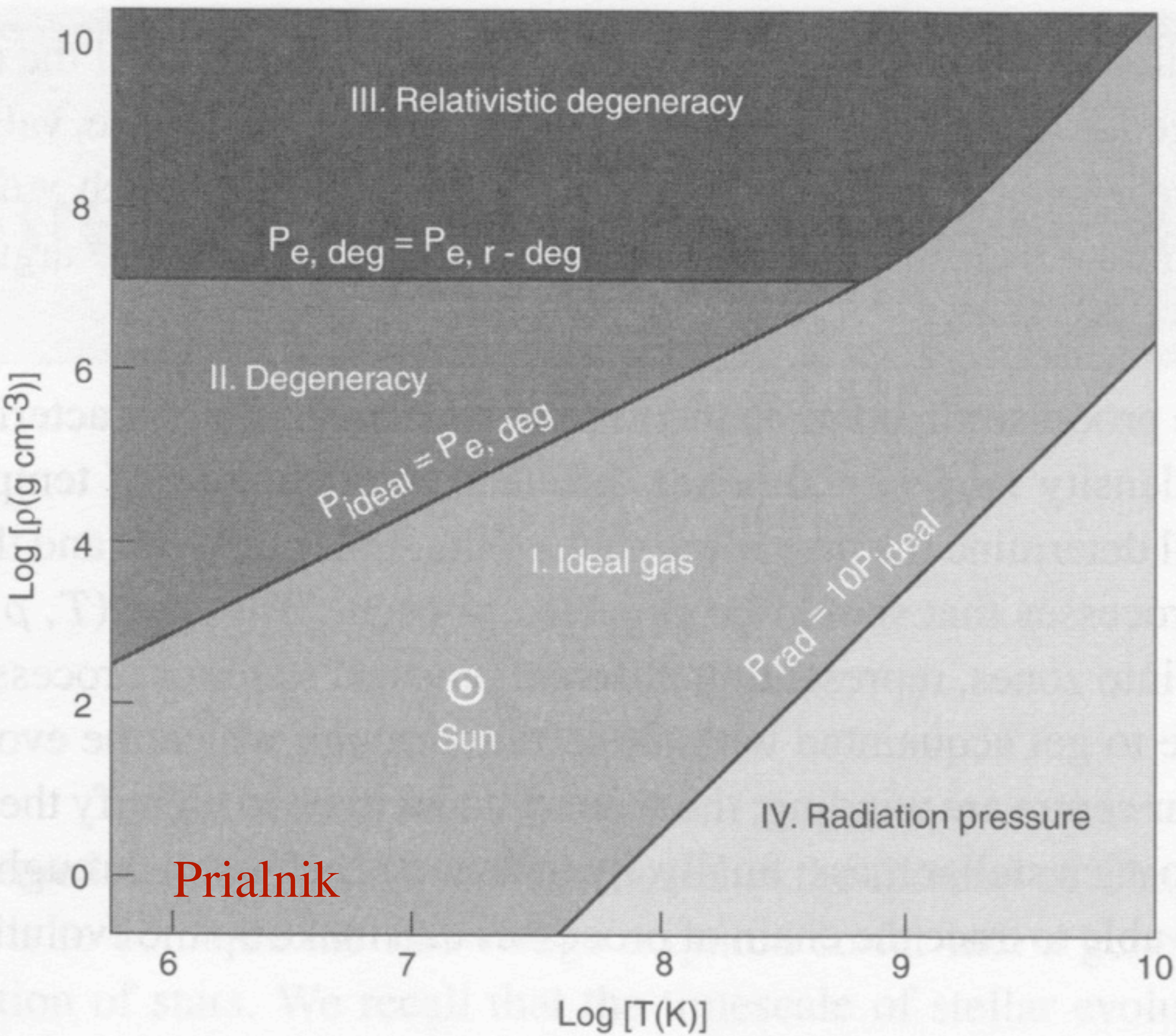
$$T_{eff} \propto R^{2.5/(2.5+a)} M^{0.5/(2.5+a)}$$

and

$$L_{eff} \propto R^{(15+2a)/(2.5+a)} M^{2/(2.5+a)}$$

Hartmann

What is the equation of state for the core?



Nuclear Burning: the triple α process

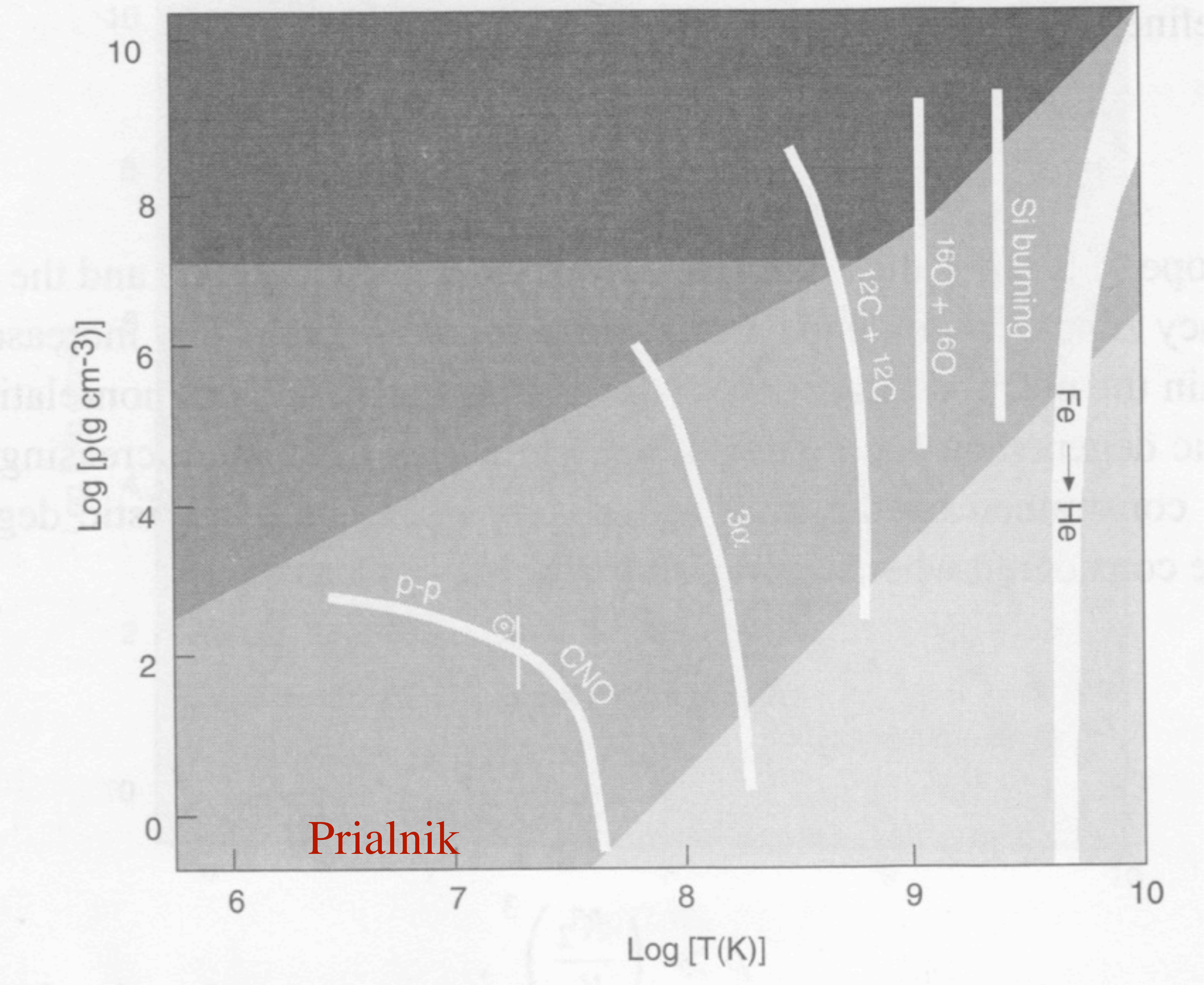
$^{4}He + ^{4}He \rightarrow ^{8}Be$

$^{8}Be + ^{4}He \rightarrow ^{12}C$

Releasing 7.275 Mev

$q = k \varrho^{2} T^{40}$

Nuclear Burning Sequences



What is the density of the core?

Hydrostatic Equilbrium gives:

$$P_c = K\frac{GM^2}{R^4} = K\left(\frac{4\pi}{3}\right)^{1/3} GM^{2/3}\rho^{4/3}$$

The Ideal Gas Law then Gives:

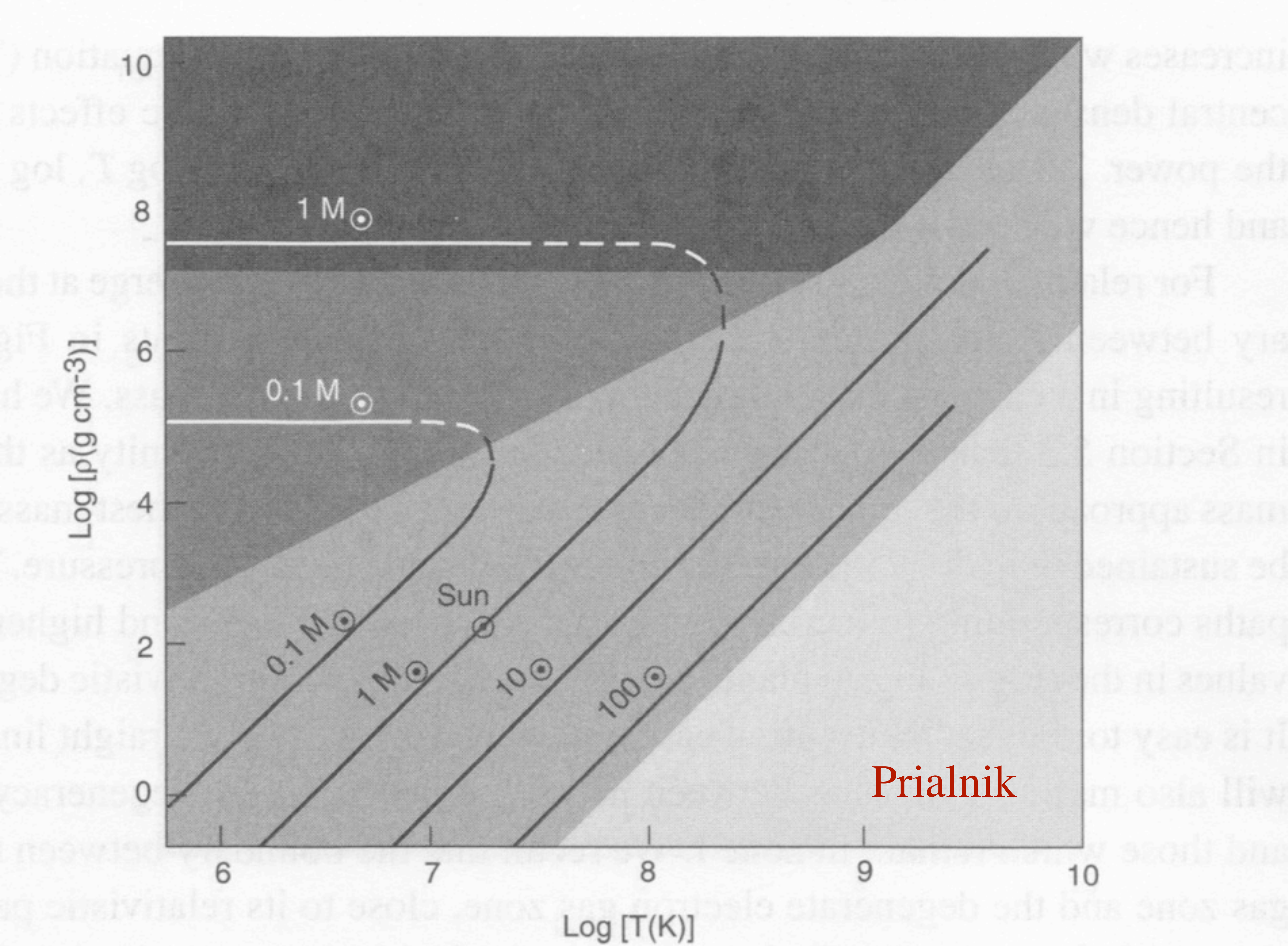
$$\rho_c = \frac{3}{4\pi}\left(\frac{k}{K_1 G\mu m_H}\right)^3 \frac{T_c^3}{M^2}$$

In contrast, non-relativistic degeneracy pressure gives:

$$P \propto \rho^{5/3}$$

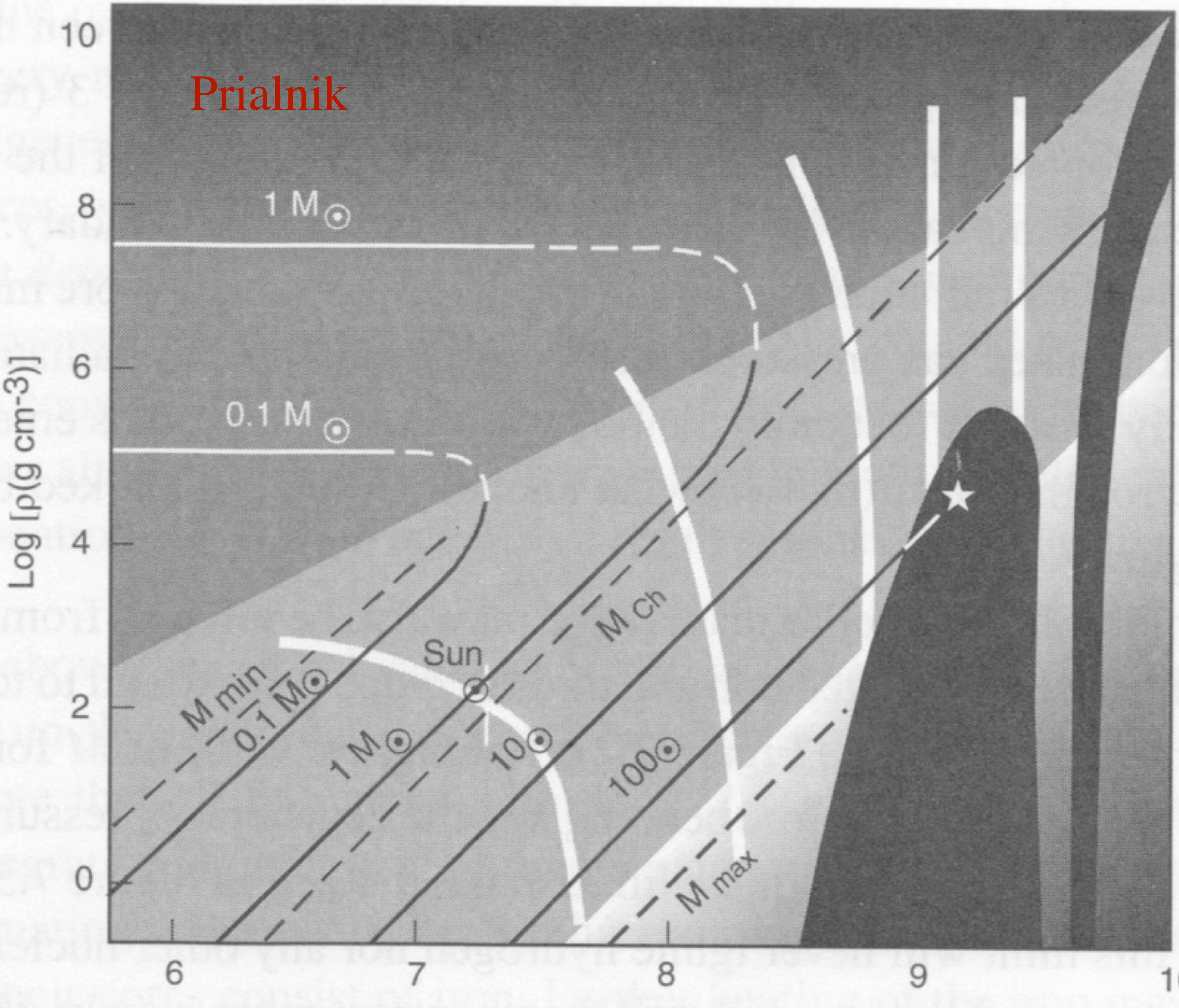
$$\rho_c = K_2 M^2$$

Evolution of Central Cores



Prialnik

When does degeneracy happen?



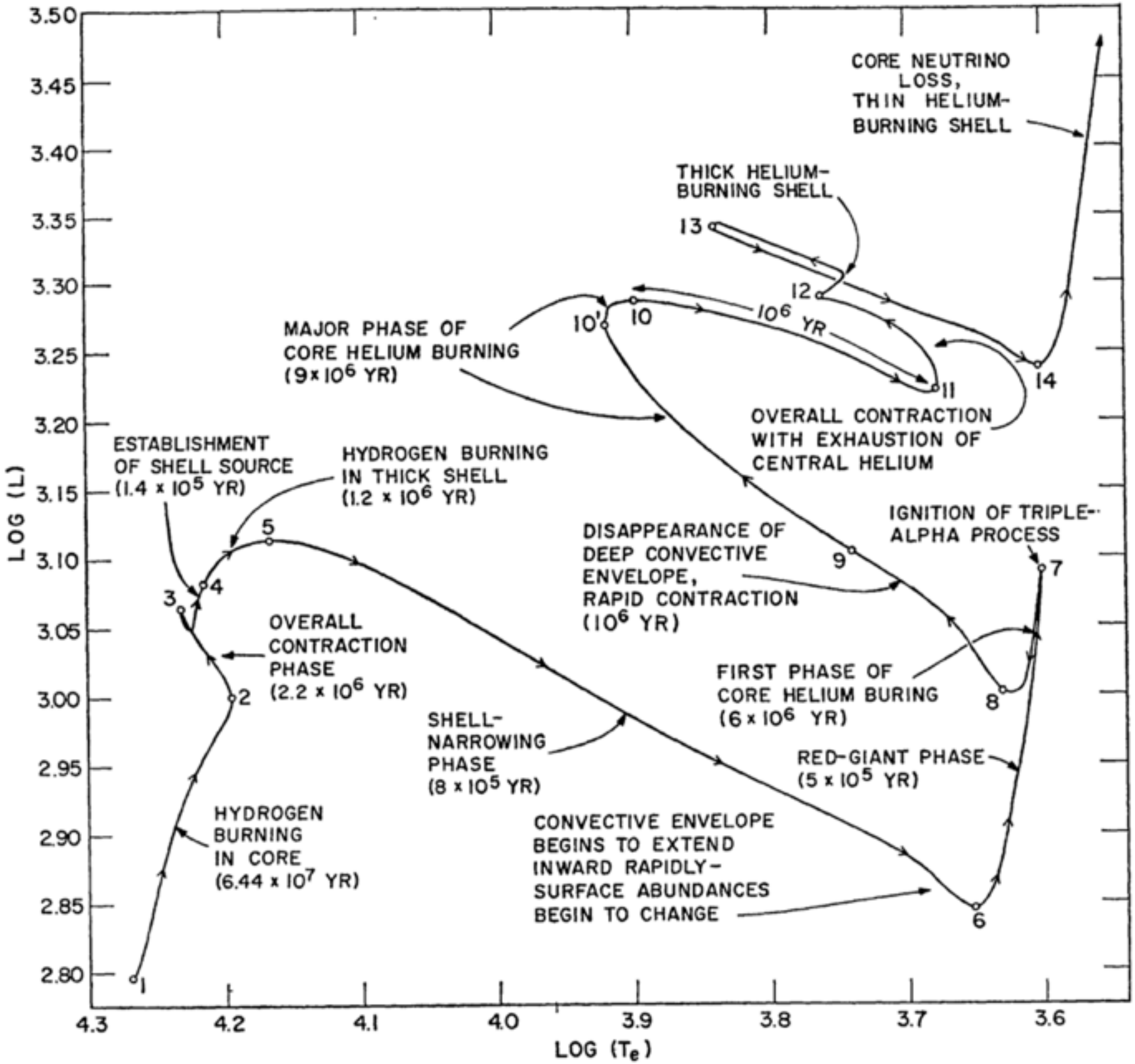


FIG. 1. The path of a metal-rich $5M_{\odot}$ star in the Hertzsprung-Russell diagram. Luminosity is in solar units, $L_{\odot}=3.86\times10^{33}$ erg/sec, and surface temperature T_e is in deg K. Traversal times between labeled points are given in years.

Iben 1967

Post-Main Sequence (so far)

- Once 10% of Hydrogen is converted into Helium, core stops producing enough nuclear energy and begins to contract.
- Hydrogen burning in shell surrounding core contributes increasing luminosity as core collapses.
- The envelope surrounding the core expands and cools to allow increased radiation (lower opacity).
- The star hits maximum allowed luminosity for a radiative star and becomes a convective star. It follows Hayashi-like track upward in luminosity as radius expands further.
- Helium flash occurs for stars with M < 2 Msun, as their cores become dominated by e^- degeneracy pressure before they start Helium burning.
- As Helium burning starts, star moves to the horizontal branch