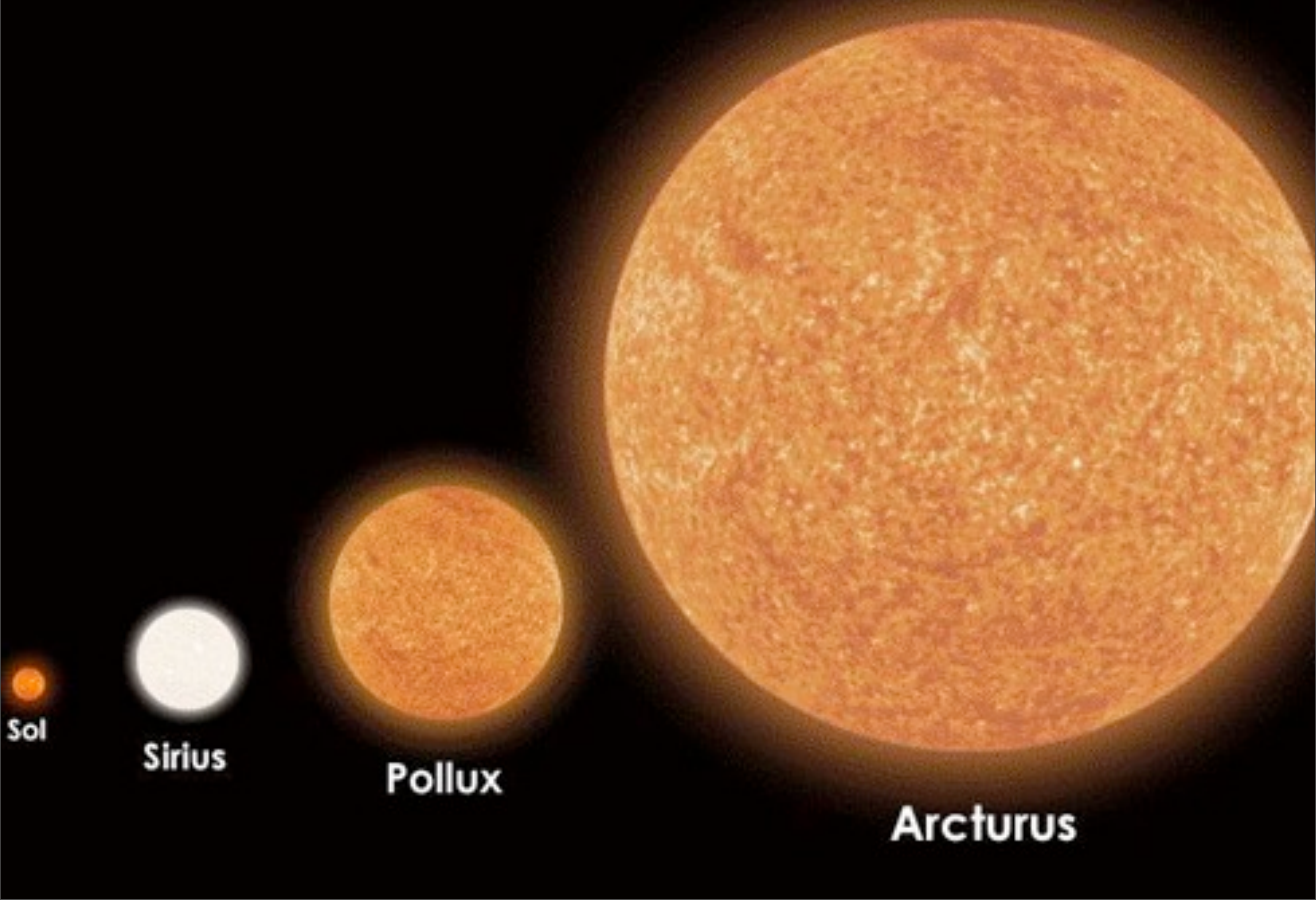
The Helium Flash and the Horizontal Branch



It's sooooo simple??

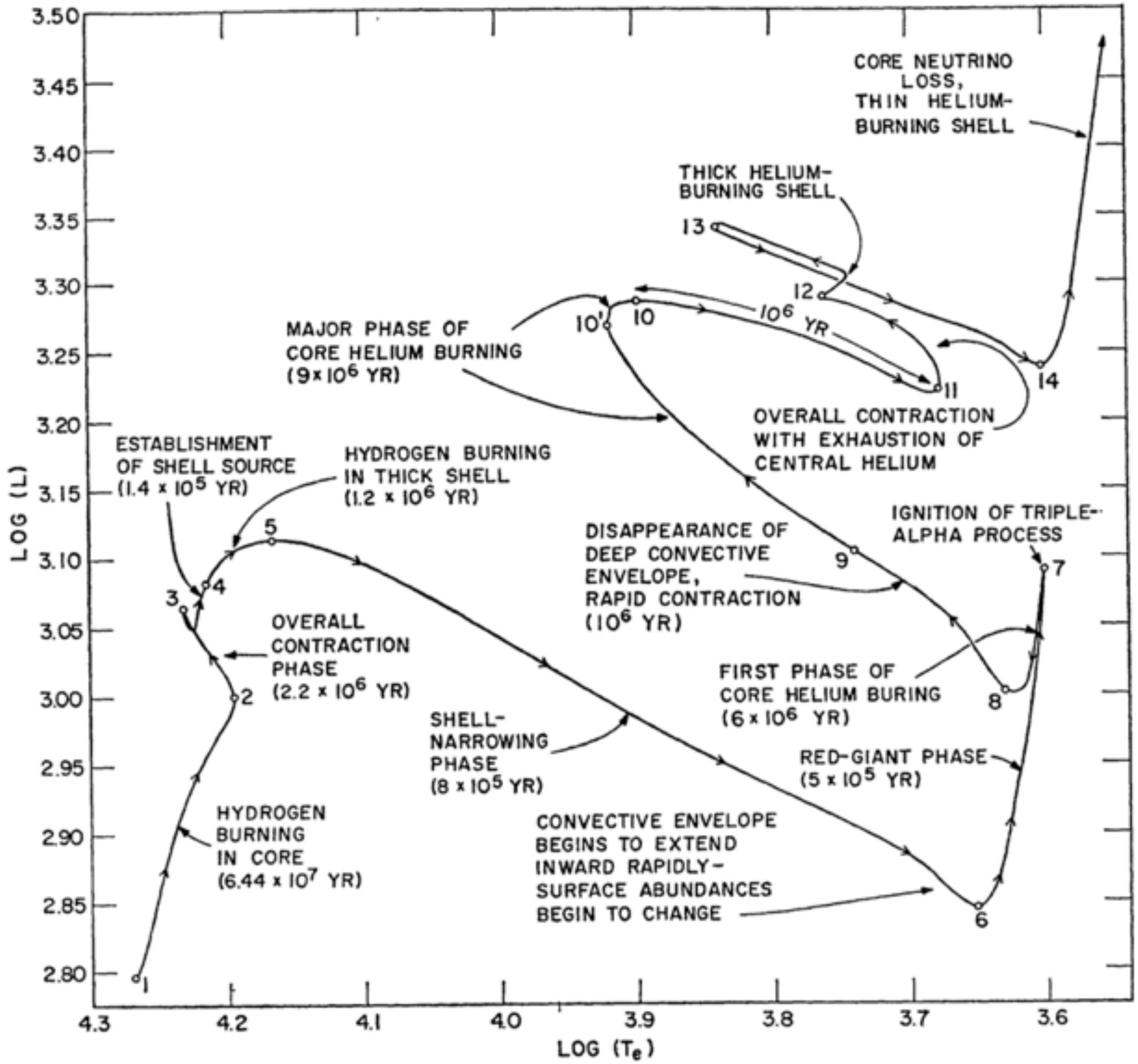
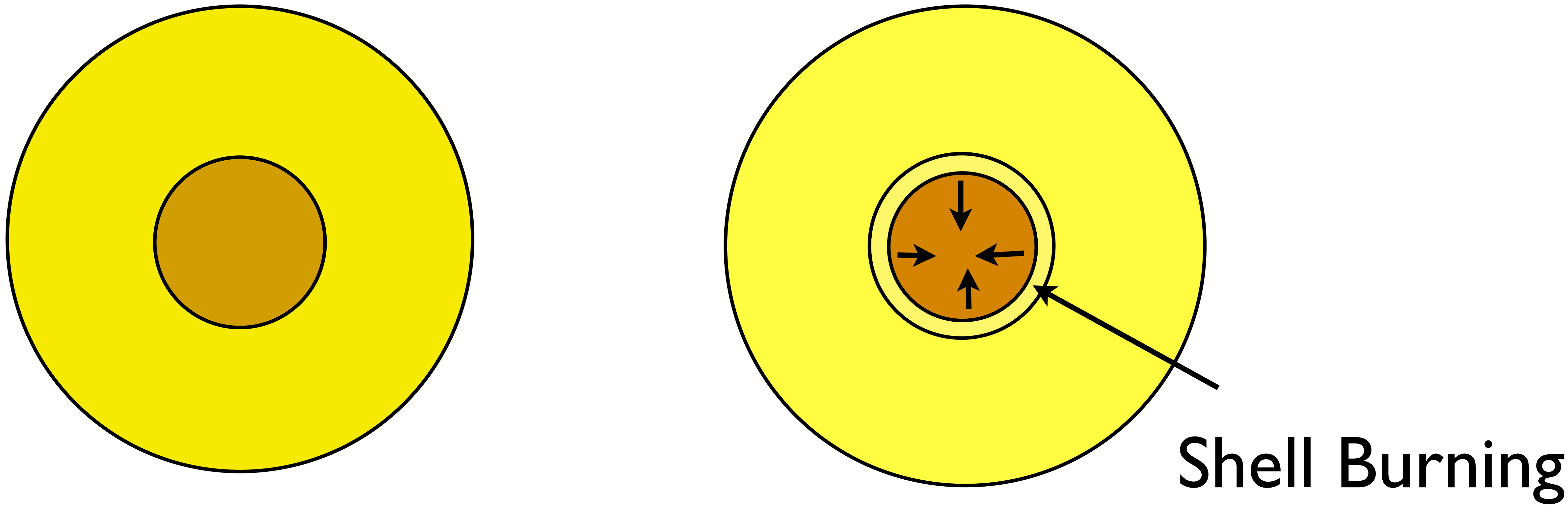


FIG. 1. The path of a metal-rich $5M_\odot$ star in the Hertzsprung-Russell diagram. Luminosity is in solar units, $L_\odot = 3.86 \times 10^{33}$ erg/sec, and surface temperature T_e is in deg K. Traversal times between labeled points are given in years.

Iben 1967

Leaving the Main Sequence

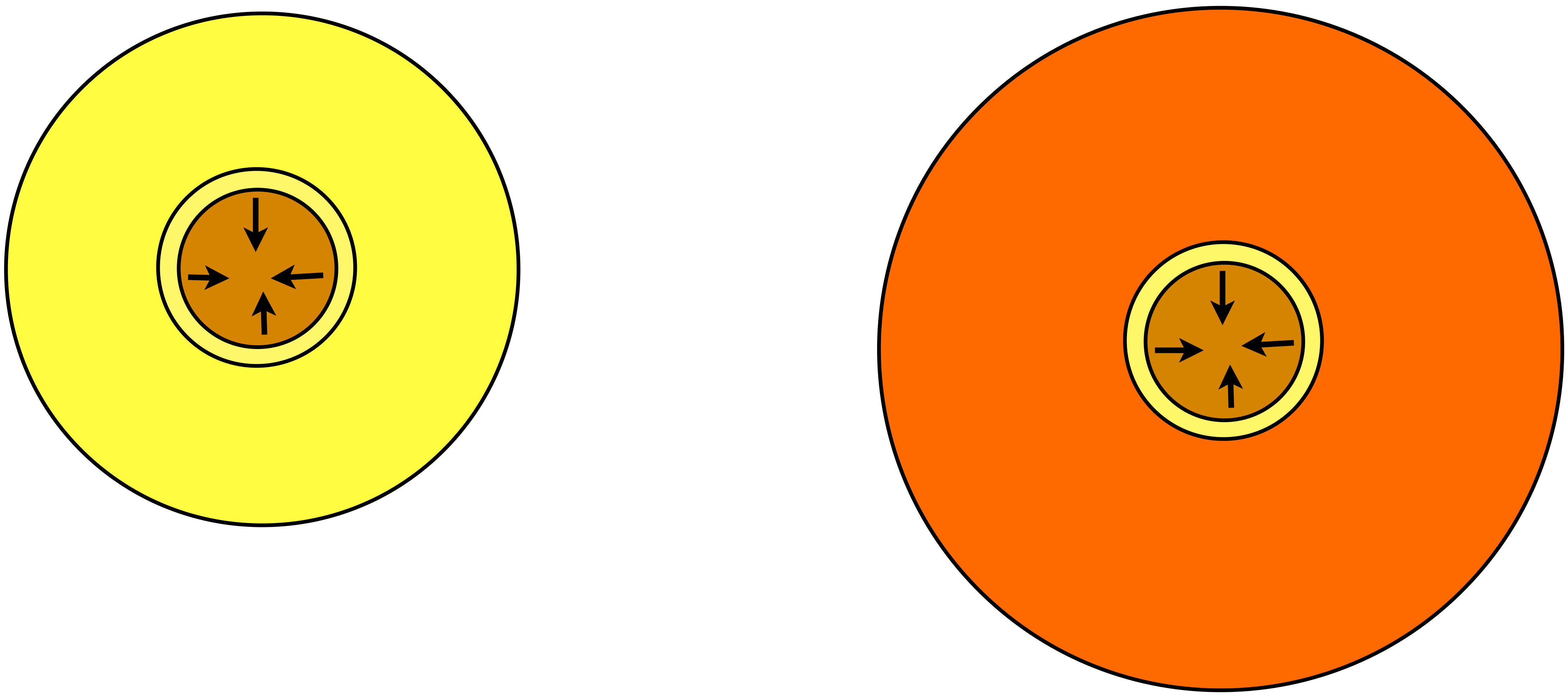
Central Helium rich core becomes gravitationally unstable and begins to contract.



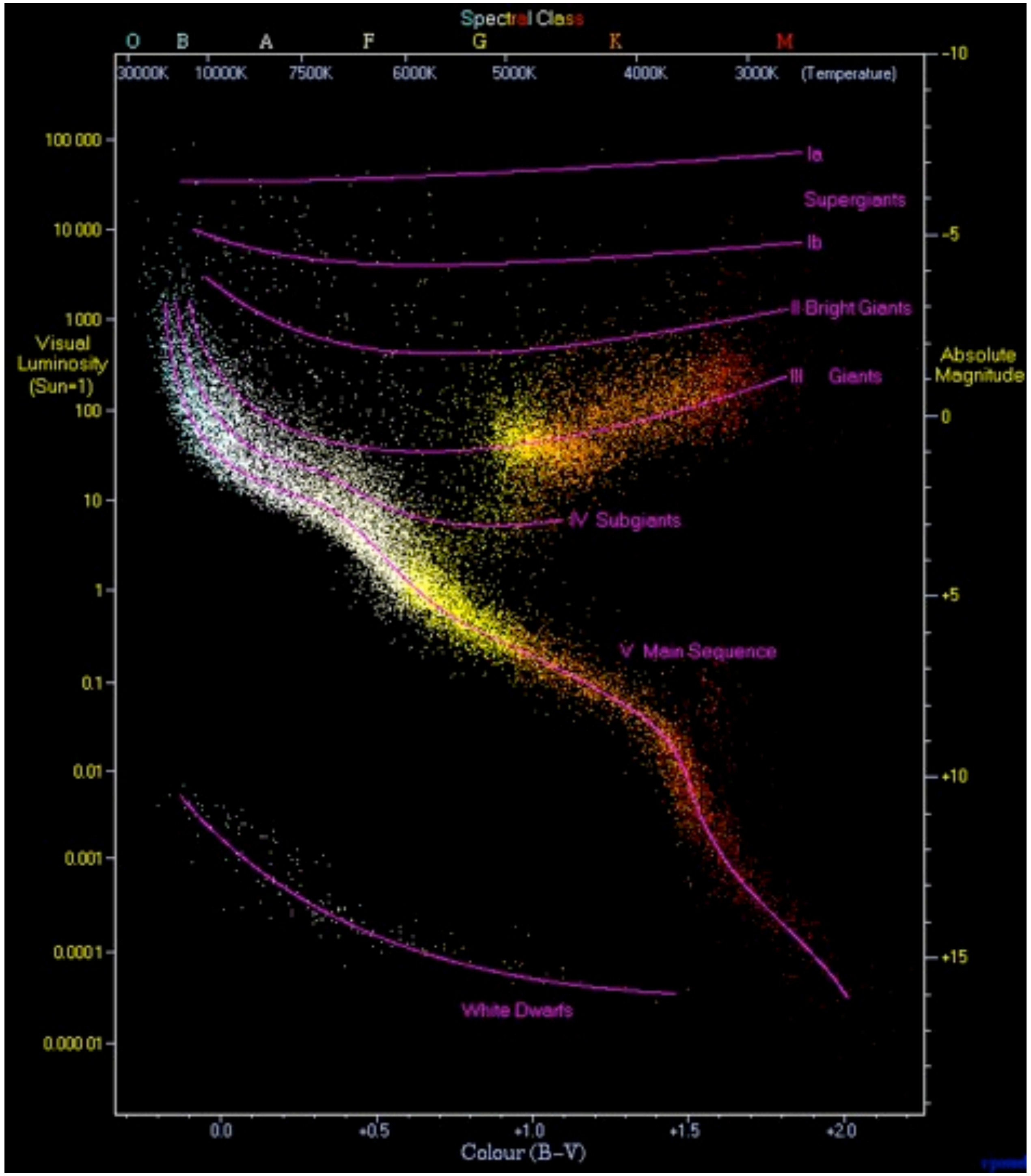
A shell of Hydrogen follows the contracting core, being compressed to higher densities and temperature. Fusion occurs in shell. Helium "ash" produced in shell falls into core.

The Sub-Giant Branch

Shell burning drives luminosity upward. Star too dense to transport energy. Energy absorbed in envelope, which expands.



Expansion facilitates more energy transport (Kramers law).



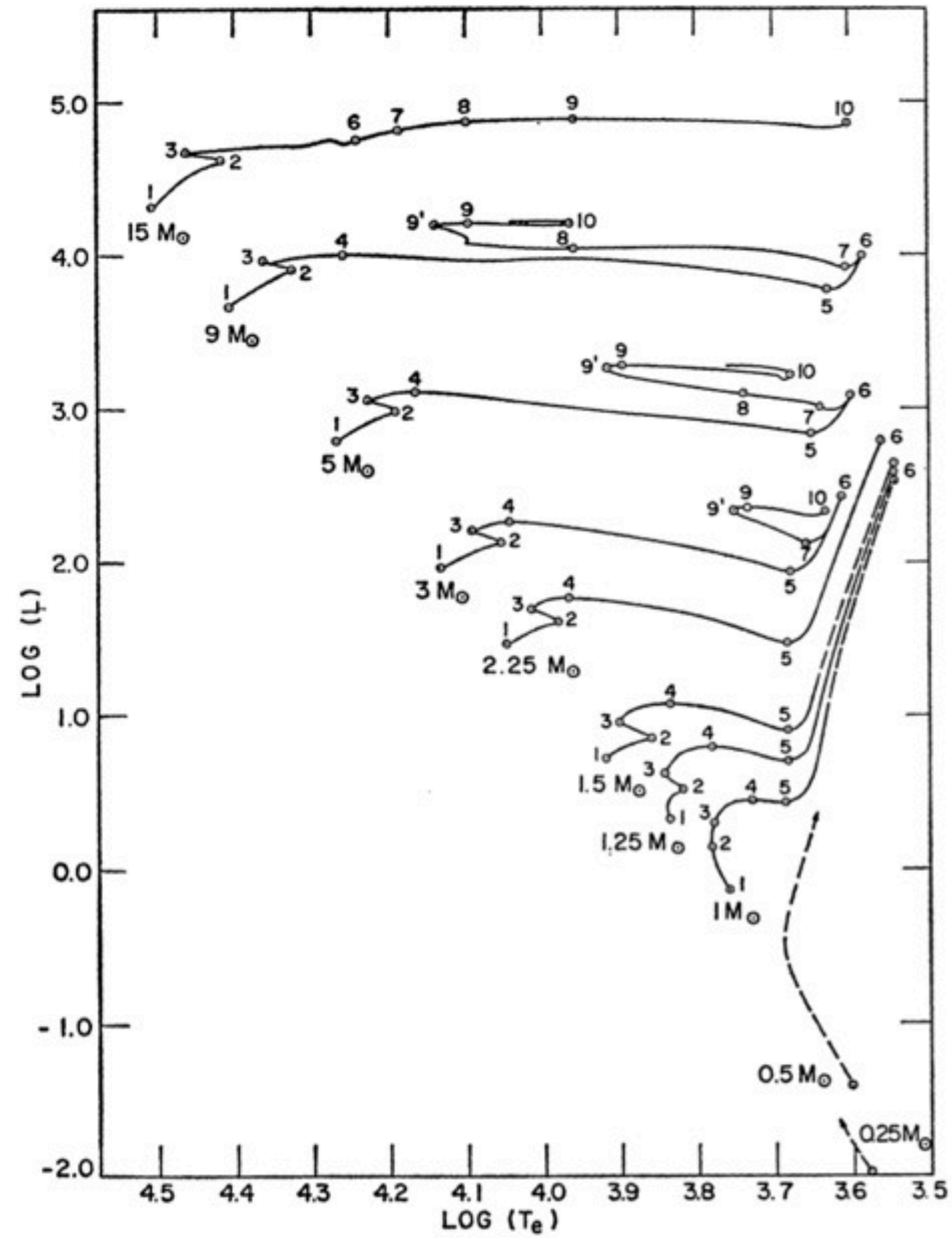


FIG. 3. Paths in the H-R diagram for metal-rich stars of mass $(M/M_\odot)=15$, 9, 5, 3, 2.25, 1.5, 1.25, 1, 0.5, 0.25. Units of luminosity and surface temperature are the same as in Figure 1. Traversal times between labeled points are given in Tables III and IV. Dashed portions of evolutionary paths are estimates.

http://www.atlasoftheuniverse.com/hr.html

http://stars.astro.illinois.edu/sow/hrd.html

There is a large pressure drop outside the core:

Assume most of the mass in the core, and $R_c << R_e$

$$\frac{dT}{dr} = -\frac{L}{4\pi r^2 K} \qquad K(\rho, T) = \frac{17}{16\pi}\left(\frac{k}{\mu m_H}\right)\frac{L}{GM}$$

imply:

$$T(r) = \frac{4}{17}\frac{GM\mu m_H}{k}\left(\frac{1}{r} - \frac{1}{R_e}\right)$$

Note: T(r) only dependent on Mass (mainly core mass)

$$\rho = \left[\frac{16\pi}{17}\frac{GMK_0}{L}\left(\frac{\mu m_H}{k}\right)\right]^{1/2} T^{13/4}$$ then implies

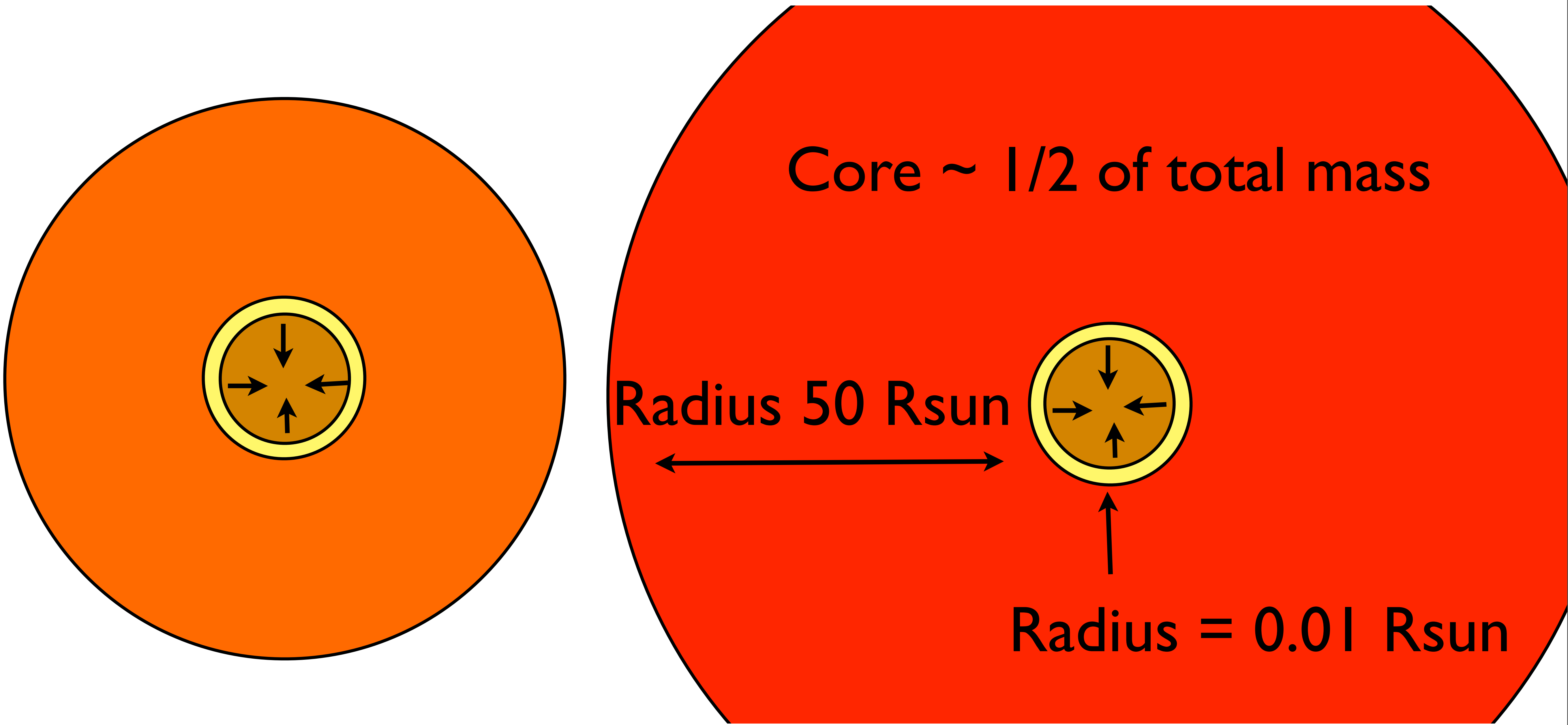
$$\rho(r) = 0.0156\left(\frac{K_0}{L}\right)^{1/2}\left(\frac{GM\mu m_H}{k}\right)^{15/4}\left(\frac{1}{r} - \frac{1}{R_e}\right)^{13/4}$$

As luminosity grows, density drops

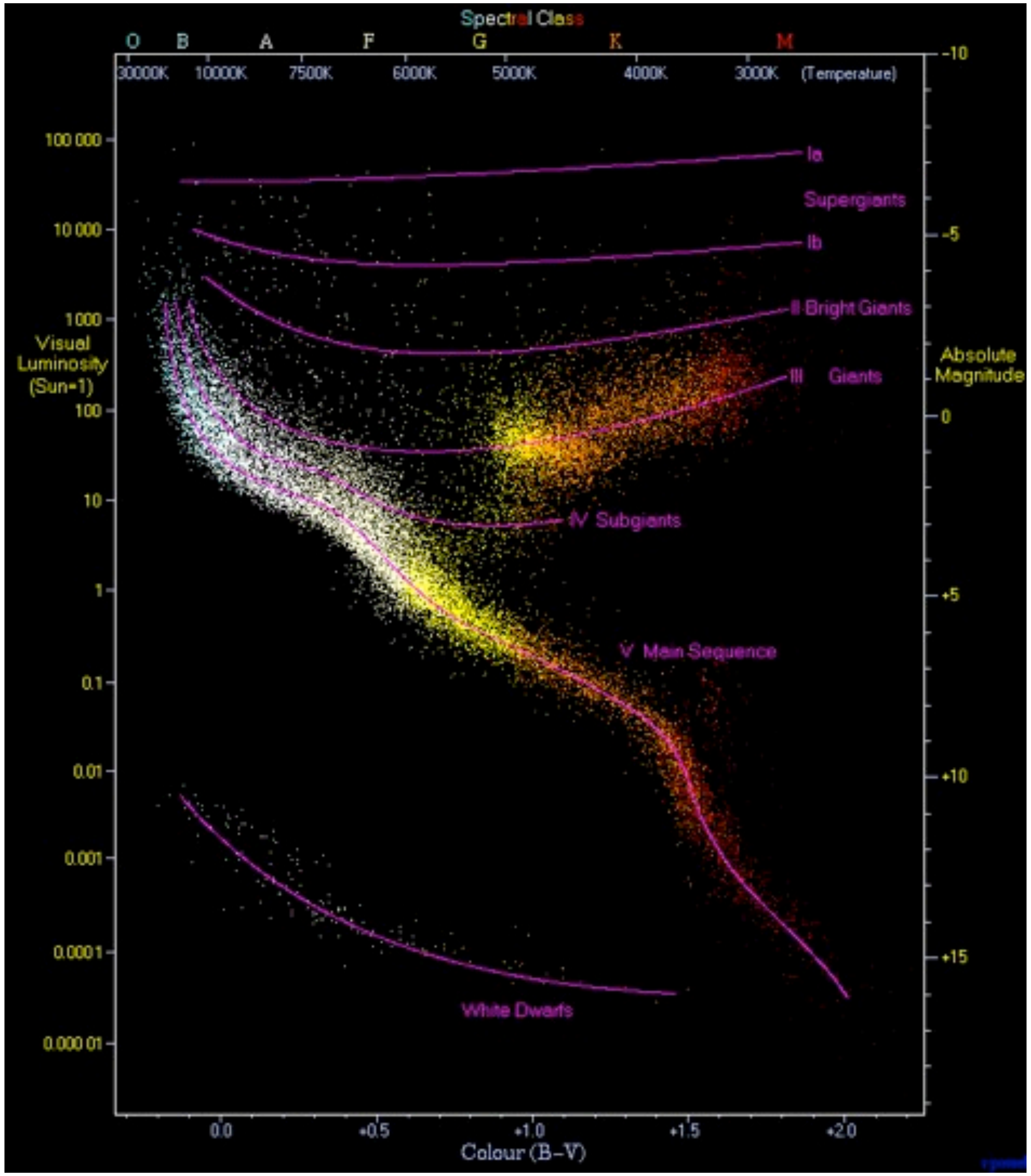
Applegate 1988

The Giant Branch

Convection turns on and the star ascends a Hayashi track to become a red giant. Mass loss may occur due to large size and luminosity.



Photospheric temperature relatively constant as radius and luminosity increase. The core becomes hotter and grows in mass as Helium ash continues to fall into the core.



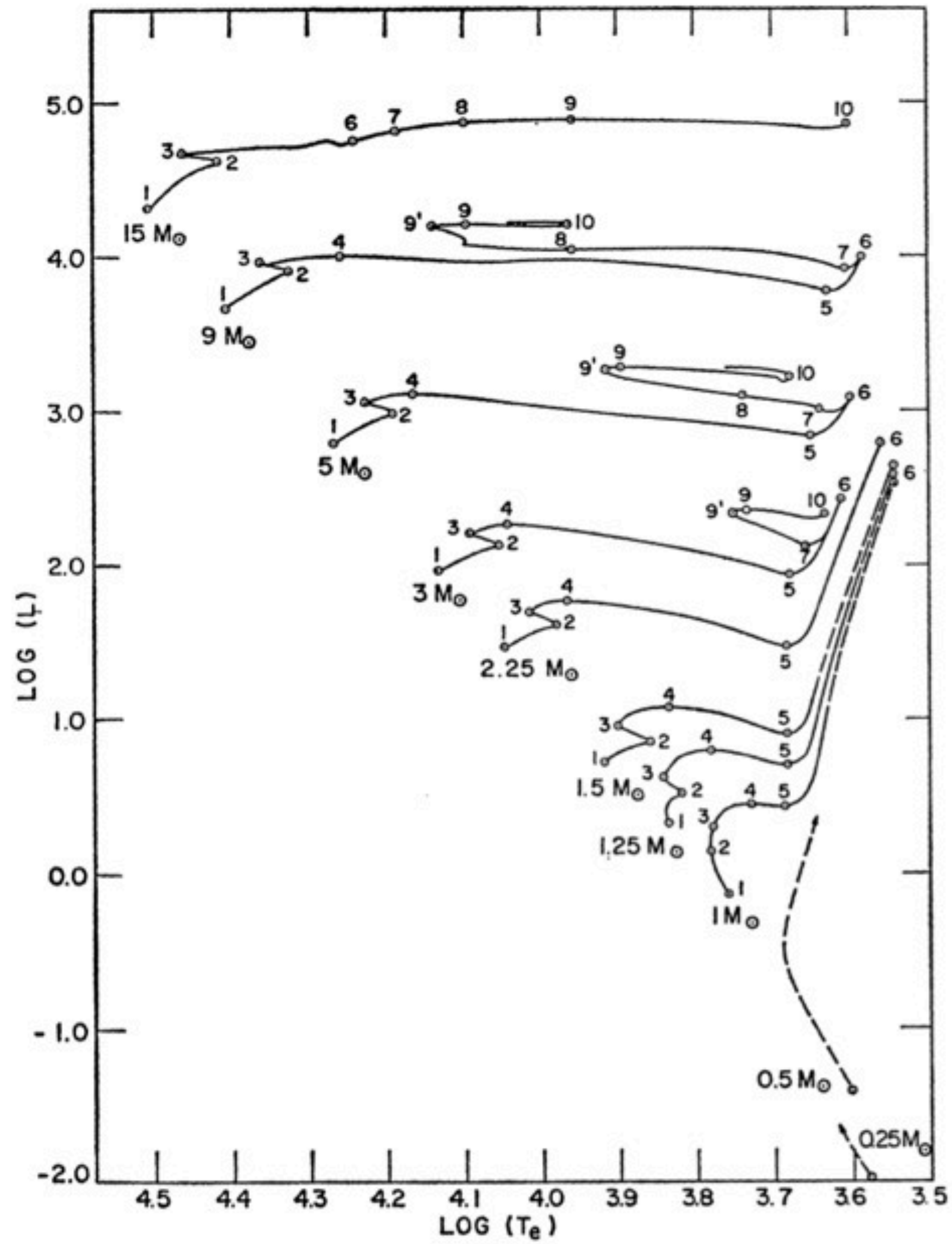


FIG. 3. Paths in the H-R diagram for metal-rich stars of mass $(M/M_\odot)=15$, 9, 5, 3, 2.25, 1.5, 1.25, 1, 0.5, 0.25. Units of luminosity and surface temperature are the same as in Figure 1. Traversal times between labeled points are given in Tables III and IV. Dashed portions of evolutionary paths are estimates.

http://www.atlasoftheuniverse.com/hr.html
http://stars.astro.illinois.edu/sow/hrd.html

On the Red Giant Branch (same as for pre-ms stars)

For a convective atmosphere, we start by setting boundary condition at the photosphere.

$$\frac{dP}{dz} = -\rho g \qquad\qquad P_{eff} = \frac{2}{3}\frac{g}{\kappa_{rm}}$$

where $\kappa_{rm} = \kappa_0 \rho T_{eff}^a$

$$P_{eff} = \frac{2}{3}\frac{G\rho M_\star}{R_\star^2 \kappa_0 \rho T_{eff}^a} = \frac{2}{3}\frac{G\rho 4\pi R_\star^3}{3R_\star^2 \kappa_0 \rho T_{eff}^a} = \kappa_0 R_\star T_{eff}^{-a}$$

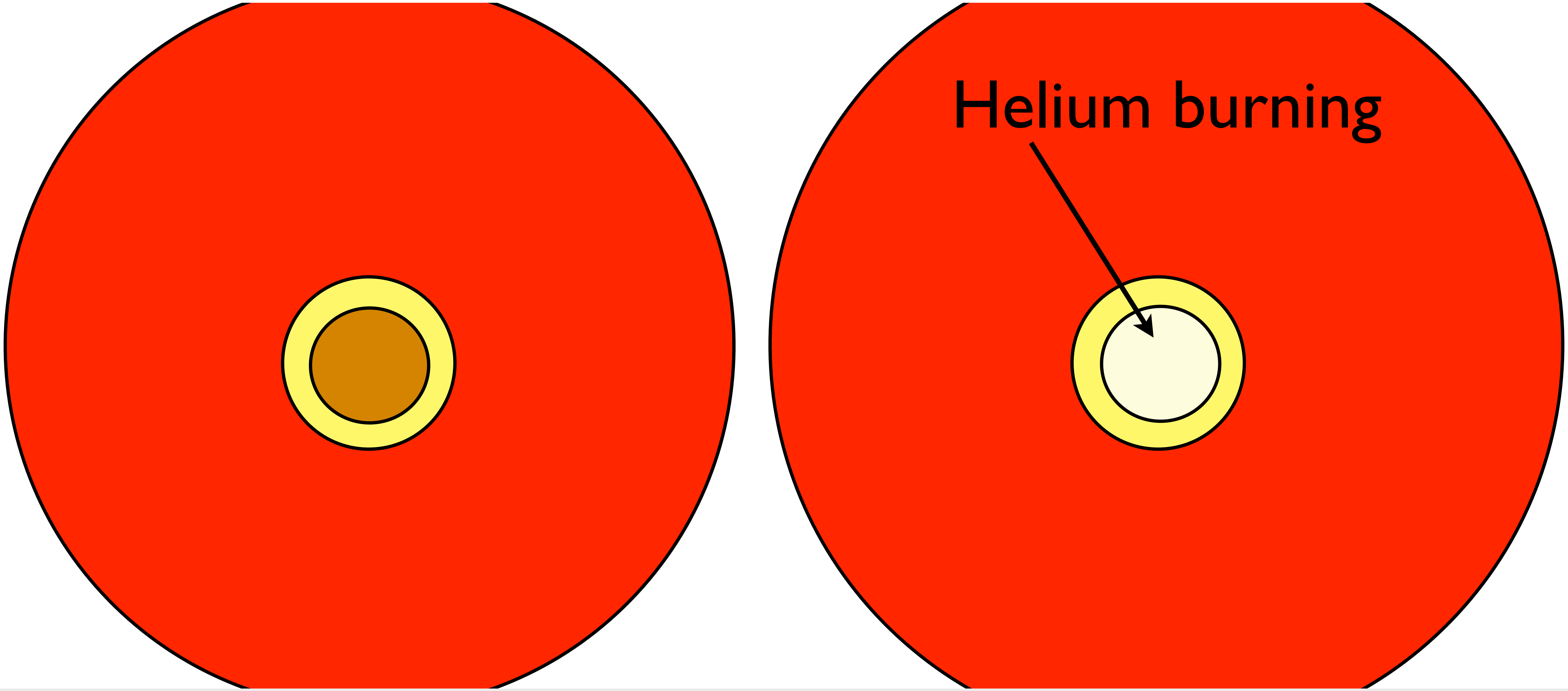
For a cool stellar atmosphere, the opacity decreases strongly with temperature due to H^- opacity:

a ~ 10, implies a small change in T_{eff} is a large change in P_{eff}: this is why the temperature doesn't change much.

Hartmann

The Helium Flash (< 2 Msun)

The central core becomes dense enough, that degeneracy pressure dominates over thermal pressure. Still, the core radiates energy and continues to contract, until Helium fusion occurs.



Once He fusion stars, temperature goes up, but core doesn't expand immediately. This creates a brief burst of energy, much of which goes into inflating the core and star..

Pressure in Degenerate Gas

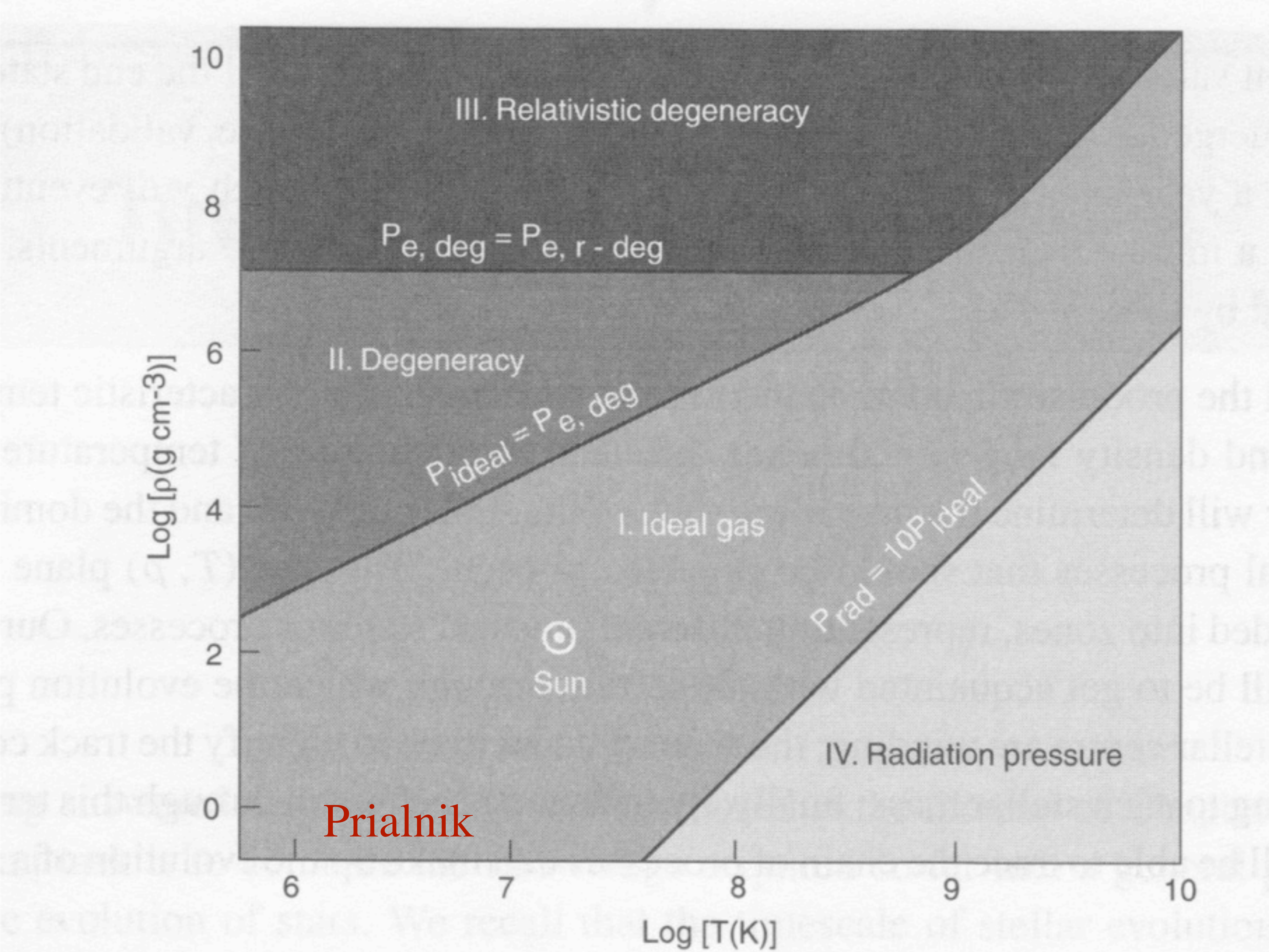
The pressure in a degenerate gas is the combination of the degeneracy pressure of the electrons and the ideal gas pressure of the ions (you should review this). For non-relativistic electrons

$$P_e = \frac{h^2}{20m_e}\left(\frac{3}{\pi}\right)^{2/3}\frac{1}{m_H^{5/3}}\left(\frac{\rho}{\mu_e}\right)^{5/3} \tag{11}$$

and for ions:

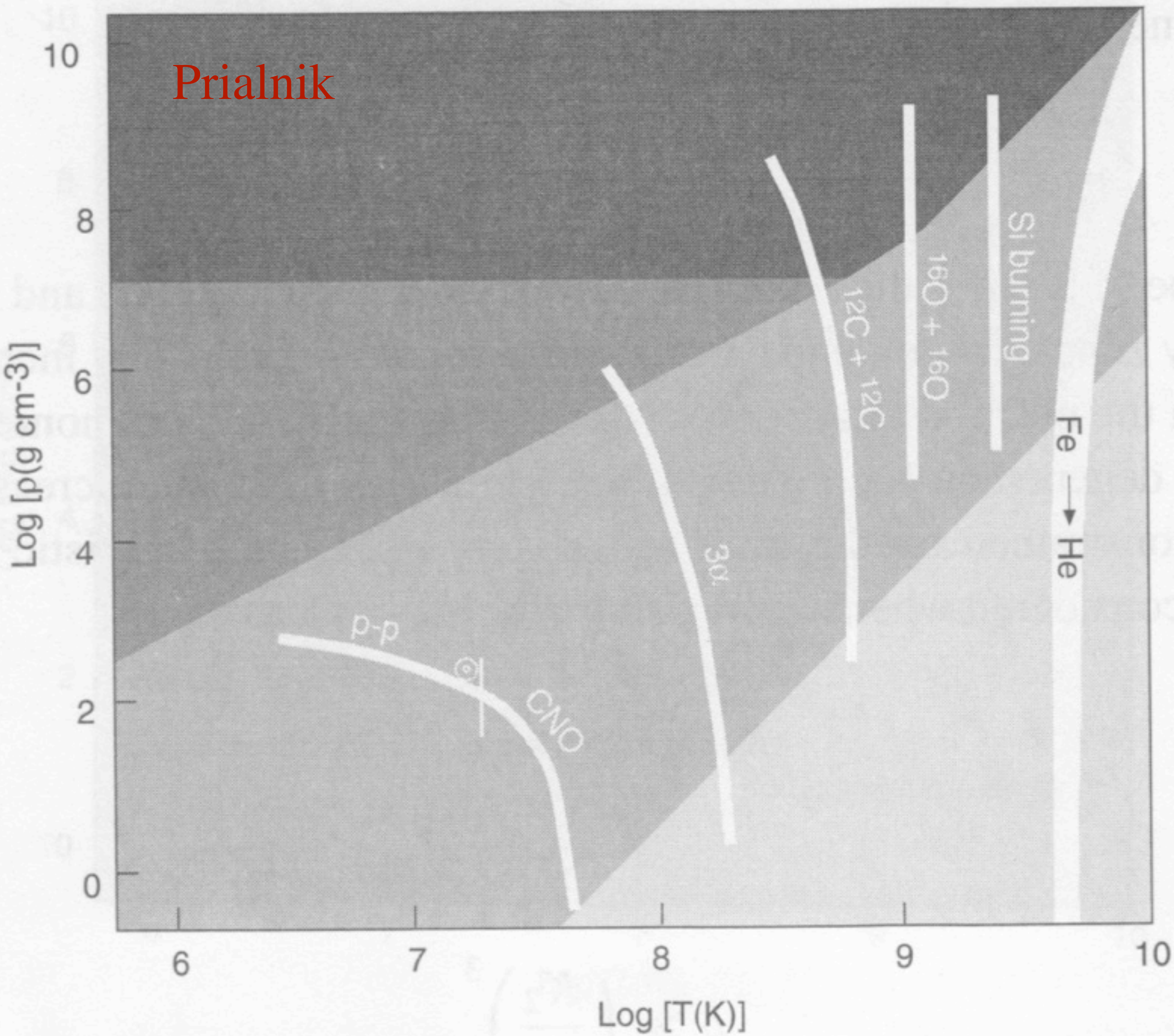
$$P_{ion} = \frac{\rho}{\mu_i}kT \tag{12}$$

What is the equation of state for the core?



Prialnik

Nuclear Burning Sequences



What is the density of the core?

Hydrostatic Equilbrium gives:

$$P_c = K\frac{GM^2}{R^4} = K\left(\frac{4\pi}{3}\right)^{1/3} GM^{2/3}\rho^{4/3}$$

The Ideal Gas Law then Gives:

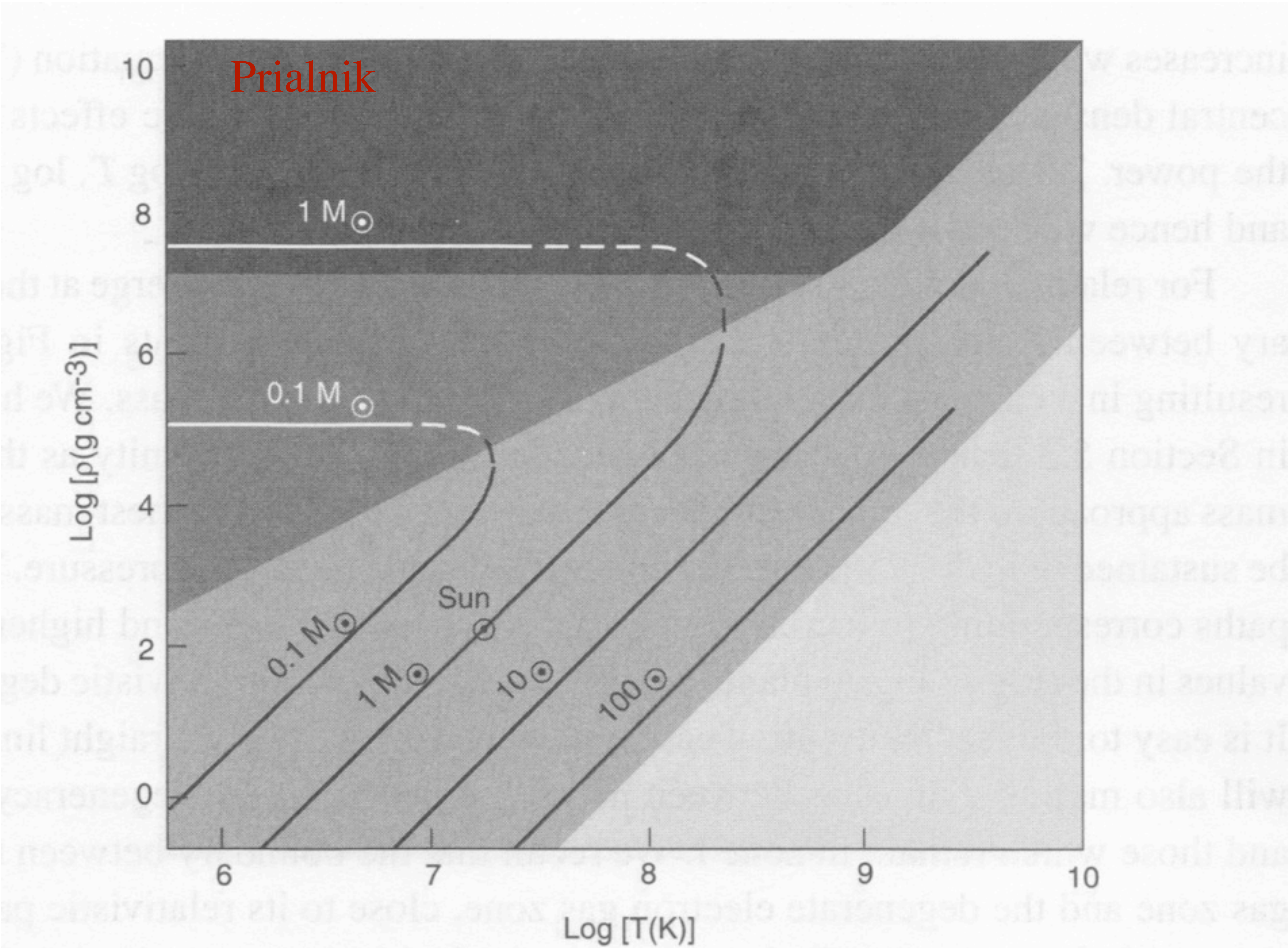
$$\rho_c = \frac{3}{4\pi}\left(\frac{k}{K_1 G\mu m_H}\right)^3 \frac{T_c^3}{M^2}$$

In contrast, non-relativistic degeneracy pressure gives:

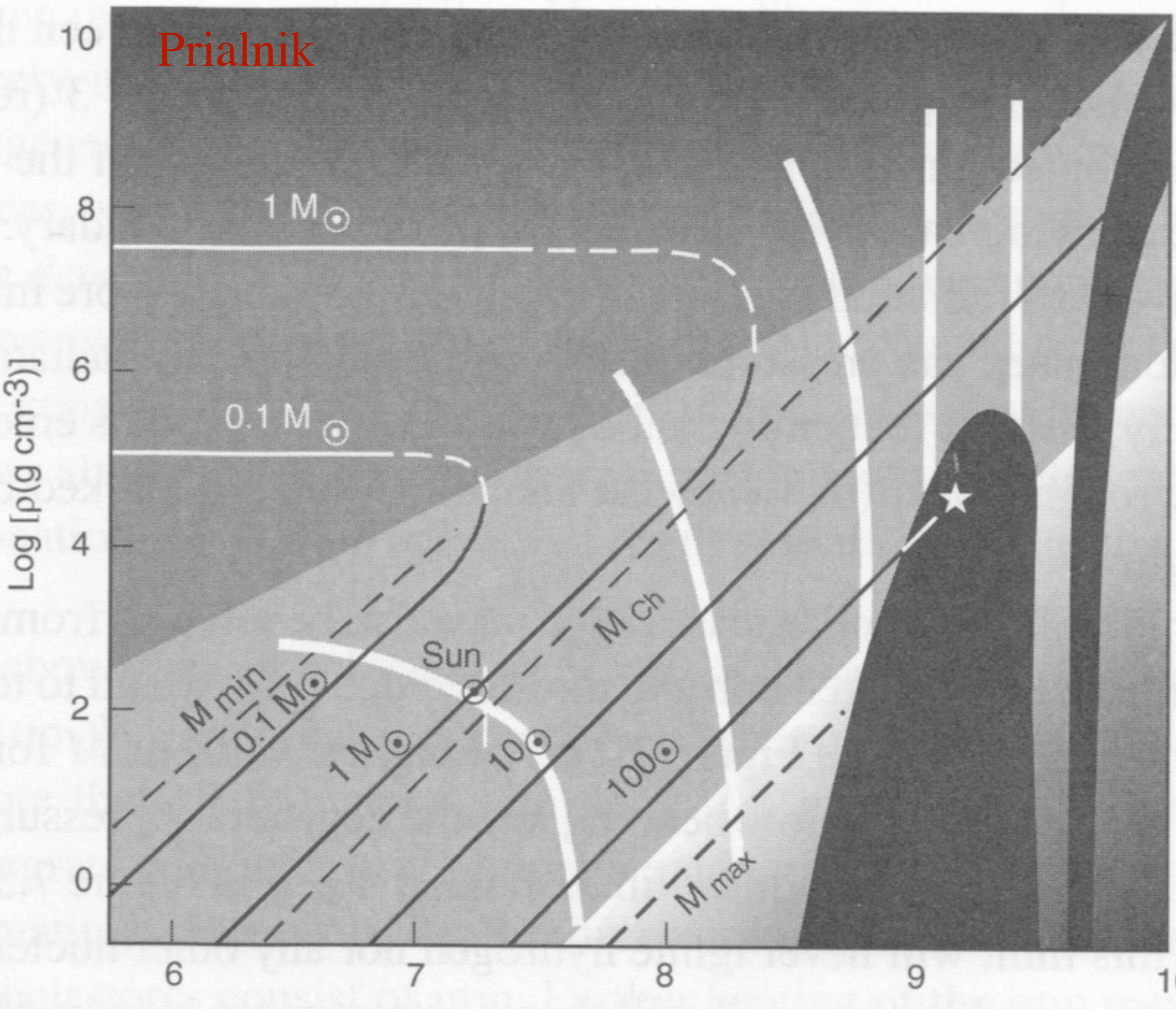
$$P \propto \rho^{5/3}$$

$$\rho_c = K_2 M^2$$

Evolution of Central Cores



When does degeneracy happen?



Thermal Instability in Degenerate Gas

The central pressure of a star is given by

$$P_c = KM^{2/3}\rho_c^{4/3} \tag{13}$$

$$\frac{dP_c}{P_c} = \frac{4}{3}\frac{d\rho}{\rho_c|} \tag{14}$$

In general, if we can write $P = \rho^a T^b$ then

$$\frac{dP_c}{P_c} = a\frac{d\rho_c}{\rho_c} + b\frac{dT_c}{T_c} \tag{15}$$

Prialnik

Thermal Instability in Degenerate Gas

combining the above equation

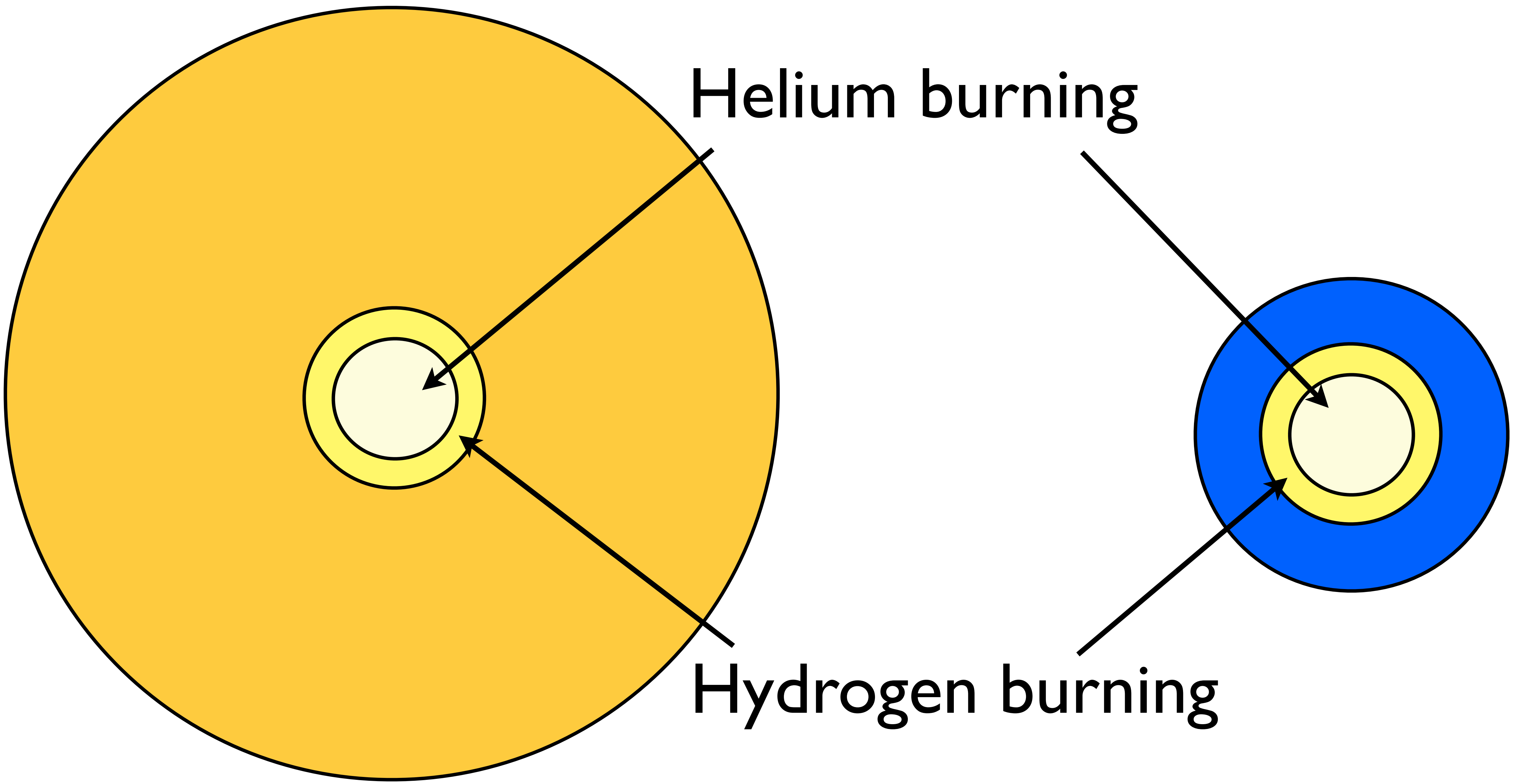
$$\left(\frac{4}{3} - a\right)\frac{d\rho_c}{\rho_c} = b\frac{dT_c}{T_c} \qquad (16)$$

If the right hand term is negative, then contraction will lead to cooling and expansion heating, the opposite that is needed for a stable star. Thus if $a > 4/3$ then the gas will be unstable. For degenerate gases, $a \geq 4/3$ and $0 < b << 1$.

Prialnik

The Horizontal Branch

The core expands and nuclear reaction rates and luminosity may decrease. The star then enters a helium burning main sequence.



At this point the star is relatively steady for 10^8 years. However, pulsations may occur. Luminosities of 50-100 Lsun.

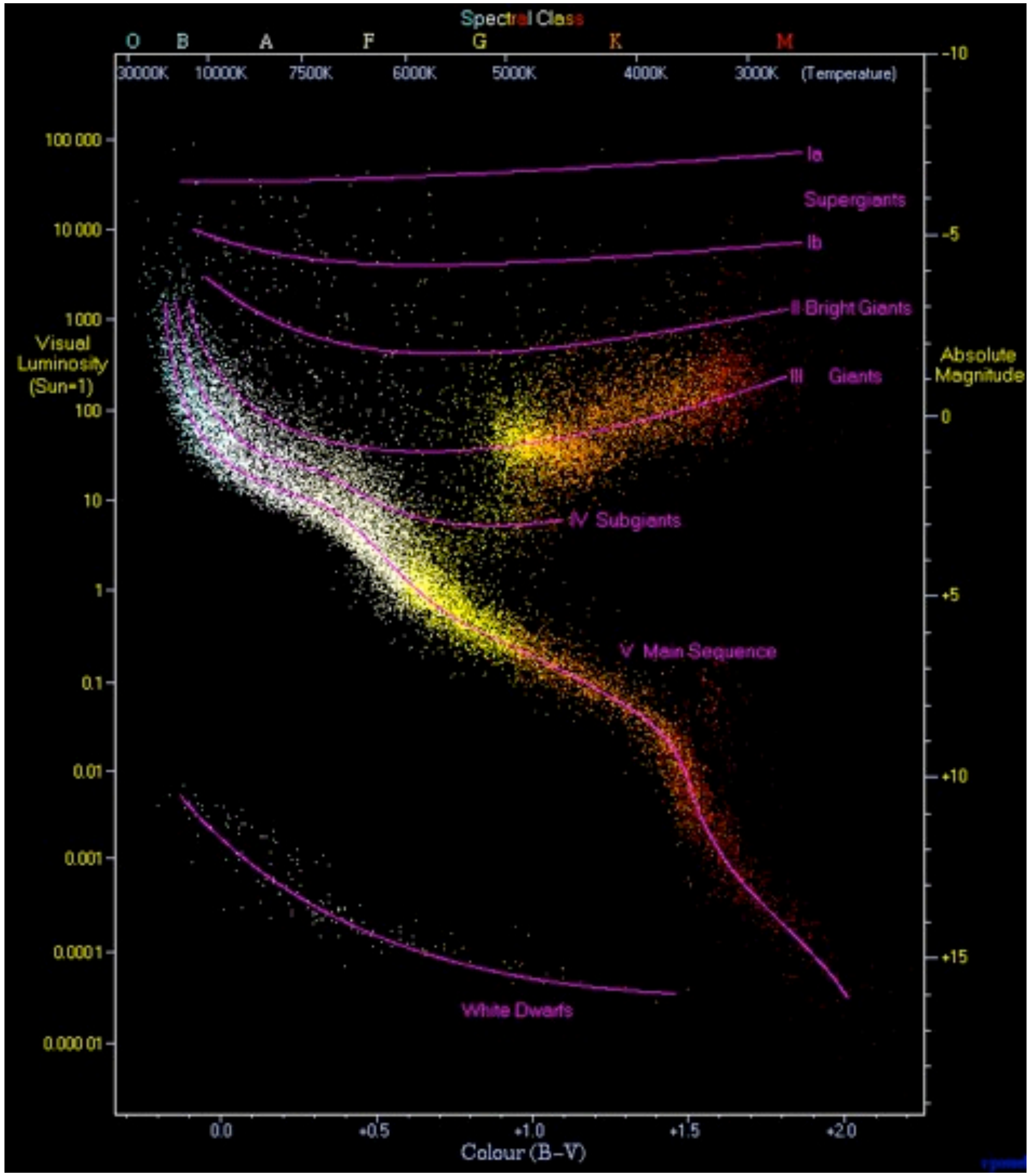
Nuclear Burning: the triple α process

$^{4}He + ^{4}He \rightarrow ^{8}Be$

$^{8}Be + ^{4}He \rightarrow ^{12}C$

Releasing 7.275 Mev

$q = k\,\rho^{2}\,T^{40}$



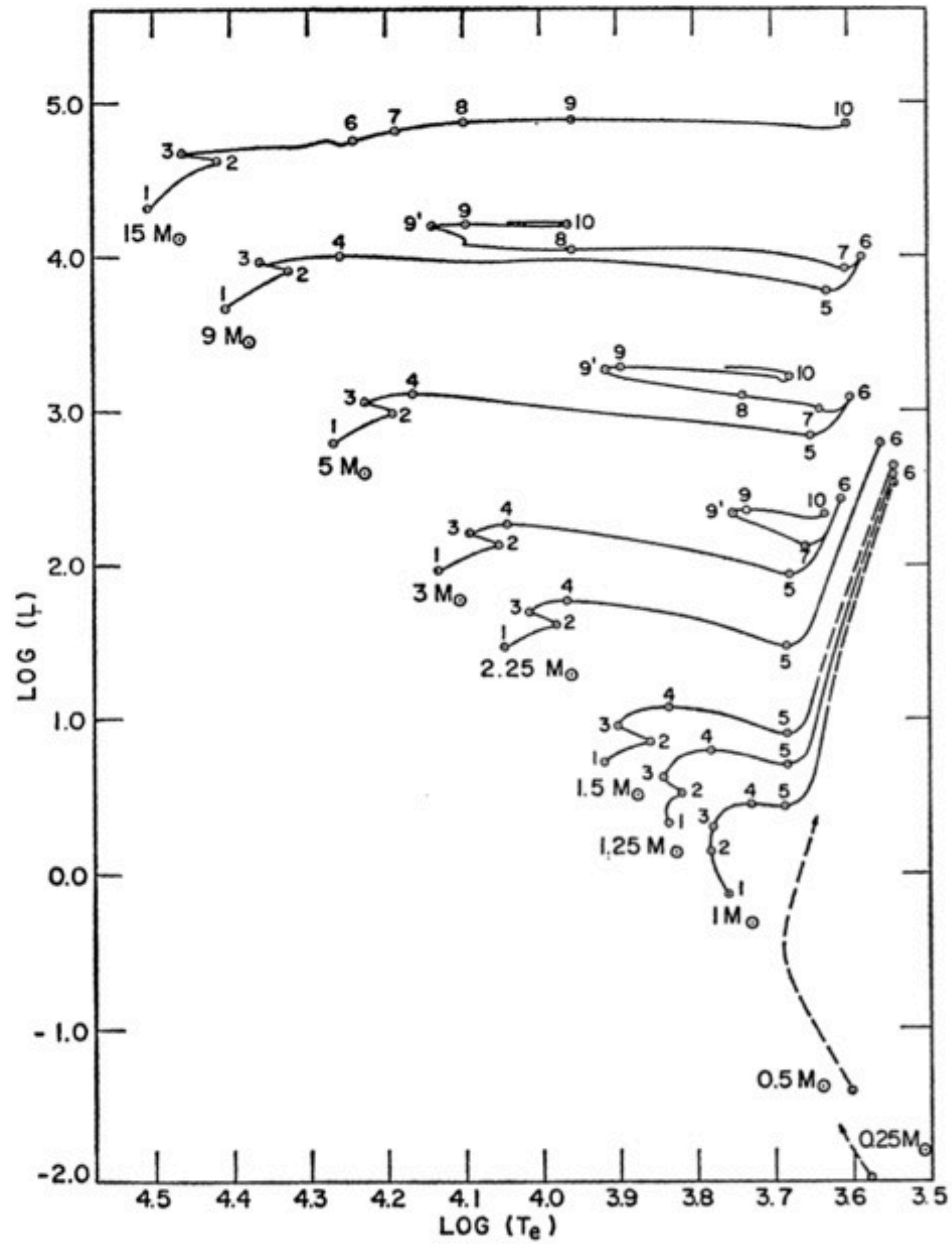
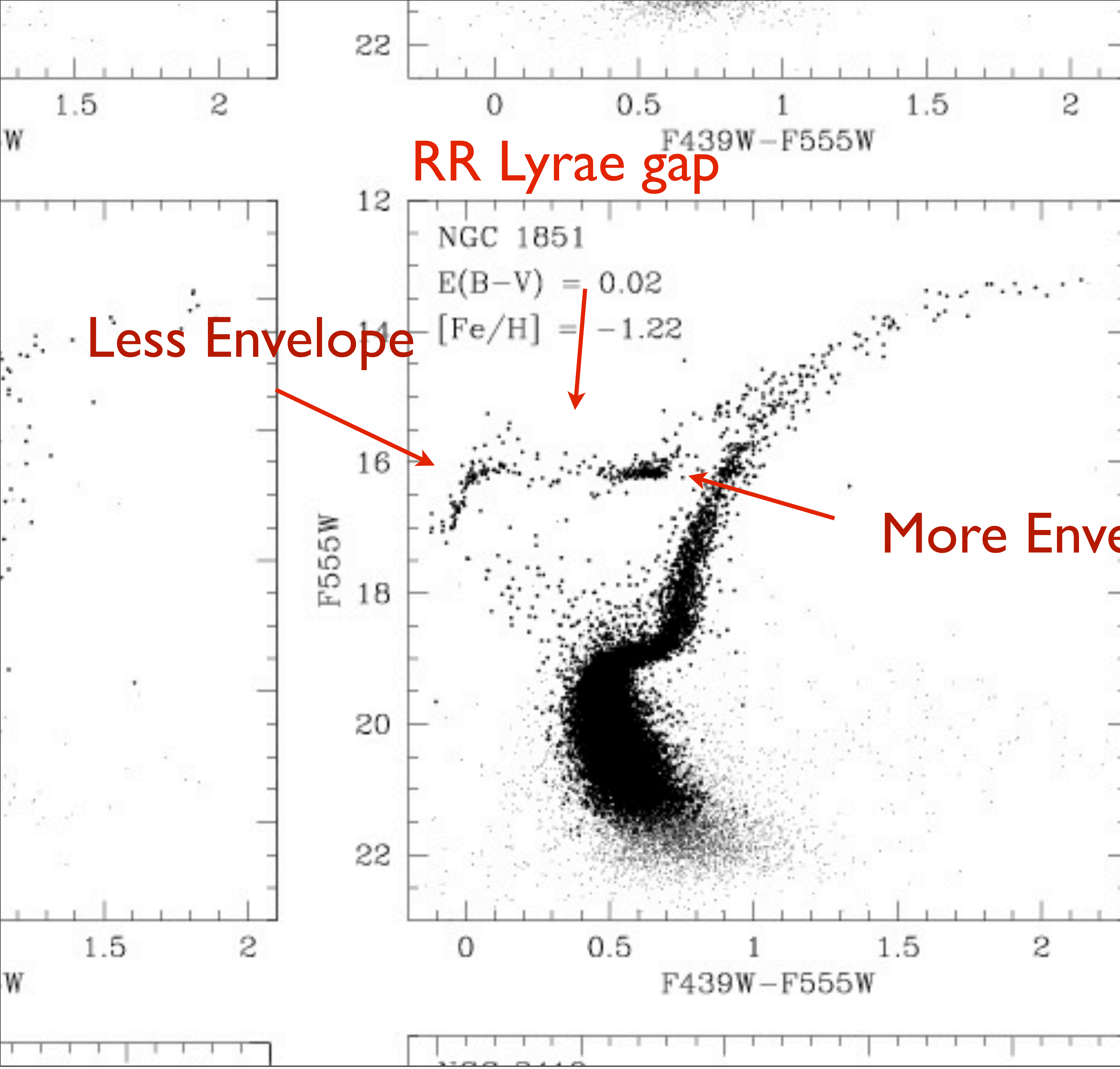


FIG. 3. Paths in the H-R diagram for metal-rich stars of mass $(M/M_\odot)=15$, 9, 5, 3, 2.25, 1.5, 1.25, 1, 0.5, 0.25. Units of luminosity and surface temperature are the same as in Figure 1. Traversal times between labeled points are given in Tables III and IV. Dashed portions of evolutionary paths are estimates.

http://www.atlasoftheuniverse.com/hr.html

http://stars.astro.illinois.edu/sow/hrd.html

Piotto et al. 2002



Dynamical Instabilities

Dynamical Stability

Consider a star in Hydrostatic equilibrium, given in the Lagrangian and Eulerian equations:

$$\frac{dP}{dm} = \frac{Gm}{4\pi r^4}, \text{ or } \frac{dP}{dr} = \rho\frac{Gm}{r^2} \tag{1}$$

which implies

$$\rho = \frac{1}{4\pi r^2}\frac{dm}{dr} \tag{2}$$

now consider a small perturbation, a contraction in the radius:

$$r' = r - \epsilon r = r(1-\epsilon) \tag{3}$$

The resulting density is

$$\rho' = \frac{1}{4\pi r^2(1-\epsilon)^2}\frac{dm}{dr}\frac{dr}{dr'} = \frac{\rho}{(1-\epsilon)^3)} \approx \rho(1+3\epsilon) \tag{4}$$

Prialnik

Dynamical Instabilities

If we consider that $P'/P = (\rho'/\rho)^{\gamma_a}$, where $\gamma_a = 5/3$ for an adiabatic, monotonic gas:

$$P'_{gas} = P(1+3\epsilon)^{\gamma} \approx P(1+3\epsilon\gamma) \qquad (5)$$

We can also calculate the pressure needed for Hydrostatic equilibrium. Start with:

$$P = \int_m^M \frac{Gm}{4\pi r^4} dm \qquad (6)$$

which when perturbed give

$$P'_h = \int_m^M \frac{Gm}{4\pi r^4 (1-\epsilon)^4} dm \approx P(1+4\epsilon) \qquad (7)$$

Prialnik

Dynamical Instabilities

For the gas to return to equilibrium

$$P'_{gas} > P'_h \tag{8}$$

which requires that:

$$P(1 + 3\epsilon\gamma_a) > P(1 + 4\epsilon) \tag{9}$$

or

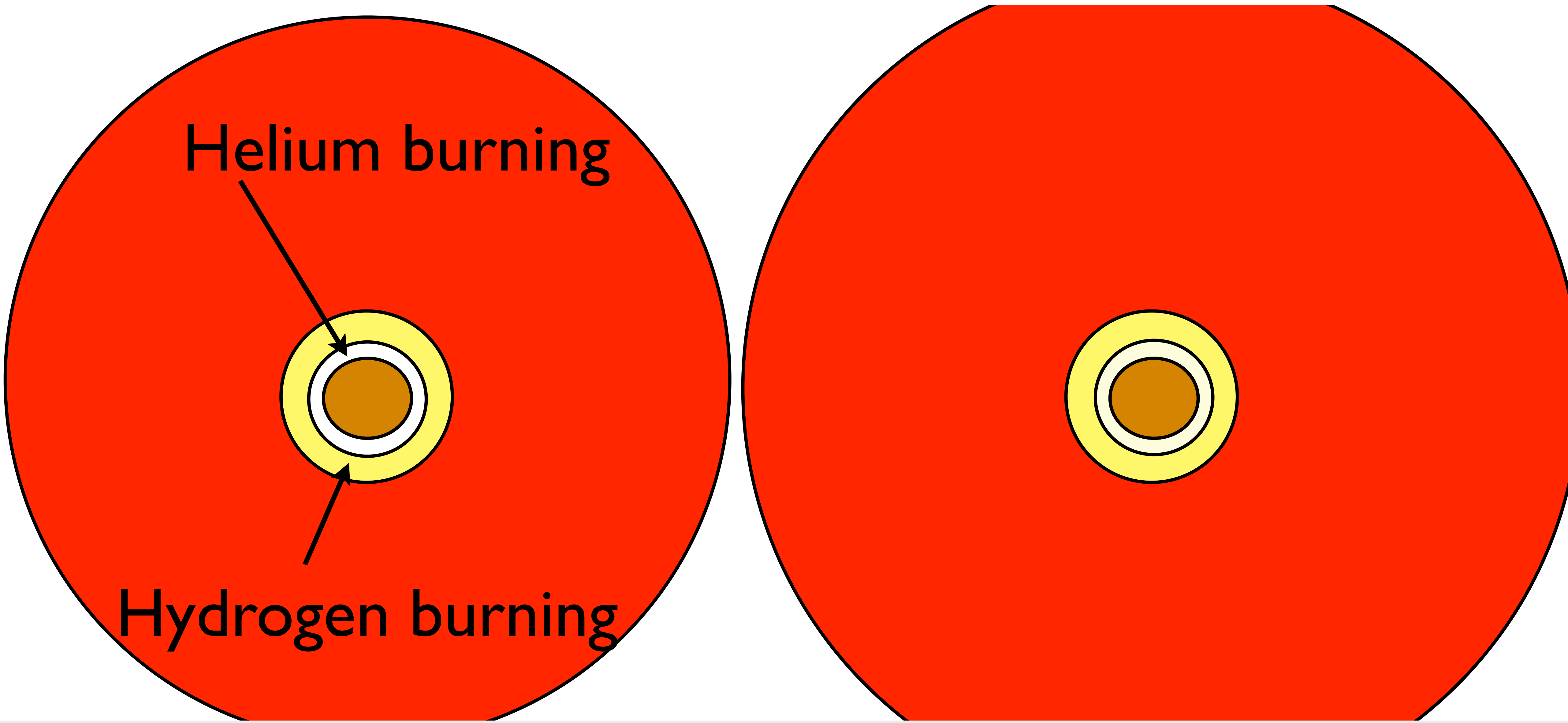
$$\gamma_a > \frac{4}{3} \tag{10}$$

This is the case for the adiabatic monotonic gas, this is satisfied. Also for a non-relativistic degenerate gas. On the other hands, for a relativistic degenerate gas or photon supported gas, $\gamma = 4/3$, which implies these are neutrally stable.

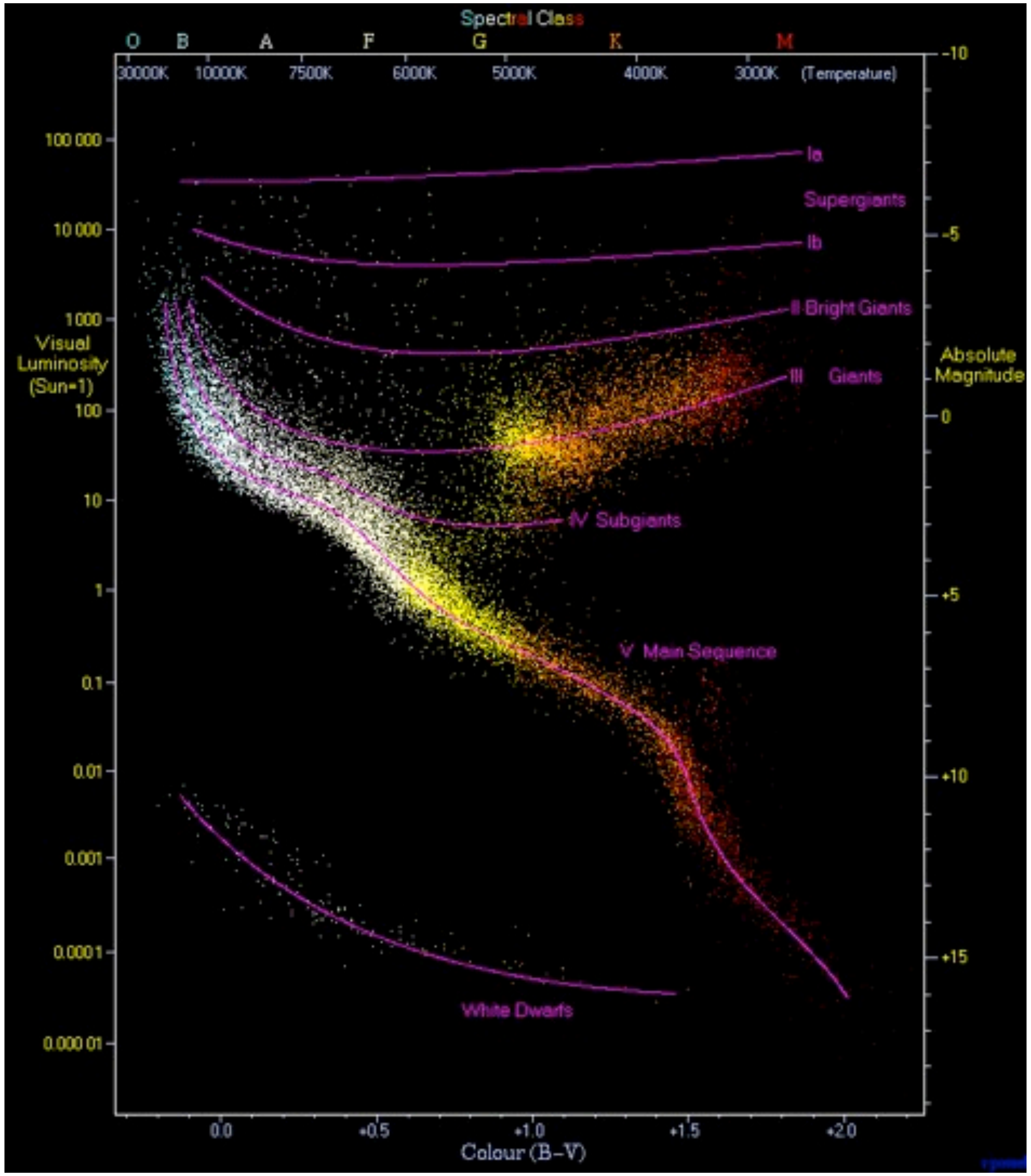
Prialnik

Asymptotic Giant Branch

Now the central carbon/oxgen core becomes unstable and starts to contract. There is now Helium and Hydrogen burning in shells.



The star may become a supergiant. However, pulsations and dust formation in the envelope may lead to the ejection of the envelope, leaving a white dwarf.



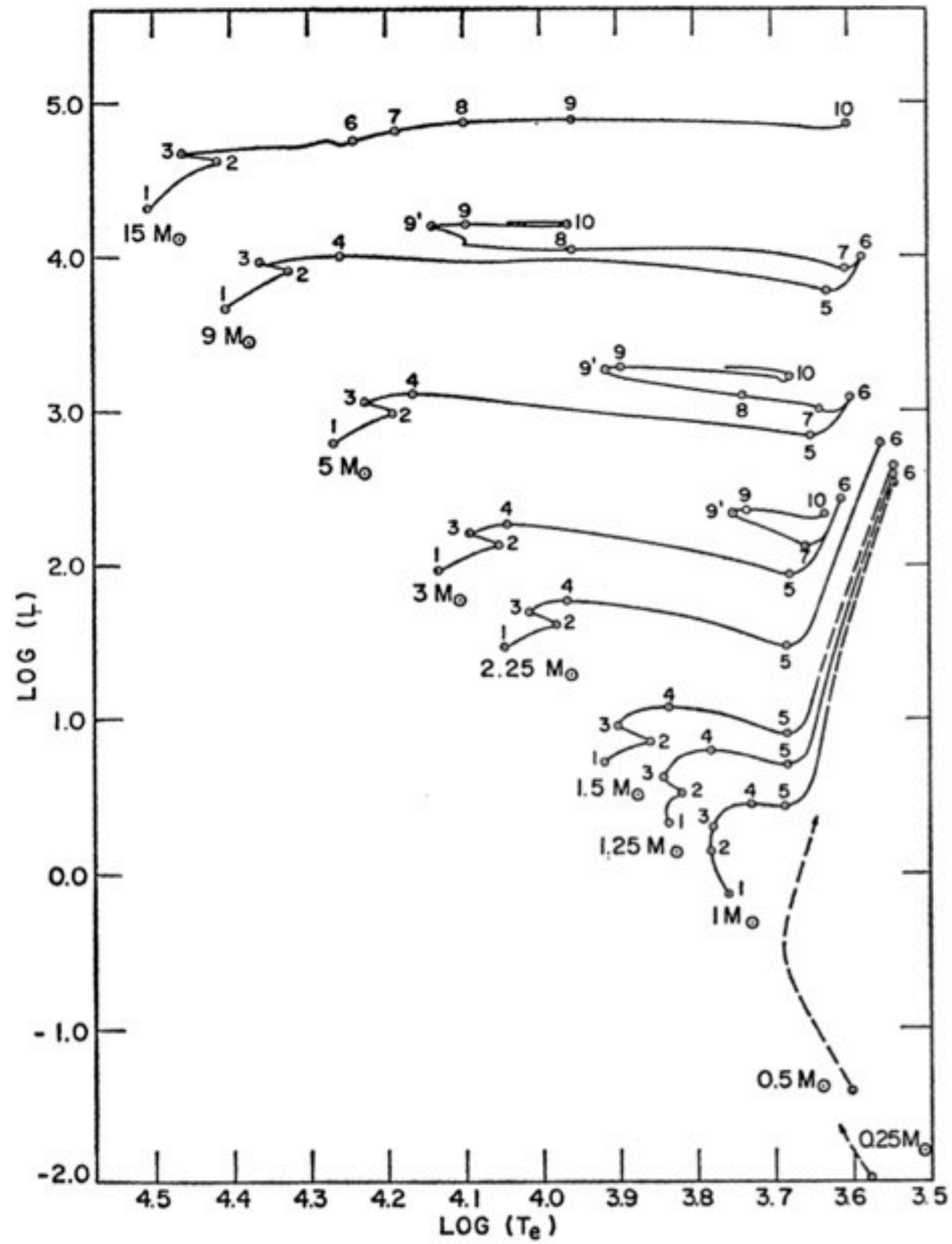


FIG. 3. Paths in the H-R diagram for metal-rich stars of mass $(M/M_\odot)=15$, 9, 5, 3, 2.25, 1.5, 1.25, 1, 0.5, 0.25. Units of luminosity and surface temperature are the same as in Figure 1. Traversal times between labeled points are given in Tables III and IV. Dashed portions of evolutionary paths are estimates.

http://www.atlasoftheuniverse.com/hr.html
http://stars.astro.illinois.edu/sow/hrd.html

Thin Shell Instabilities

In Hydrostatic Equilibrium

$$\frac{dP}{P} = -4\frac{dr}{r} \tag{17}$$

if we take the shell as $\Delta m = 4\pi r^2 l\rho$

$$\frac{d\rho}{\rho} = -\frac{dl}{l} = -\frac{dr}{l} = -\frac{dr}{r}\frac{r}{l} \tag{18}$$

$$\frac{dP}{P} = 4\frac{l}{r}\frac{d\rho}{\rho} \tag{19}$$

Prialnik

Using:

$$\frac{dP}{P} = a\frac{d\rho}{\rho} + b\frac{dT}{T} \tag{20}$$

we get

$$\left(4\frac{l}{r} - a\right)\frac{d\rho}{\rho} = b\frac{dT}{T} \tag{21}$$

For thermal stability:

$$4\frac{l}{r} > a \tag{22}$$

As $l/r \rightarrow 0$, this is no longer satisfied.

Prialnik

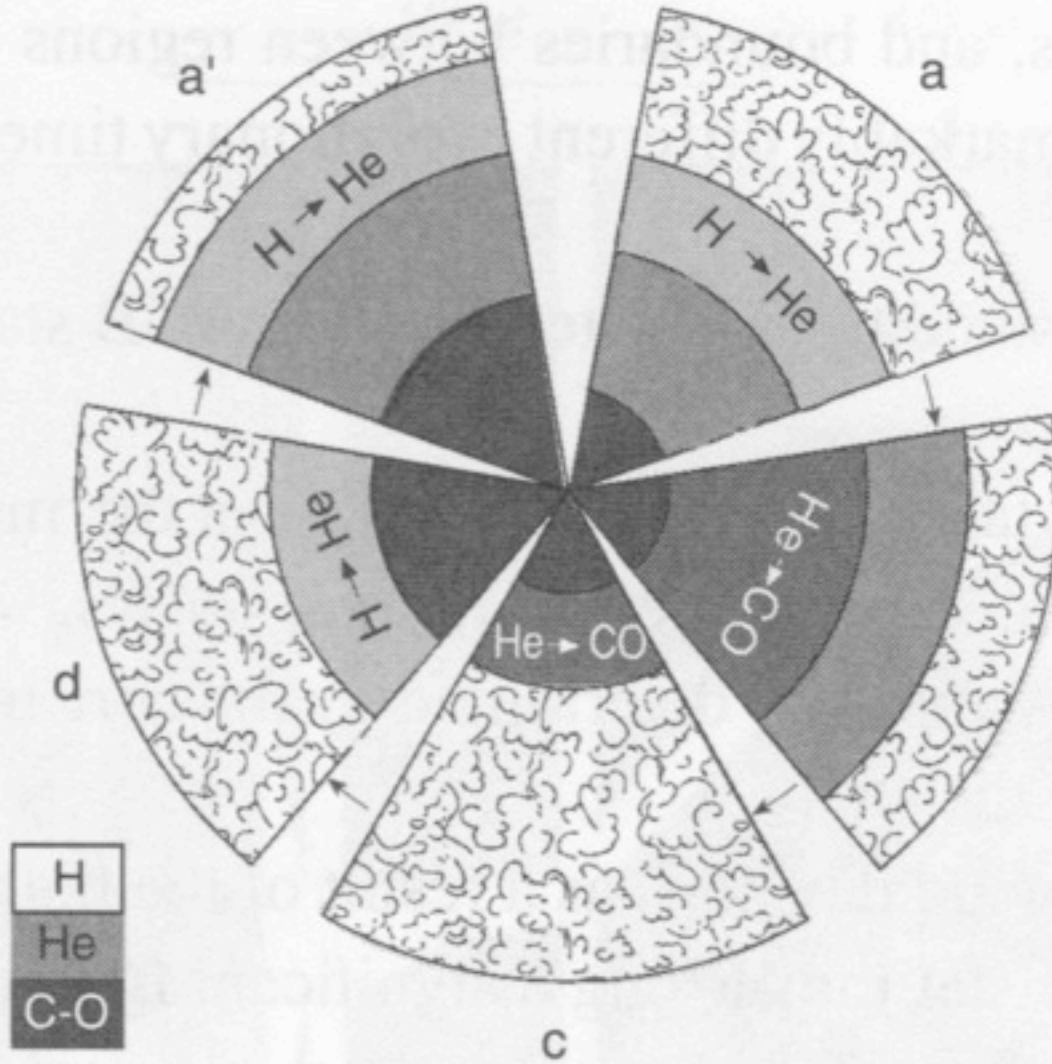


Figure 8.10 Sketch of the progress of a thermal pulse cycle through its different stages (not in scale). Hydrogen is burning during stages a and d, while helium is burning during stages b and c. When, in stage c, the outer convective zone extends inward beyond the helium shell burning boundary, hydrogen and helium burning products are mixed into the envelope and dredged up to the surface. Stage a′ is the same as a, except that the carbon-oxygen core has grown at the expense of the envelope.

Prialnik

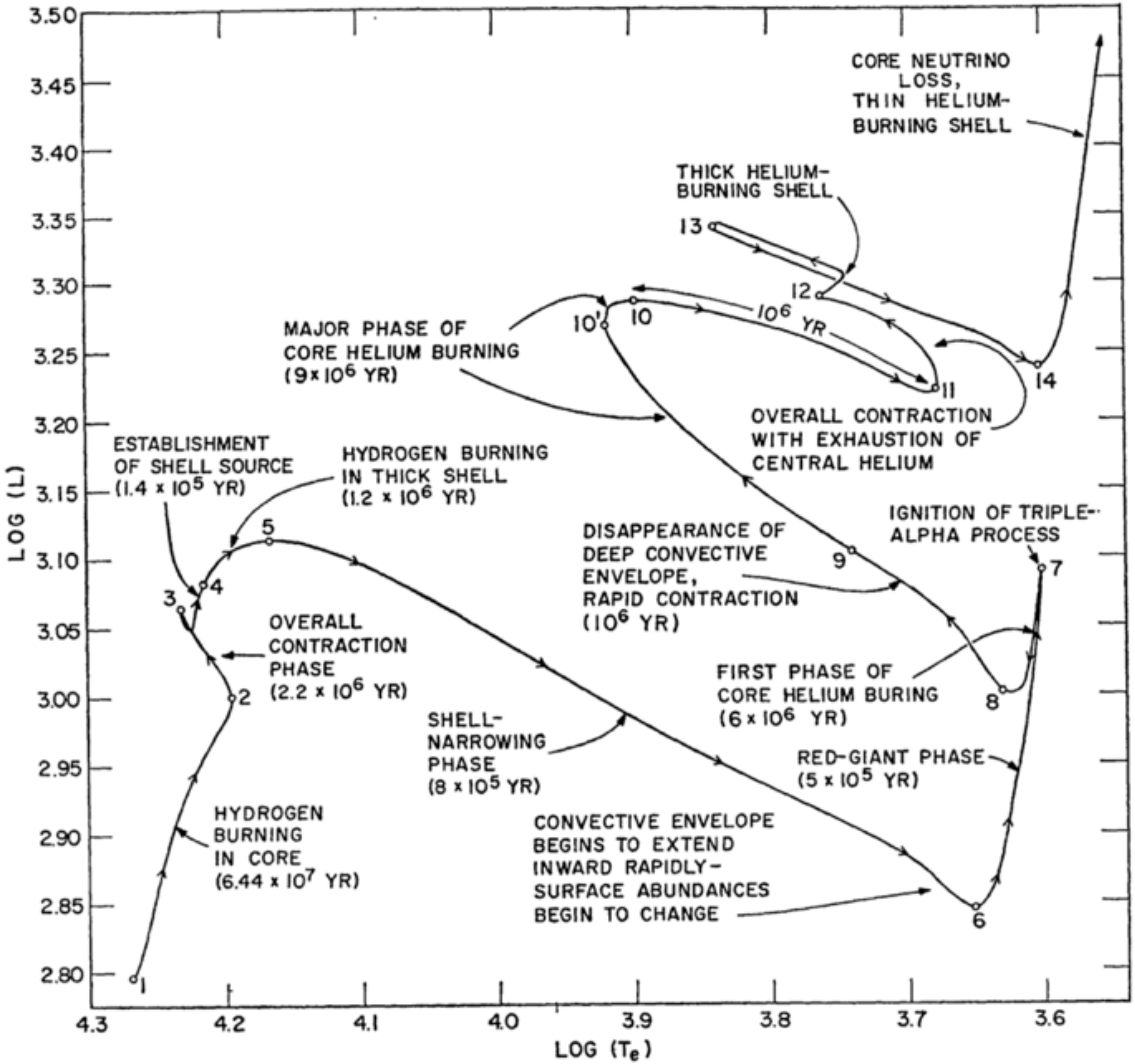


FIG. 1. The path of a metal-rich $5M_{\odot}$ star in the Hertzsprung-Russell diagram. Luminosity is in solar units, $L_{\odot}=3.86\times10^{33}$ erg/sec, and surface temperature T_e is in deg K. Traversal times between labeled points are given in years.

Iben 1967

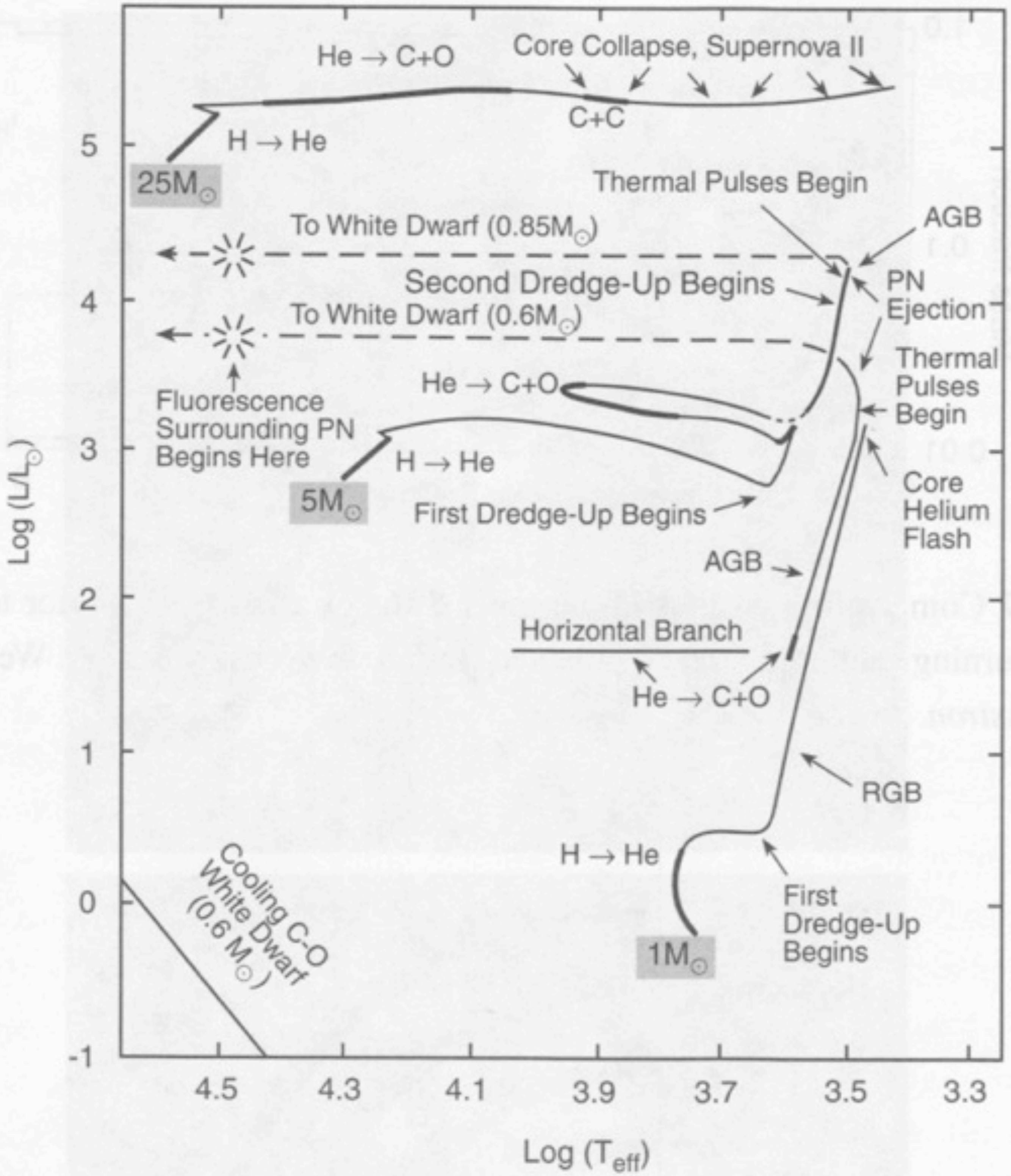


Figure 8.19 Evolutionary tracks of $1M_\odot$, $5M_\odot$, and $25M_\odot$ star models in the H–R diagram. Thick segments of the line denote long, nuclear burning, evolutionary phases. The turnoff points from the AGB are determined empirically [from I. Iben Jr. (1985), *Quart. J. Roy. Astron. Soc.*, 26].

Prialnik

Summary

As 1-2 Msun stars evolve up the Hayashi tracks on the red giant branch, the dominant pressure becomes electron degeneracy pressure.

When Helium burning is achieved, the increase in temperature does not immediately cause an increase in pressure, since degeneracy pressure is independent of temperature.

This causes the Helium flash. The total increase in power is enormous (10^{11} the stars normal output) but the observed luminosity change is lower since it takes 100-1000s of years for the energy to leak out.

The star then enters horizontal branch or red clump phase where there is Helium core burning. The horizontal branch is best visible in lower metalicity population II objects. The Effective temperature of the object depends on the amount of envelope remaining.

Depletion of Helium leads to AGB phase. Thin shell instabilities due to double shell burning can lead to large pulses in luminosity.