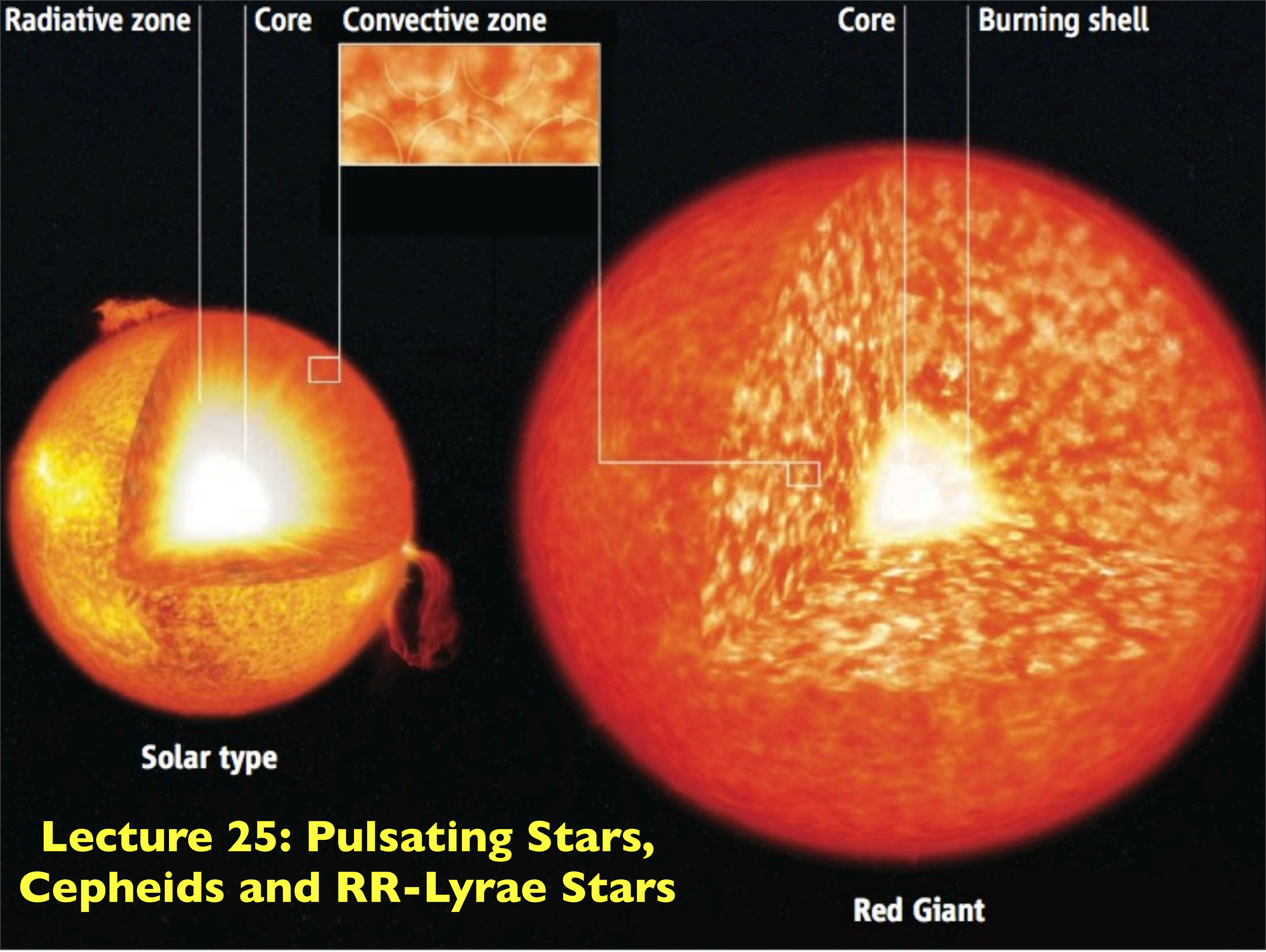


Lecture 25: Pulsating Stars, Cepheids and RR-Lyrae Stars

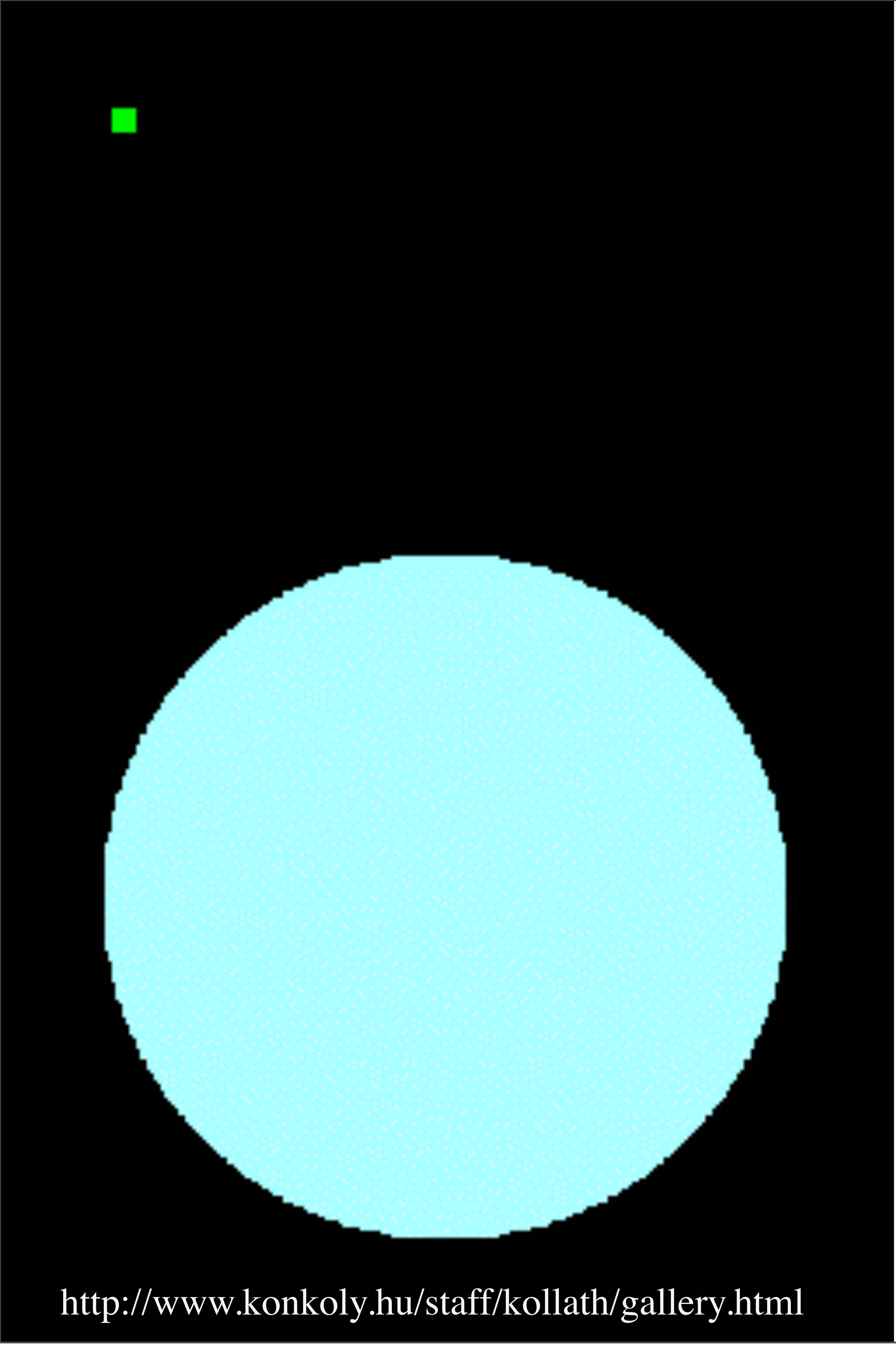
Stellar Pulsations & Oscillations

- Pulsations can leads to periodic variability in stars, although pulsations in the Sun make only small changes in luminosity.
- Sun shows over a million modes.
- Modes in Sun are excited by convection.
- Non-adiabatic effects are needed for large luminosity changes.

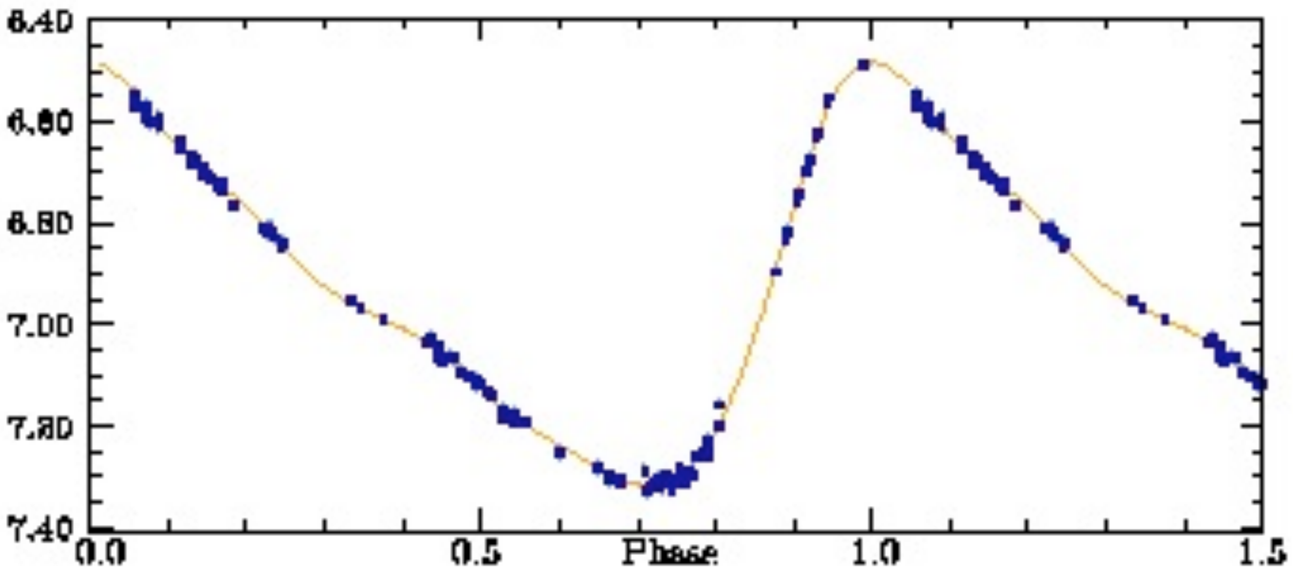
Stellar Oscillations in Red Giants

http://apod.nasa.gov/apod/ap110408.html

Stellar Pulsations with Cepheids

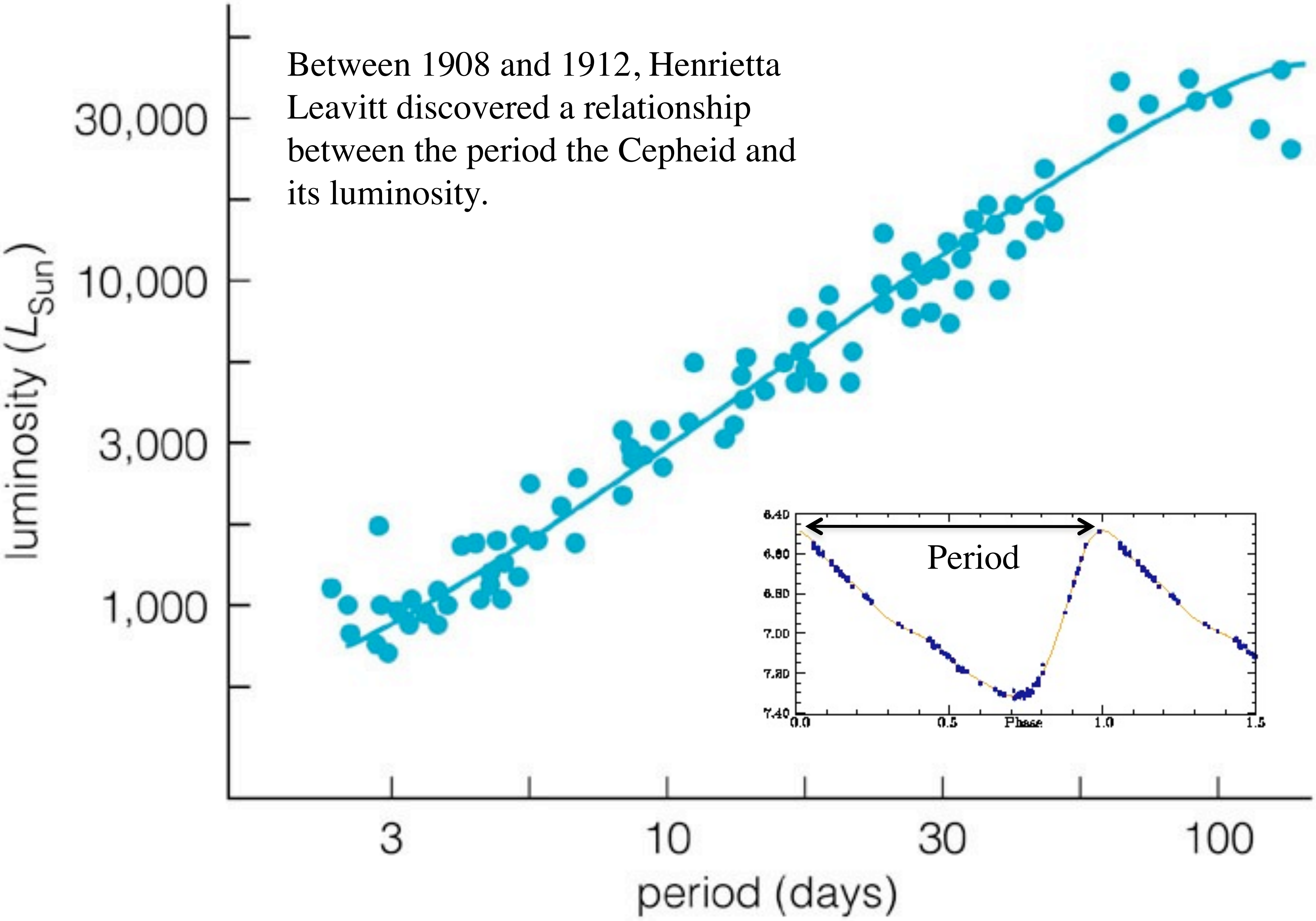


http://www.konkoly.hu/staff/kollath/gallery.html



http://www.calstatela.edu/faculty/kaniol/a360/cepheids.htm

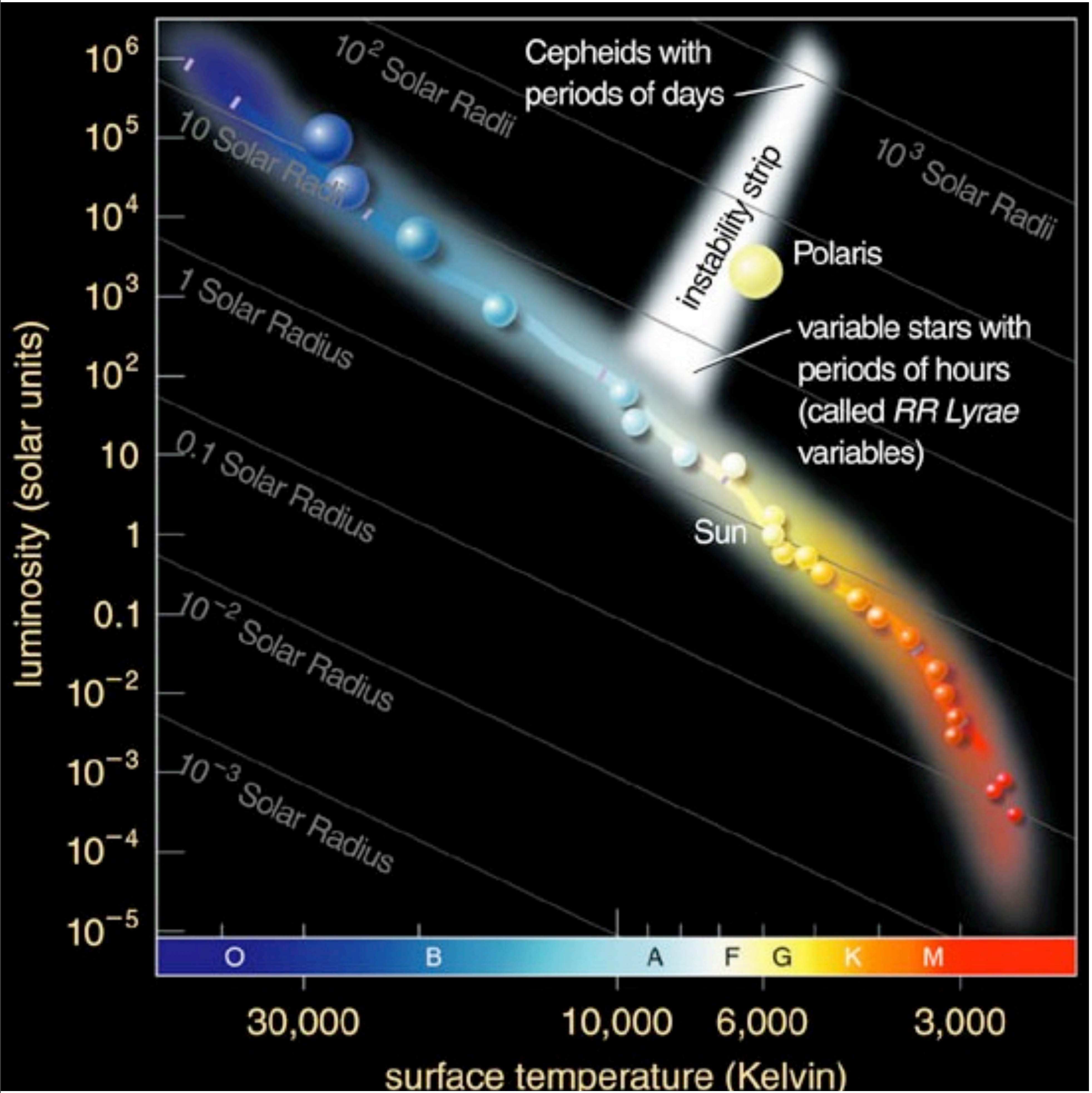
Cepheids pulsate with a period between 1 and 100 days. The pulsation causes changes in brightness which can be easily measured.



Between 1908 and 1912, Henrietta Leavitt discovered a relationship between the period the Cepheid and its luminosity.

This meant that Cepheids are a powerful standard candle

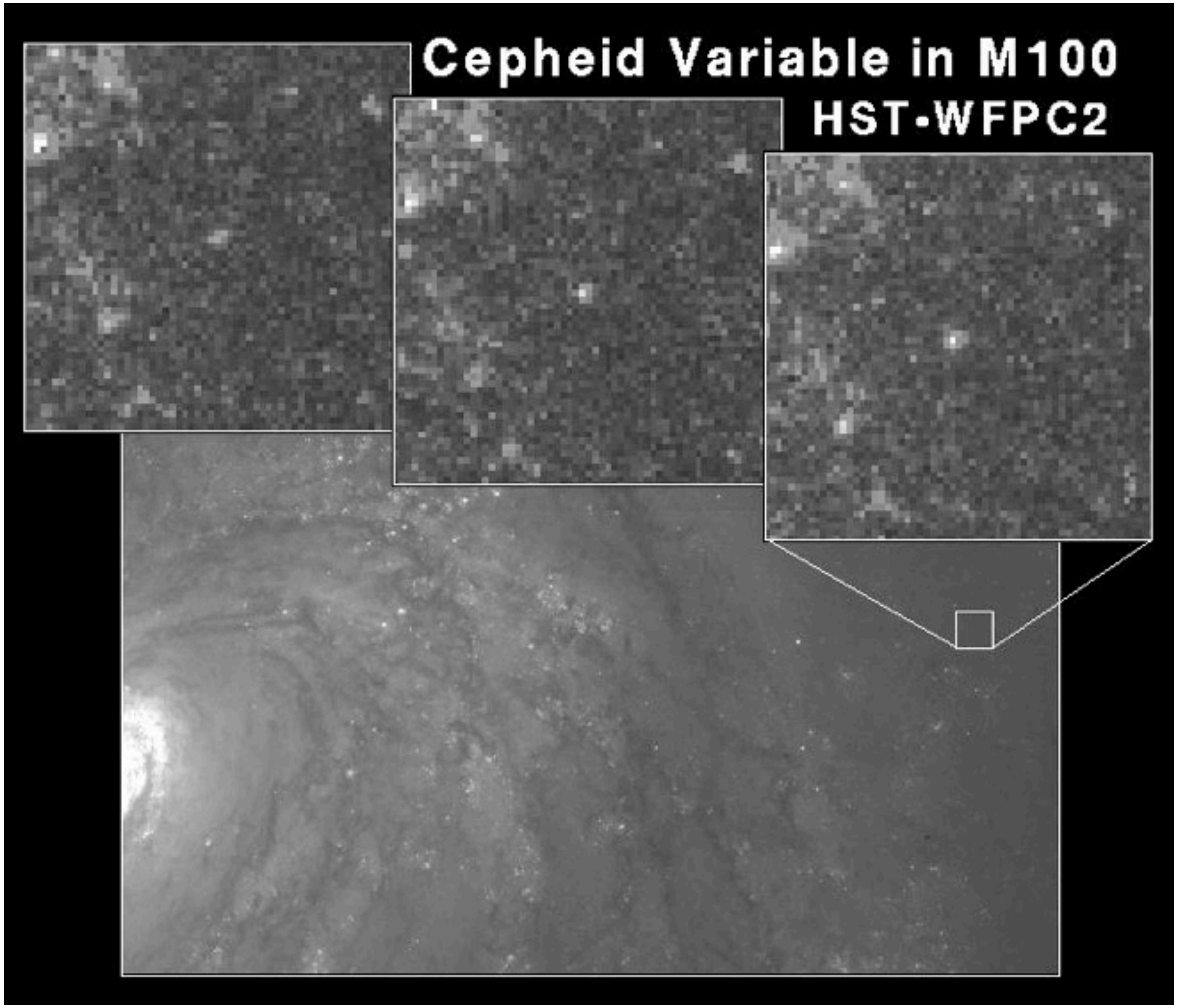
What is a well known Cepheid in the Sky?



Cepheid variable stars are very luminous, making them excellent standard Candles.

Classical Cepheids are Population I Objects found in Disks

Cepheids as Standard Candles

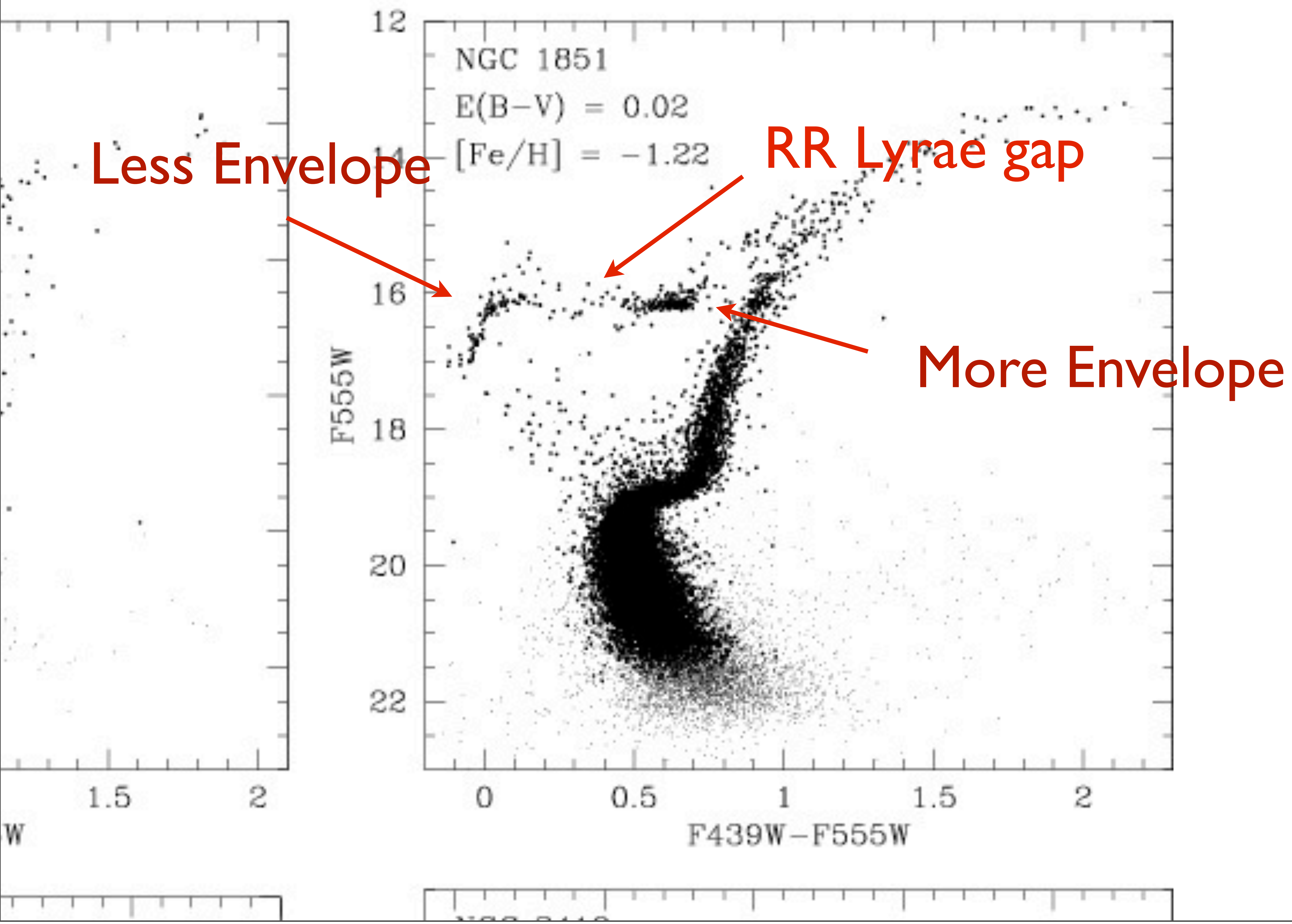


Because Cepheids are bright, they can be detected in distant galaxies. As we learned, Hubble used Cepheids to obtain the first measurements of the distances of the M31 and M33 galaxies.

With the Hubble space telescope, astronomers can now measure the periods of Cepheids in distant galaxies.

RR Lyrae stars are population II objects found in globular clusters

Piotto et al. 2002

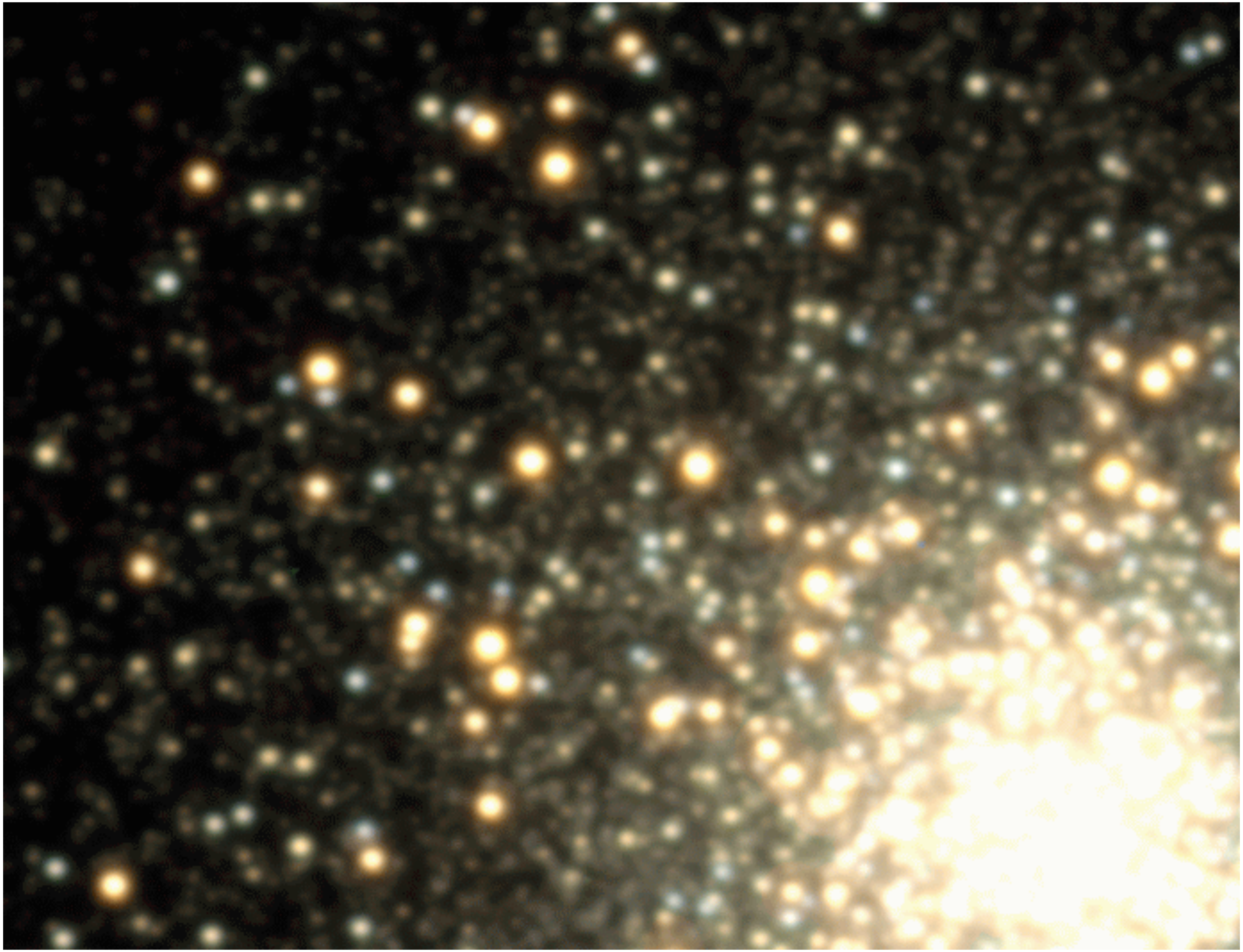


Less Envelope

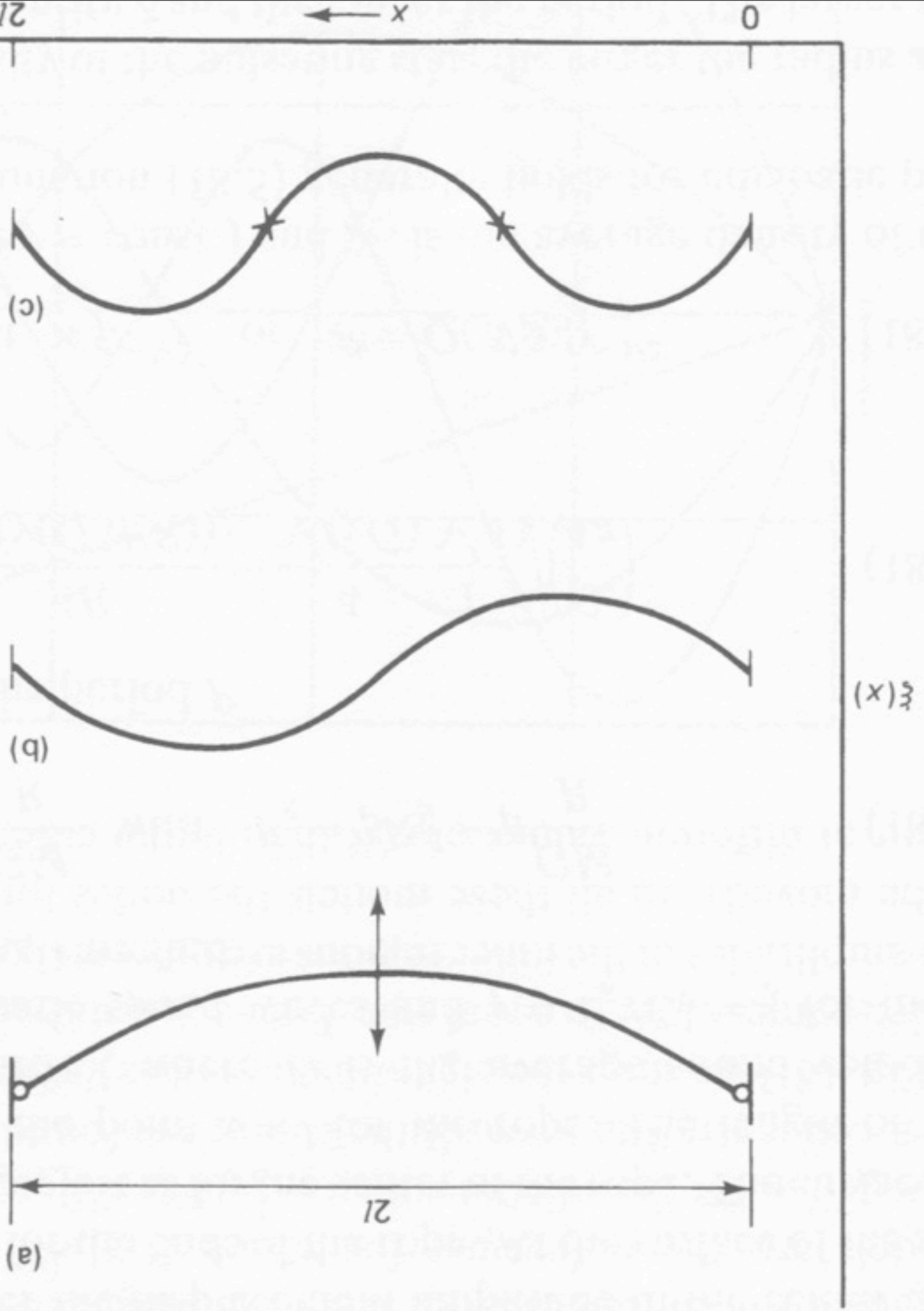
RR Lyrae gap

More Envelope

RR-Lyrae Stars in M3



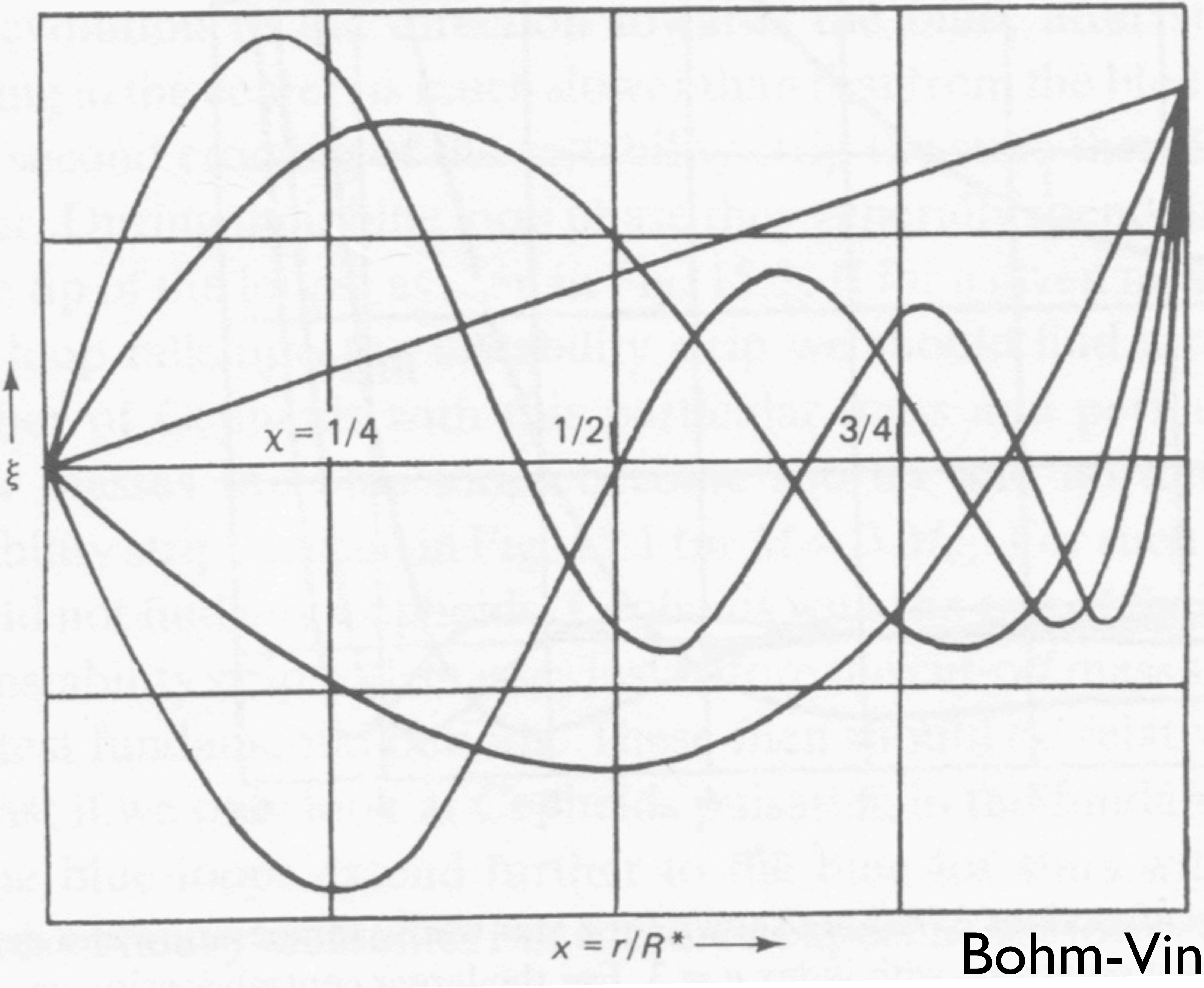
https://www.cfa.harvard.edu/~jhartman/M3_movies.html



Basic theory of Stellar Pulsations

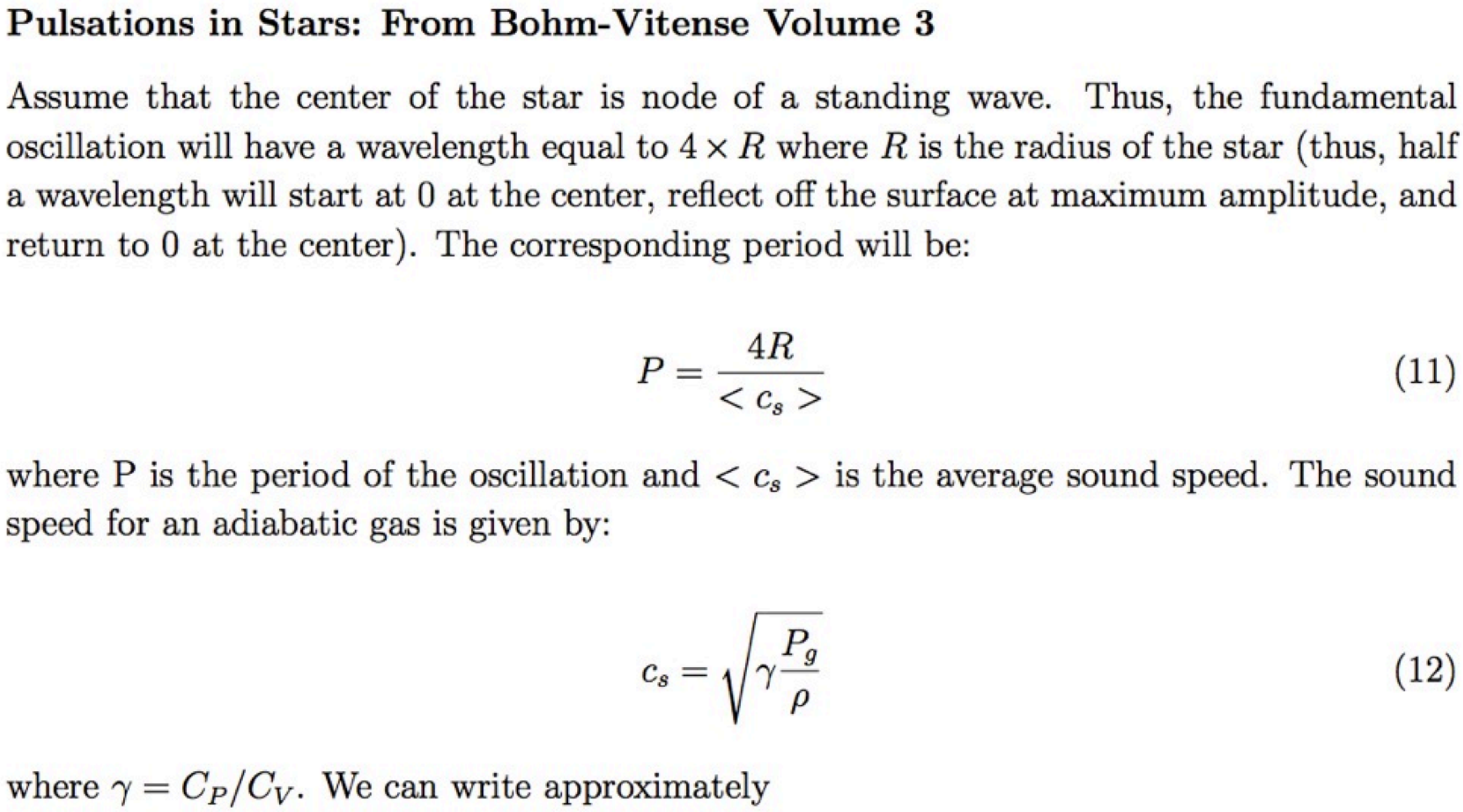
Bohm-Vintense

Eigenvectors for a constant density star



Bohm-Vintense

Basic theory of Stellar Pulsations



Pulsations in Stars: From Bohm-Vitense Volume 3

Assume that the center of the star is node of a standing wave. Thus, the fundamental oscillation will have a wavelength equal to $4 \times R$ where R is the radius of the star (thus, half a wavelength will start at 0 at the center, reflect off the surface at maximum amplitude, and return to 0 at the center). The corresponding period will be:

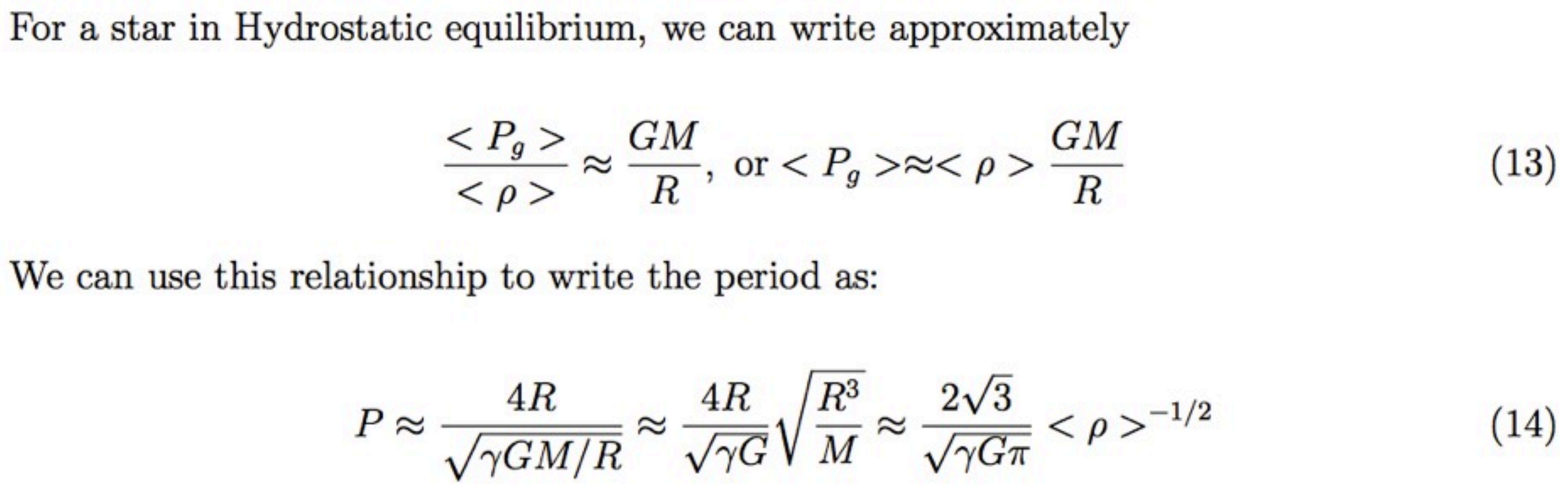
$$P = \frac{4R}{< c_s >} \tag{11}$$

where P is the period of the oscillation and $< c_s >$ is the average sound speed. The sound speed for an adiabatic gas is given by:

$$c_s = \sqrt{\gamma \frac{P_g}{\rho}} \tag{12}$$

where $\gamma = C_P/C_V$. We can write approximately

Basic theory of Stellar Pulsations



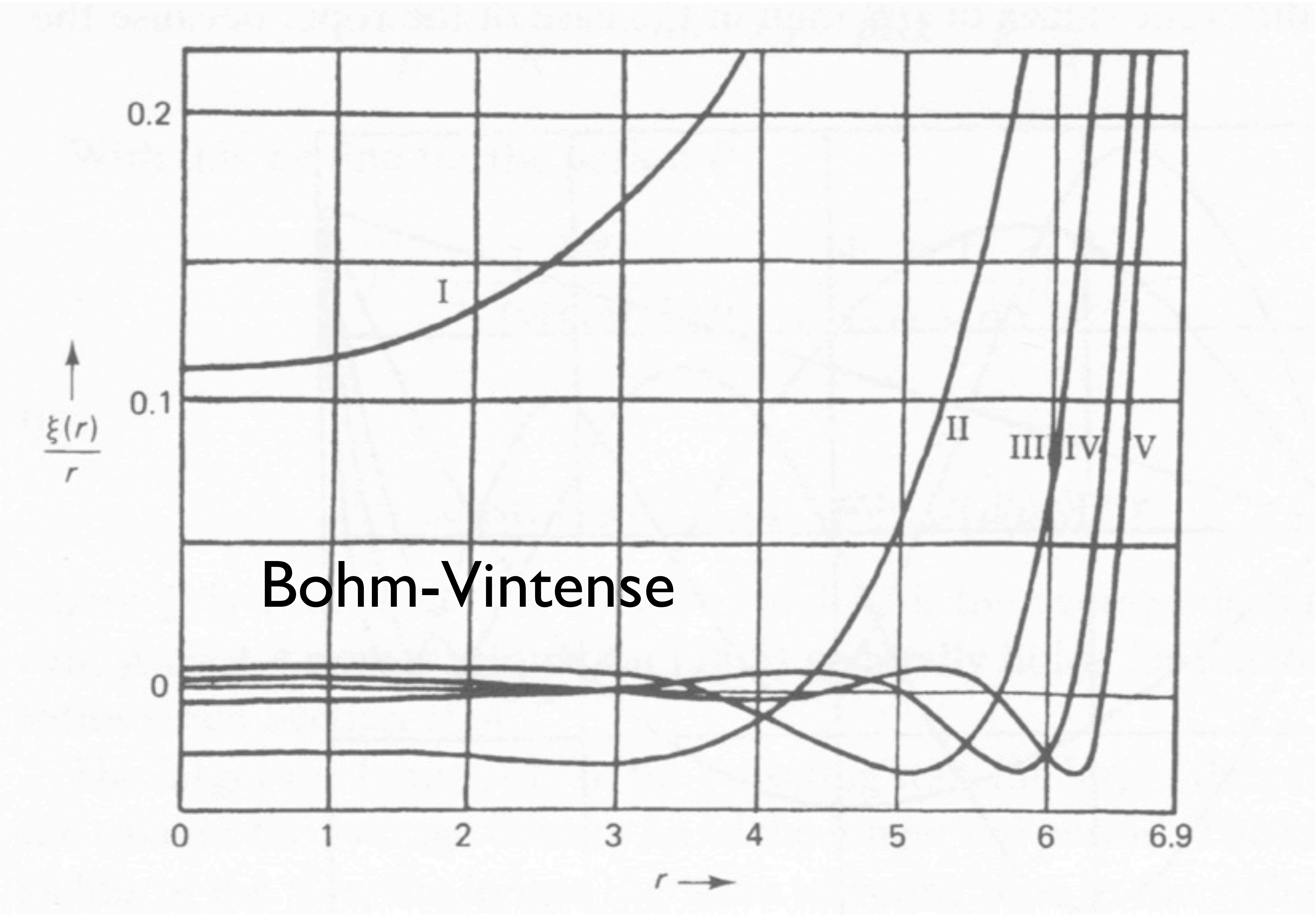
For a star in Hydrostatic equilibrium, we can write approximately

$$\frac{< P_g >}{< \rho >} \approx \frac{GM}{R}, \text{ or } < P_g > \approx < \rho > \frac{GM}{R} \tag{13}$$

We can use this relationship to write the period as:

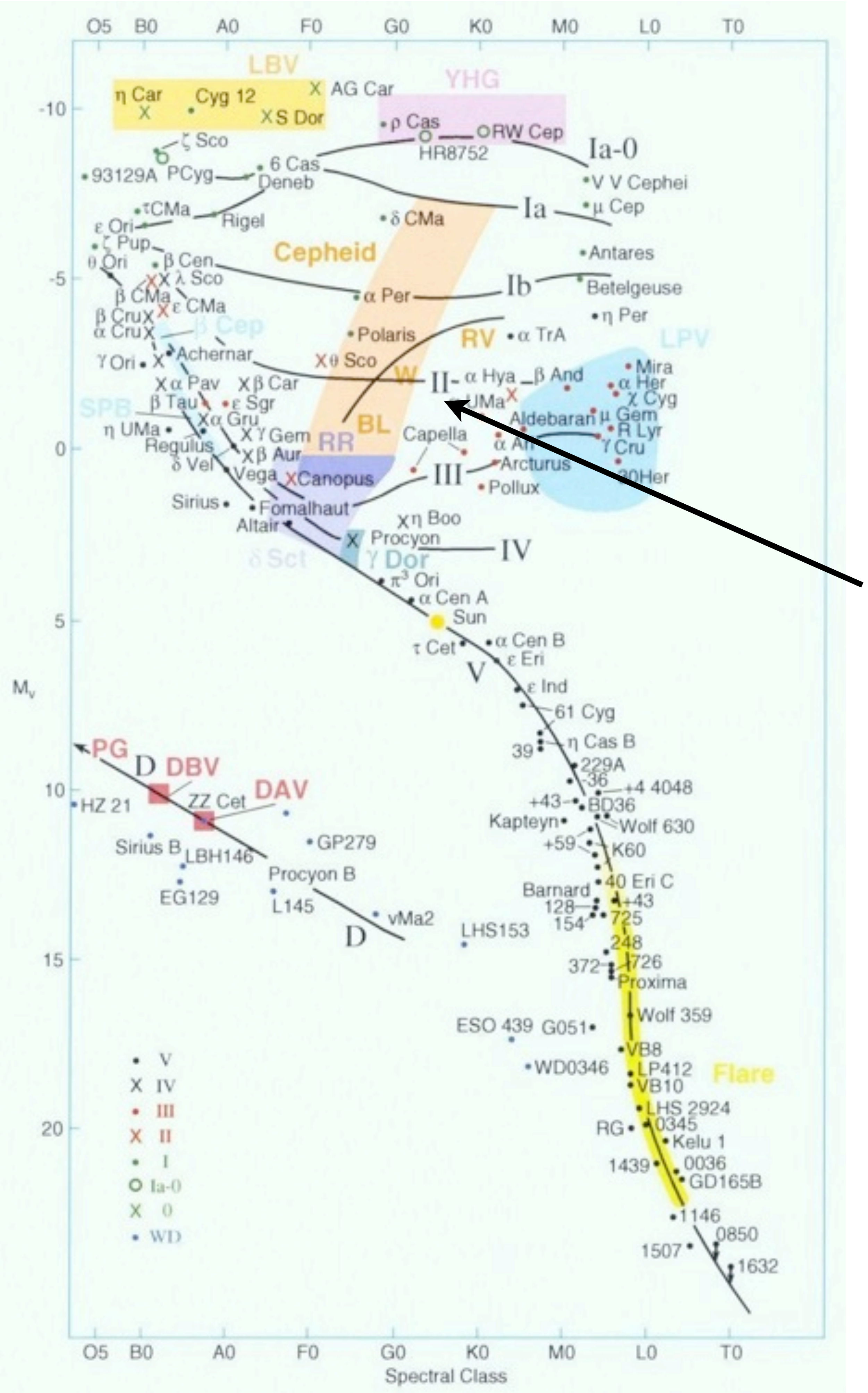
$$P \approx \frac{4R}{\sqrt{\gamma GM/R}} \approx \frac{4R}{\sqrt{\gamma G}} \sqrt{\frac{R^3}{M}} \approx \frac{2\sqrt{3}}{\sqrt{\gamma G \pi}} < \rho >^{-1/2} \tag{14}$$

Eigenvectors for a star with density gradient



Density and temperature gradients in star modify the frequencies of the overtones. Thus, observing the harmonics provides a probe of stellar interiors.

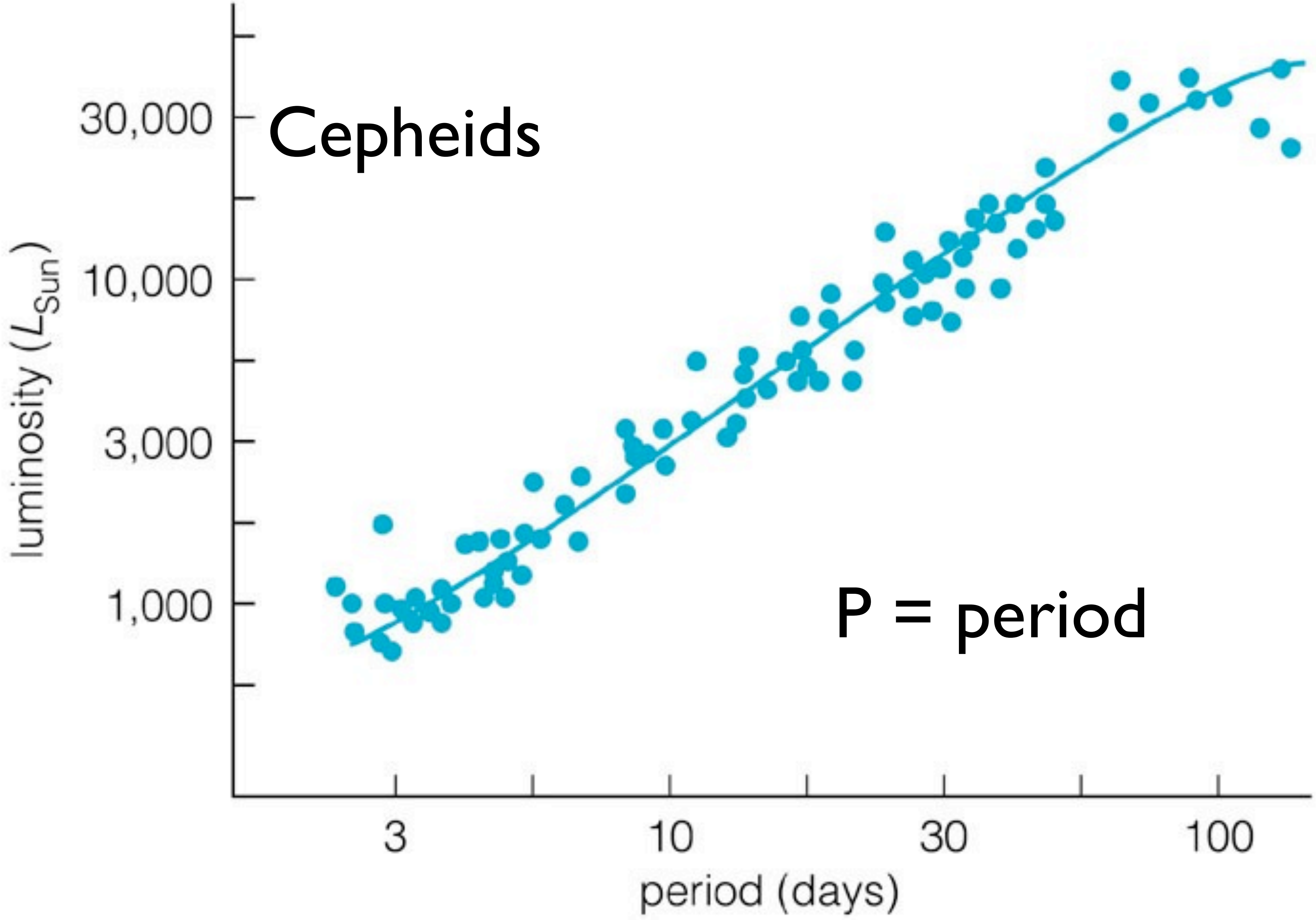
The Instability Strip



Cepheids and RR-Lyra stars appear in instability strip

Note: strip has a small relatively range in temperature but a large range in luminosity.

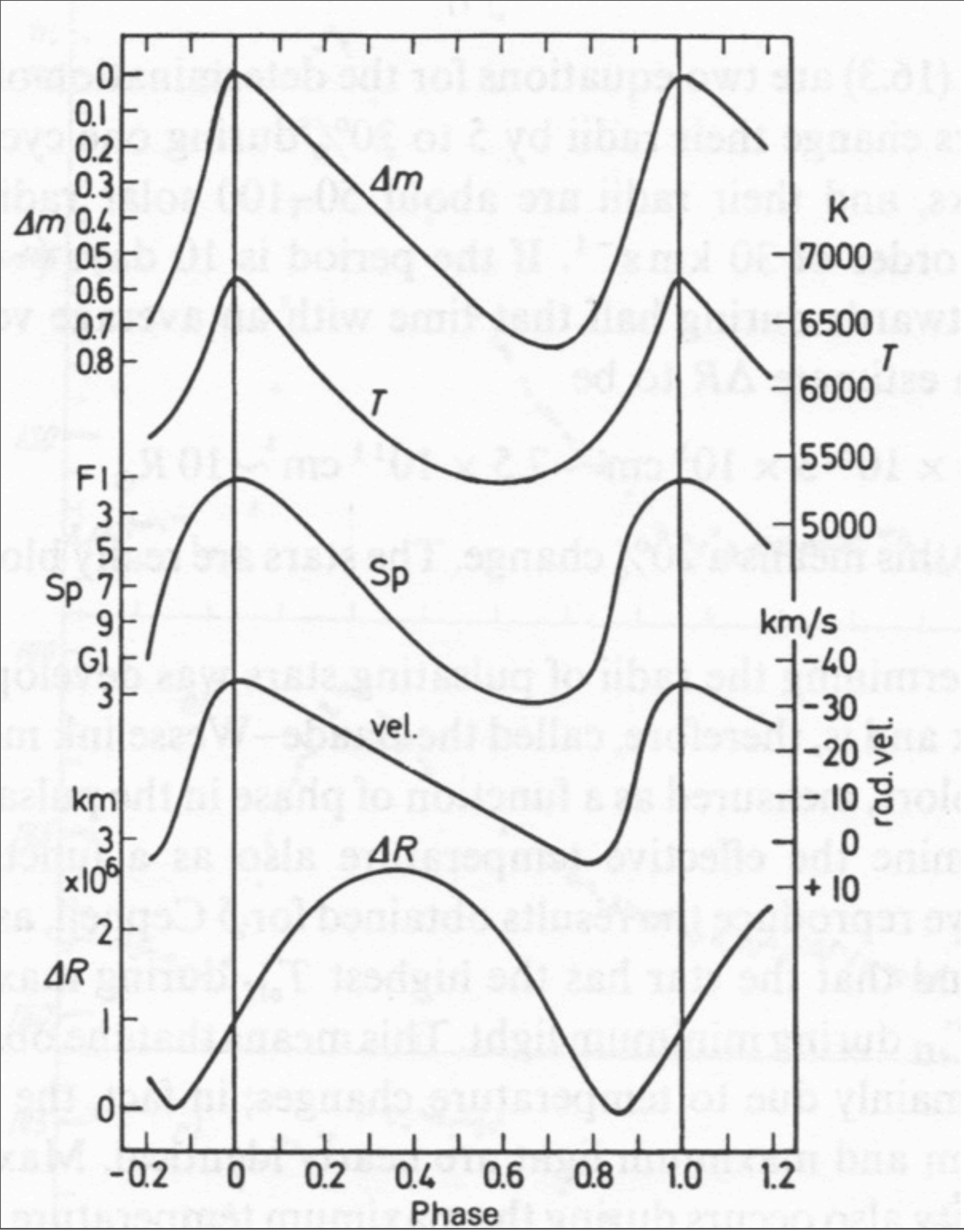
For a constant temperature, higher luminosity implies a larger radius and a larger Period



Cepheids

P = period

$$P \approx \frac{4R}{\sqrt{\gamma GM/R}} \approx \frac{4R}{\sqrt{\gamma G}}\sqrt{\frac{R^3}{M}} \approx \frac{2\sqrt{3}}{\sqrt{\gamma G\pi}} < \rho >^{-1/2} \qquad (14$$

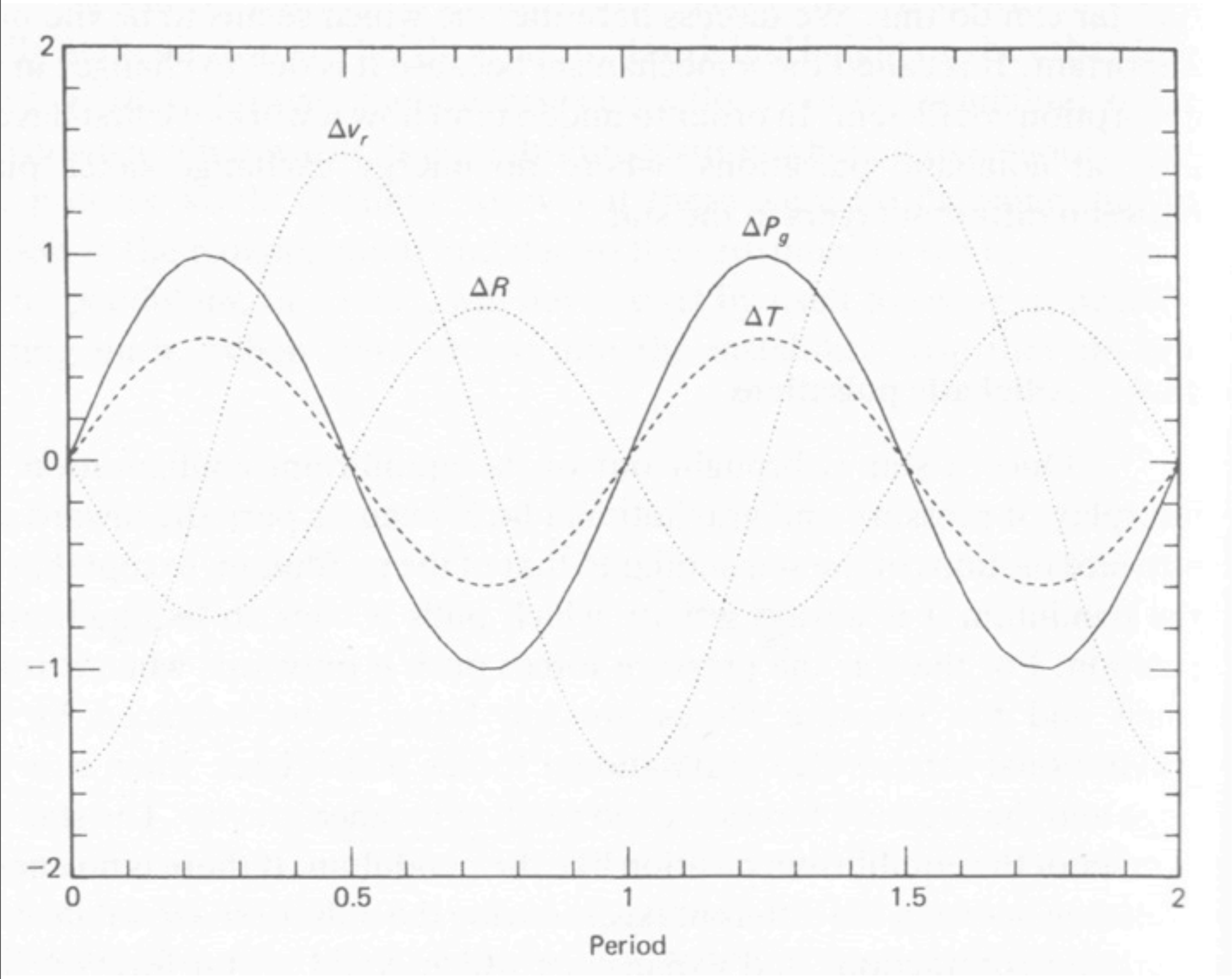


Cepheid Pulsations

Initial radii are
50-100 Rsun
Radius can change
by 10 Rsun!!

Bohm-Vintense

Adiabatic Pulsations

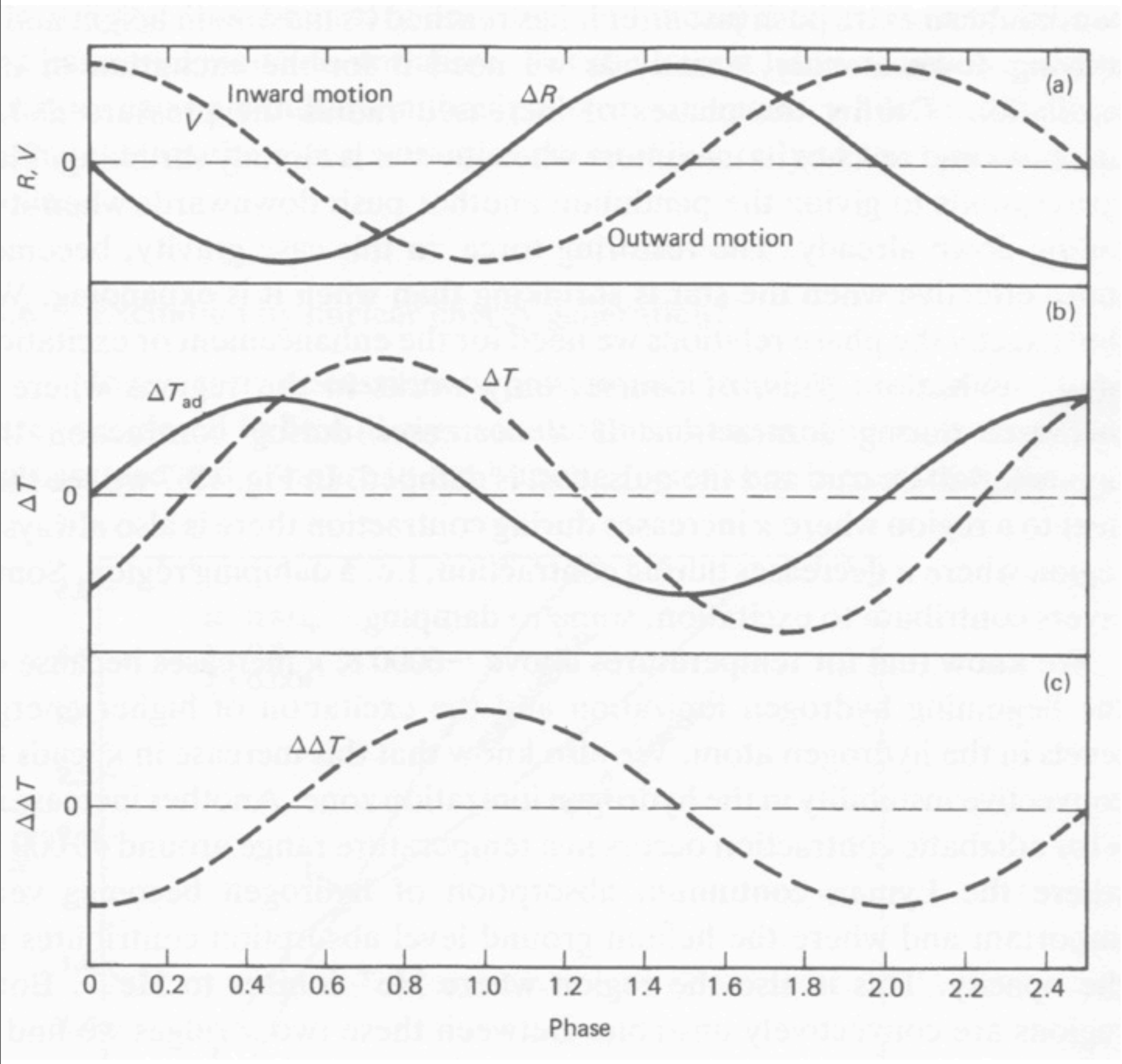


These pulsations would not cause large changes in radius or luminosity and would dampen as energy is radiated into space.

We need a mechanism to drive the pulsations.

Bohm-Vintense

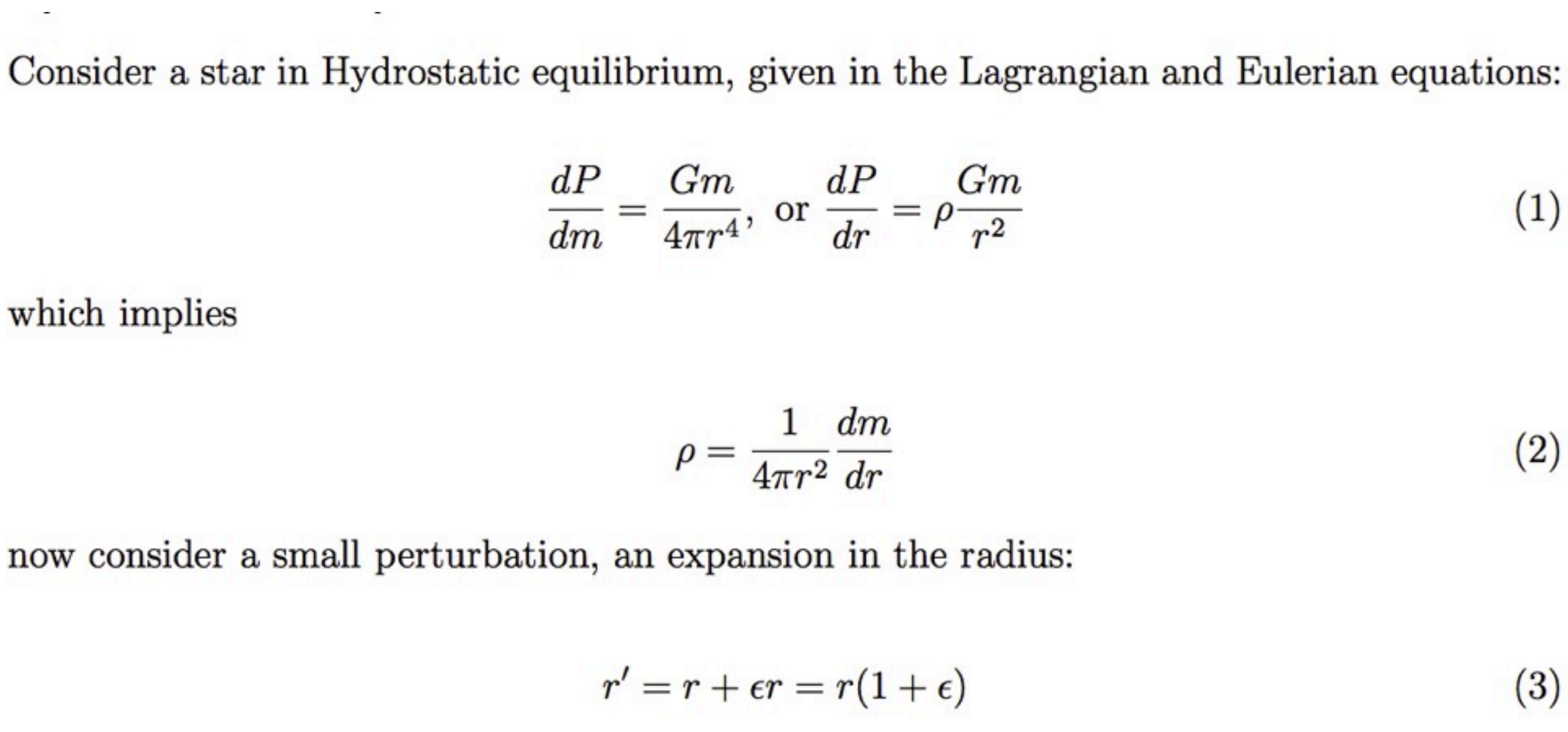
Amplified Pulsations in Cepheids



Higher temperatures during outward motion lead to amplified pulsations

Bohm-Vintense

Dynamical Instabilities



Consider a star in Hydrostatic equilibrium, given in the Lagrangian and Eulerian equations:

$$\frac{dP}{dm} = \frac{Gm}{4\pi r^4}, \text{ or } \frac{dP}{dr} = \rho\frac{Gm}{r^2} \qquad (1)$$

which implies

$$\rho = \frac{1}{4\pi r^2}\frac{dm}{dr} \qquad (2)$$

now consider a small perturbation, an expansion in the radius:

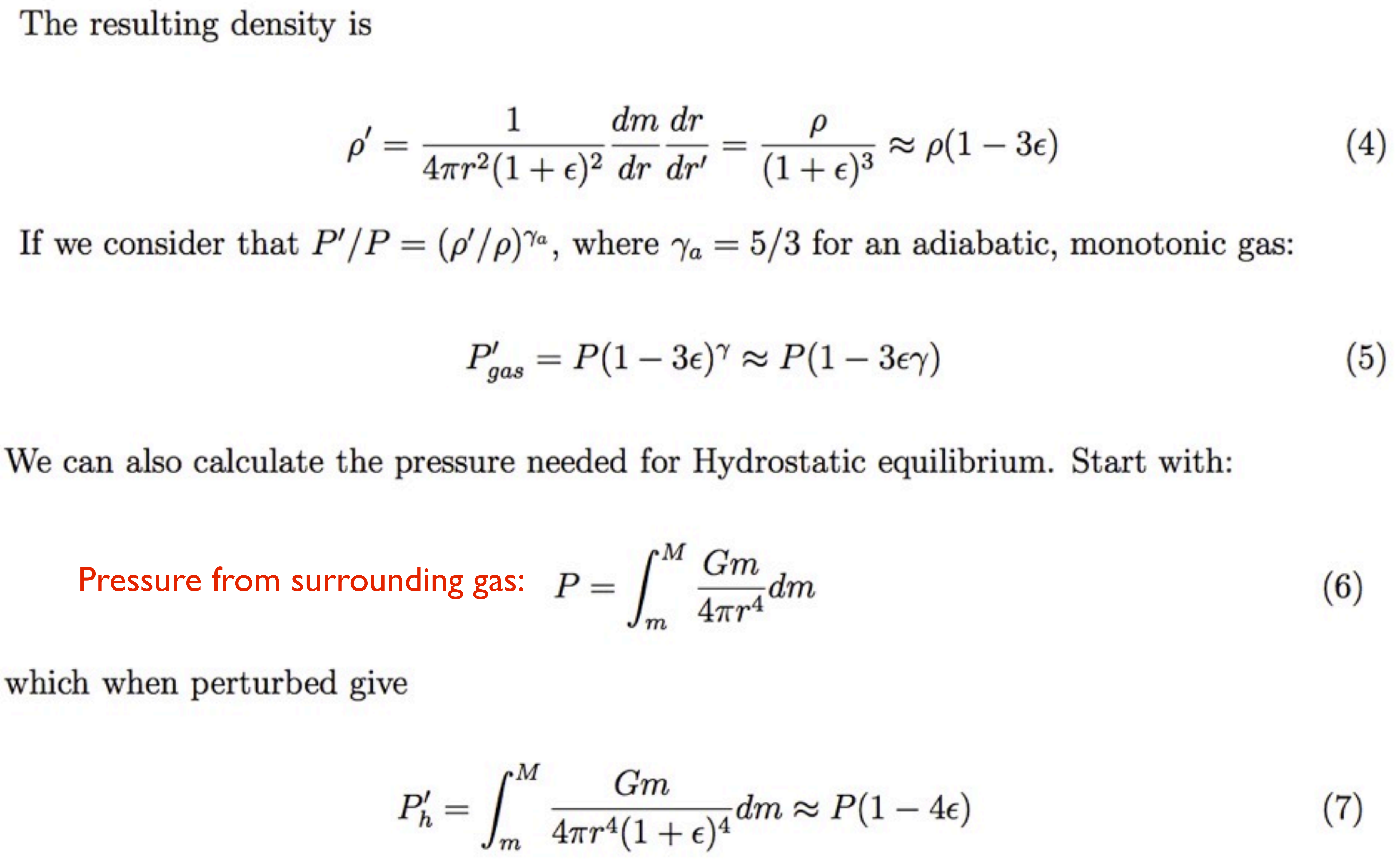
$$r' = r + \epsilon r = r(1+\epsilon) \qquad (3)$$

How does this change the pressure of the gas?
How does this change the pressure from the surrounding gas?

From Prialnik

Dynamical Instabilities

From Prialnik



The resulting density is

$$\rho' = \frac{1}{4\pi r^2(1+\epsilon)^2}\frac{dm}{dr}\frac{dr}{dr'} = \frac{\rho}{(1+\epsilon)^3} \approx \rho(1-3\epsilon) \qquad (4)$$

If we consider that $P'/P = (\rho'/\rho)^{\gamma_a}$, where $\gamma_a = 5/3$ for an adiabatic, monotonic gas:

$$P'_{gas} = P(1-3\epsilon)^{\gamma} \approx P(1-3\epsilon\gamma) \qquad (5)$$

We can also calculate the pressure needed for Hydrostatic equilibrium. Start with:

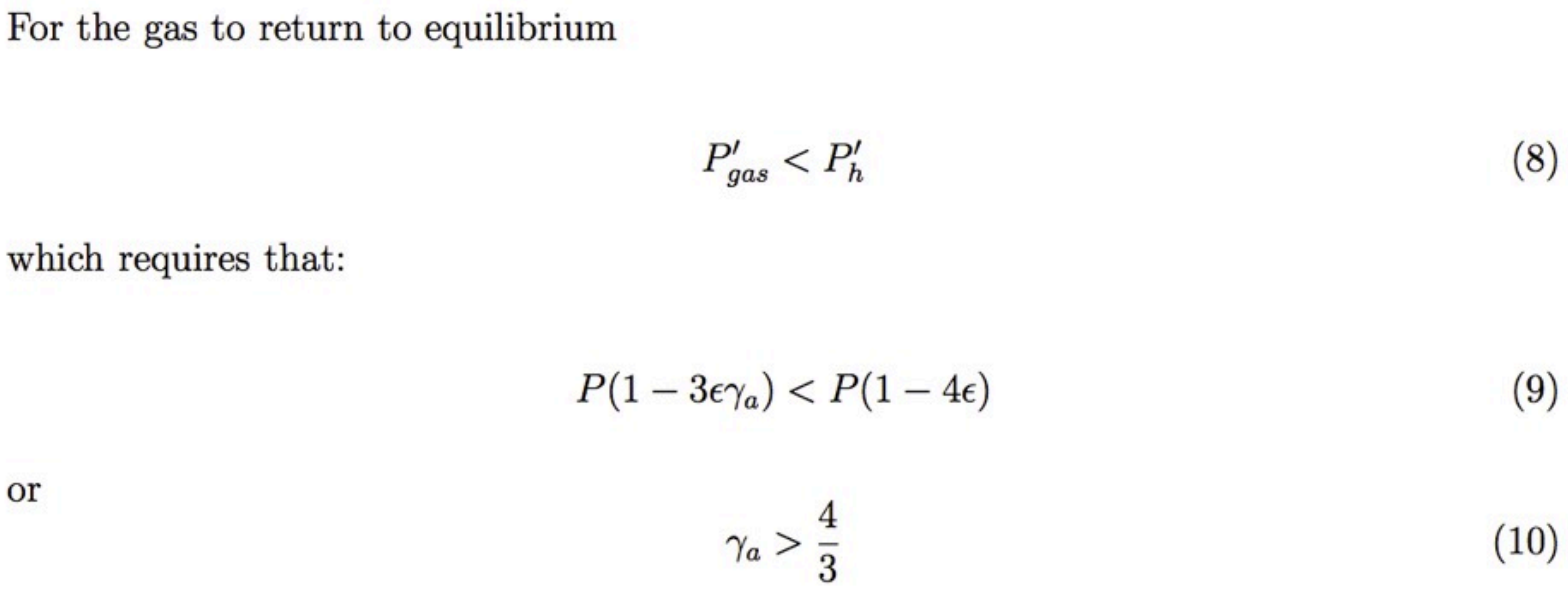
Pressure from surrounding gas: $$P = \int_m^M \frac{Gm}{4\pi r^4} dm \qquad (6)$$

which when perturbed give

$$P'_h = \int_m^M \frac{Gm}{4\pi r^4(1+\epsilon)^4} dm \approx P(1-4\epsilon) \qquad (7)$$

This pressure decreases since gravity weakens slightly if we expand the star!!

Dynamical Instabilities



For the gas to return to equilibrium

$$P'_{gas} < P'_h \tag{8}$$

which requires that:

$$P(1 - 3\epsilon\gamma_a) < P(1 - 4\epsilon) \tag{9}$$

or

$$\gamma_a > \frac{4}{3} \tag{10}$$

If $\gamma < 4/3$ then the gas pressure will increase faster than the pressure from the surrounding gas, leading to further expansion.

From Prialnik

Amplification in Cepheids: The κ mechanism

As gas contracts, temperature and pressure increase.

The causes κ to increase (rising on κ mountain)

The result is radiation is trapped, increasing heating.

Pressure does not decrease as fast with decreasing density
($\gamma < 4/3$)

The excess heating drives expansion beyond equilibrium value.

Excess heating ends when density and temperature push star to other side of κ mountain. At this point, the oscillations may be dampled by the gas radiating heat into space.

The κ Mountain

$$F = \frac{4}{3}\left(\frac{1}{\kappa}\right)\frac{dB}{dz}$$

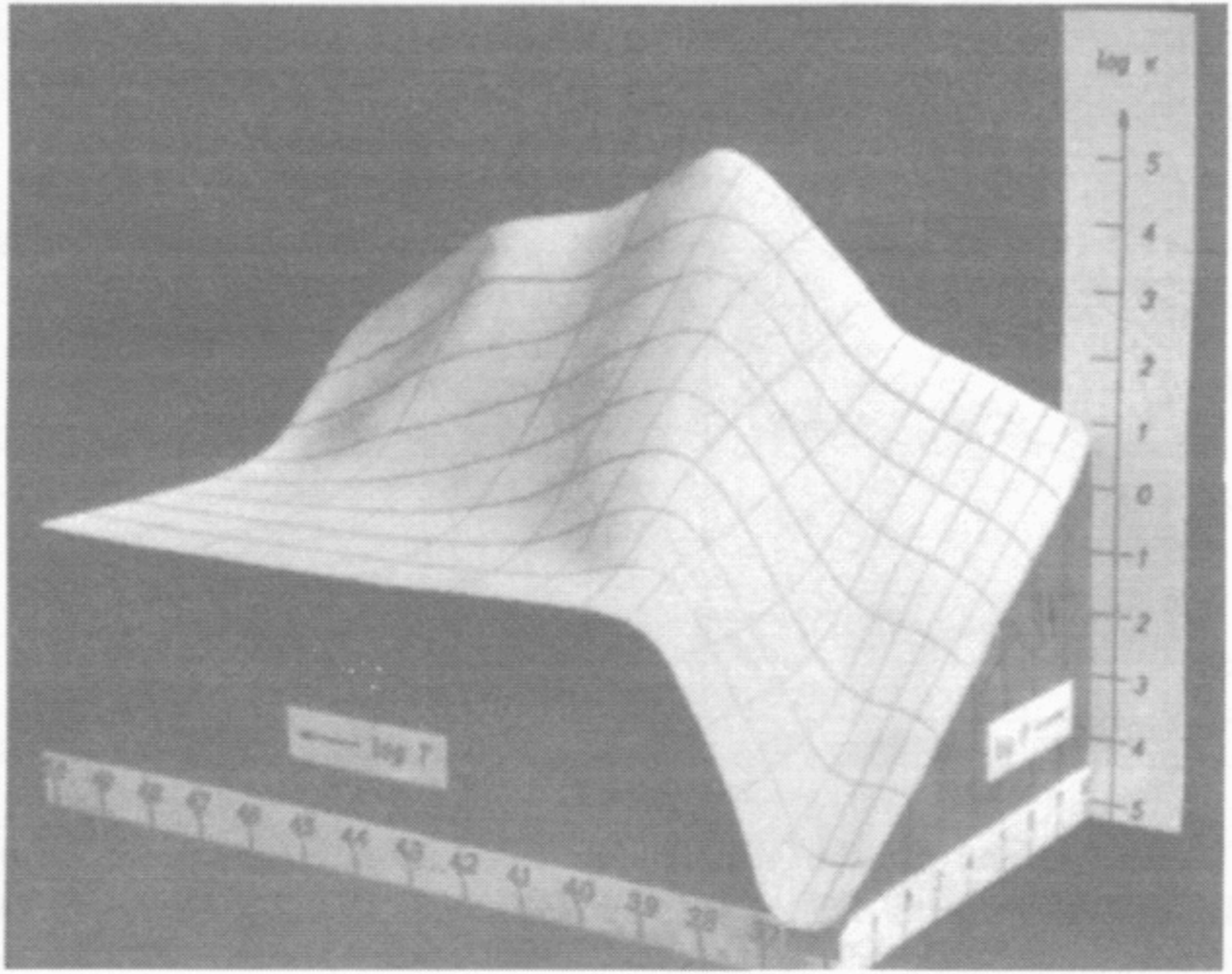
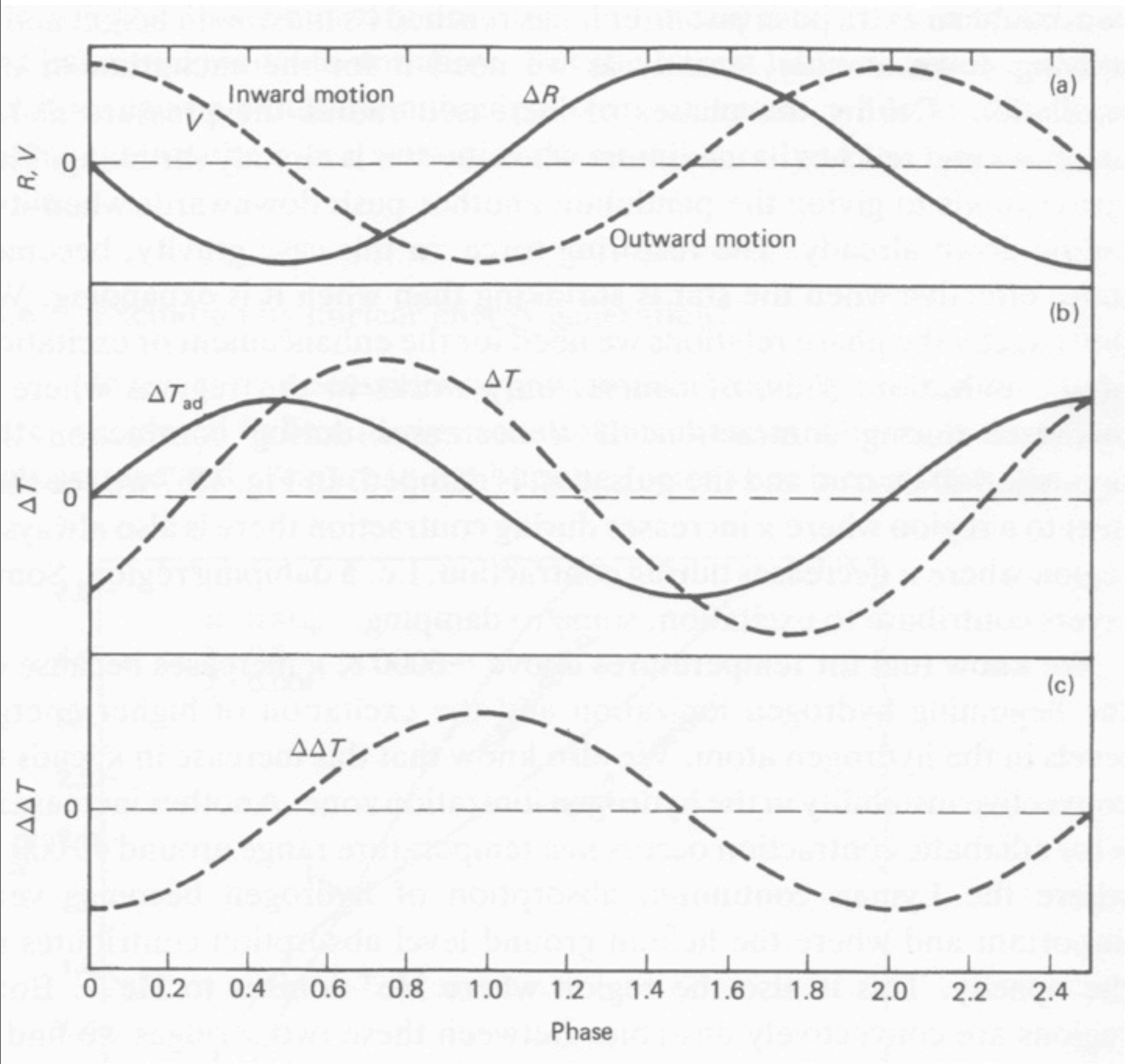


Fig. 18.7. The κ mountain as constructed by Baker and Kippenhahn (1962).

Bohm-Vintense

Amplified Pulsations in Cepheids



Higher temperatures during outward motion lead to amplified pulsations

Bohm-Vintense

Amplification in Cepheids: Ionization

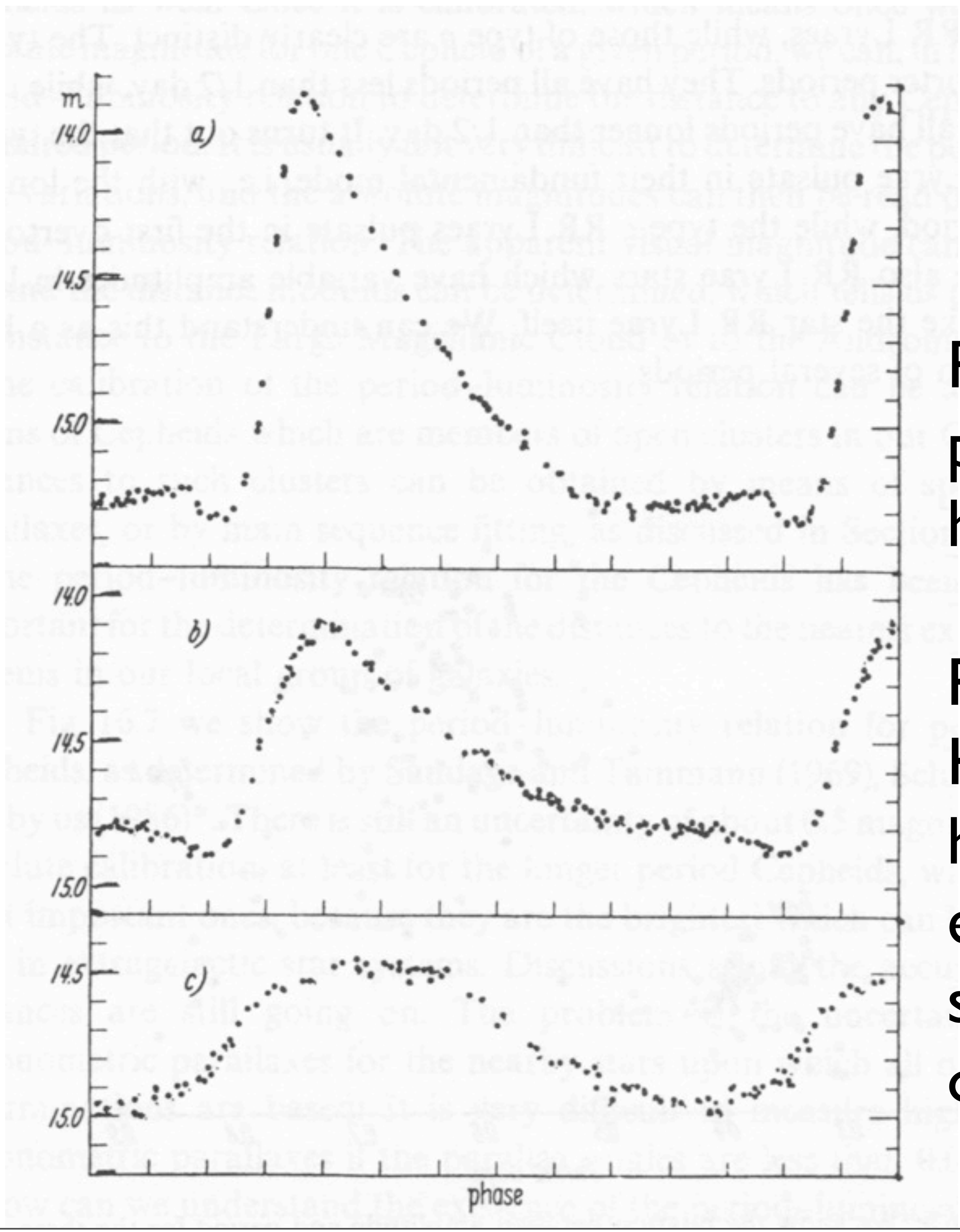
In adiabatic gas higher density implies higher pressure following adiabatic law.

As ionized gas compresses, the number of particles decreases due to recombination.

Pressure does not increase as fast with increasing density ($\gamma < 4/3$)

This is probably a secondary effect.

RR Lyrae Oscillations



RR Lyrae - solar mass population II stars on horizontal branch.

Population I stars during Helium core burning have convective envelopes and are not susceptible to RR-lyrae oscillations

Bohm-Vintense

RR Lyrae models
(Christy et al. 1966)

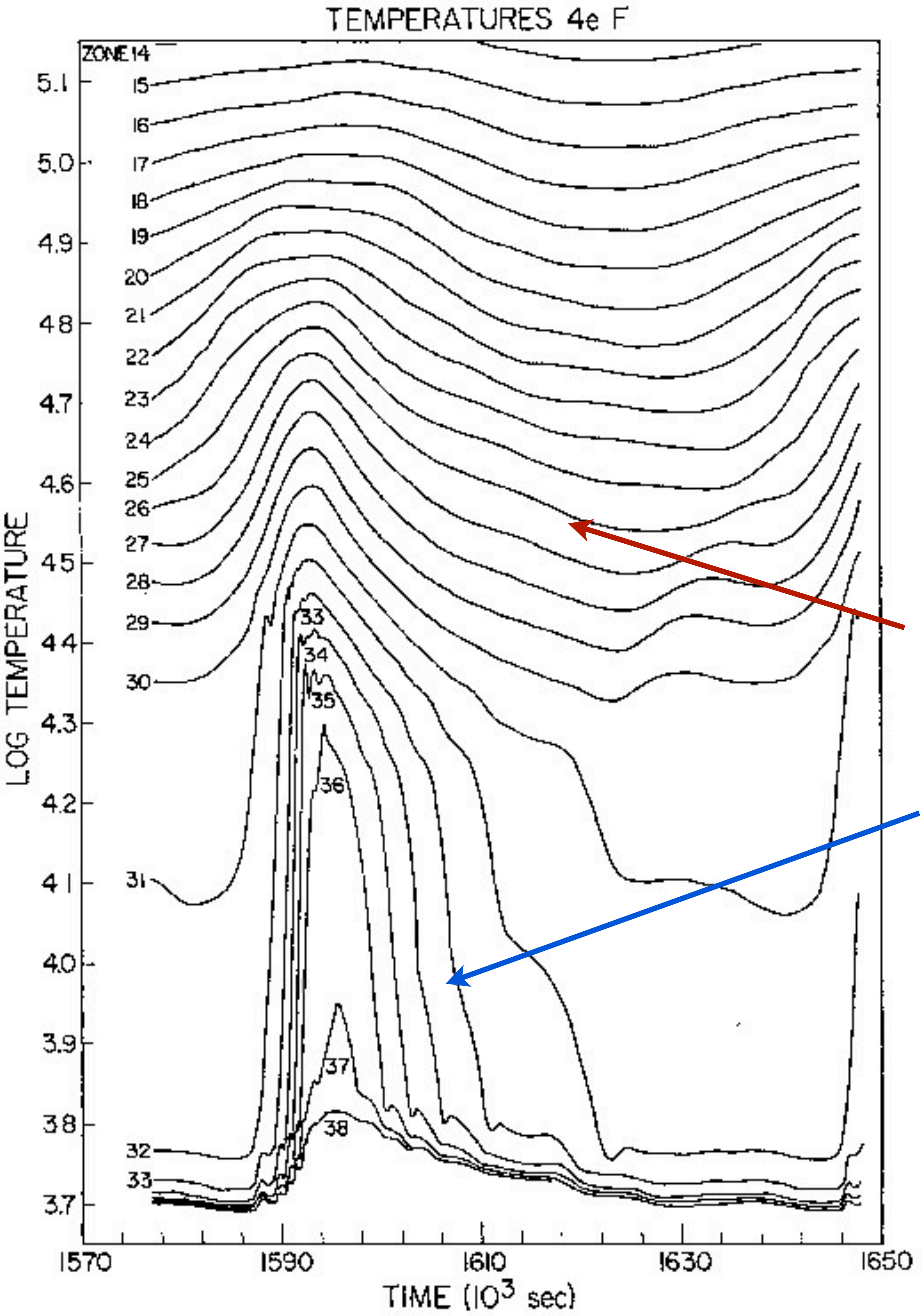


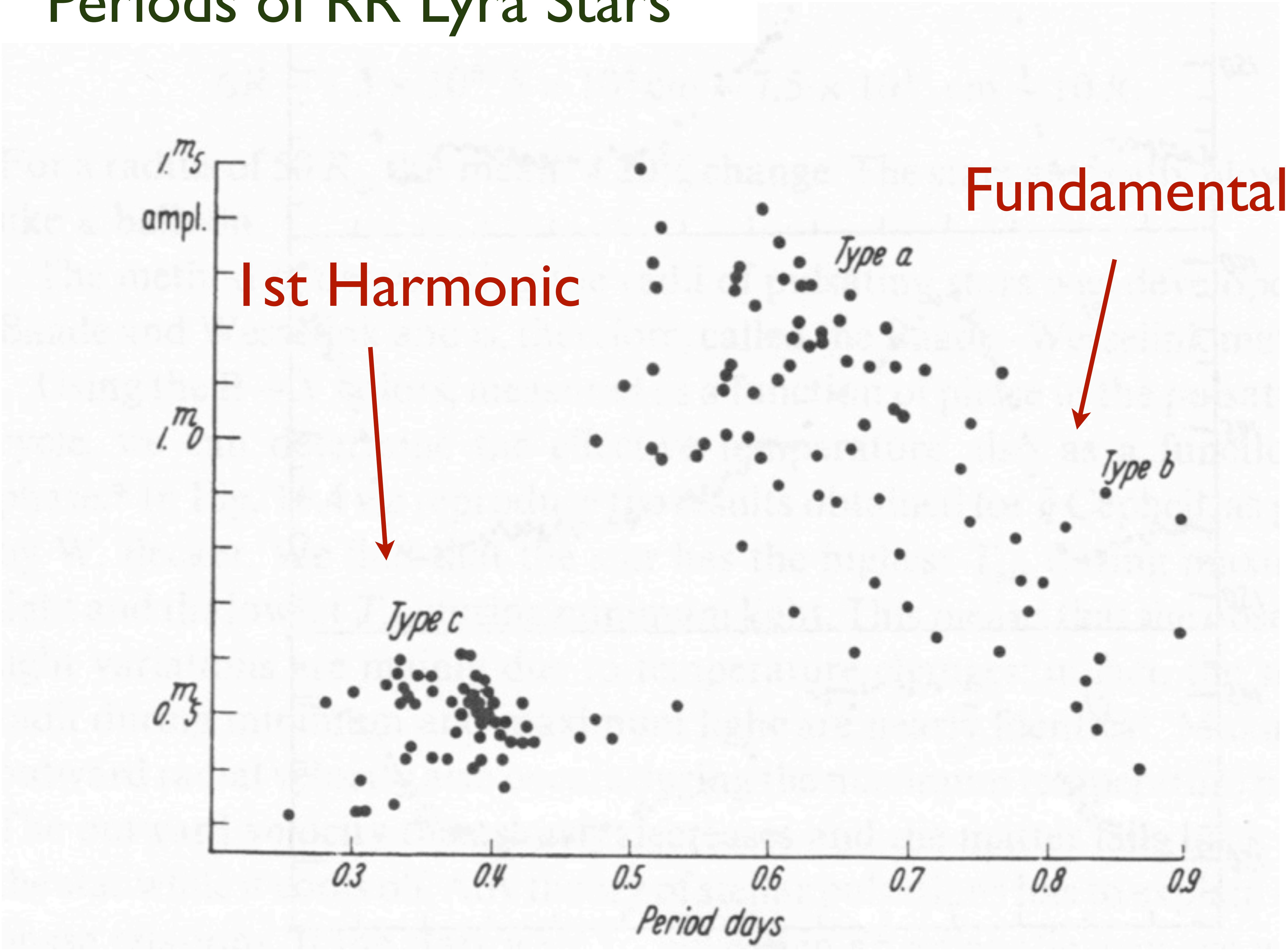
FIG. 1.—The temperature variation at various depths to 10^6 °K for model 4e F

Temperature

Ionization of HeII

Ionization of HeI and HI

Periods of RR Lyra Stars

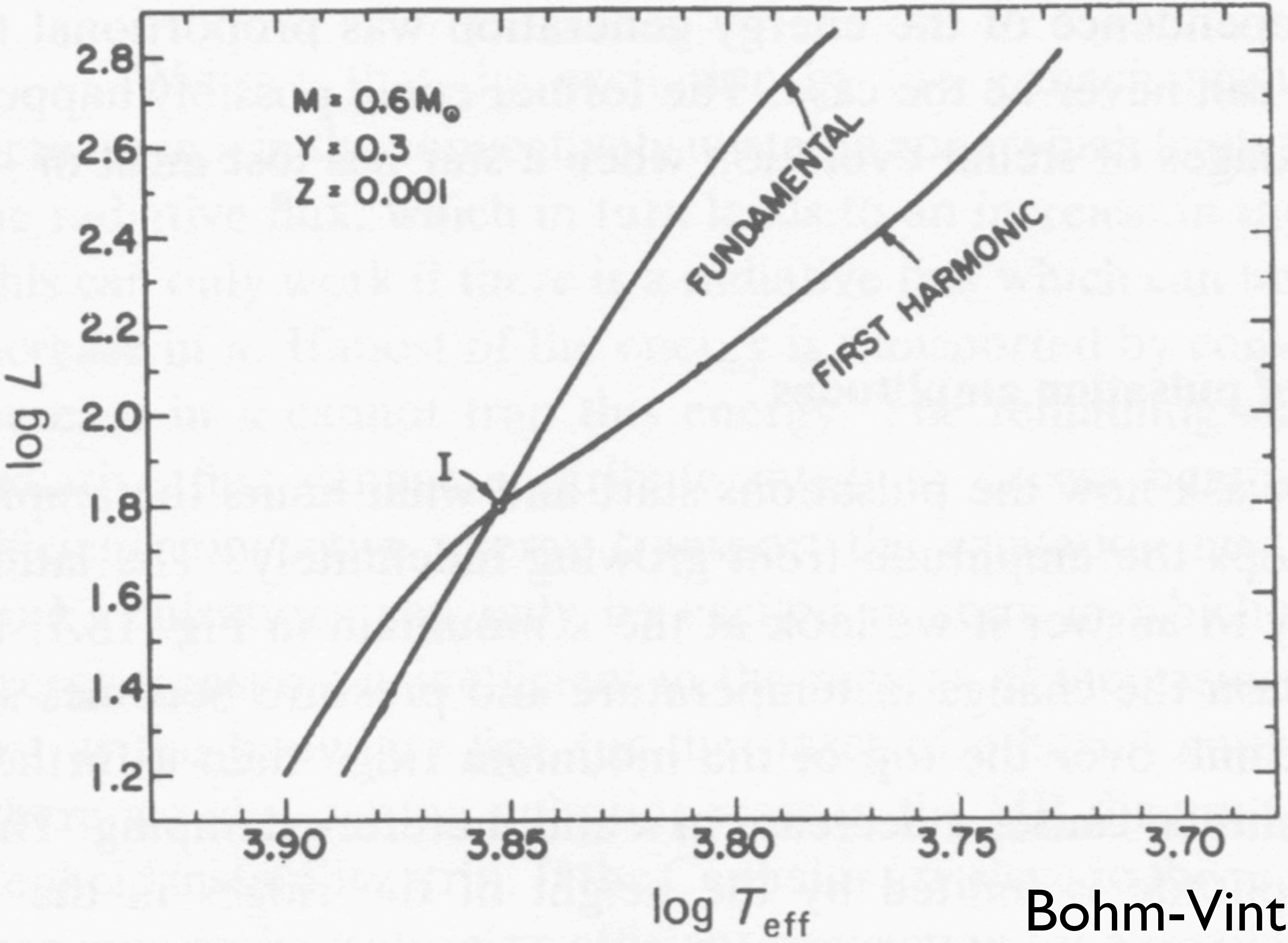


Fundamental

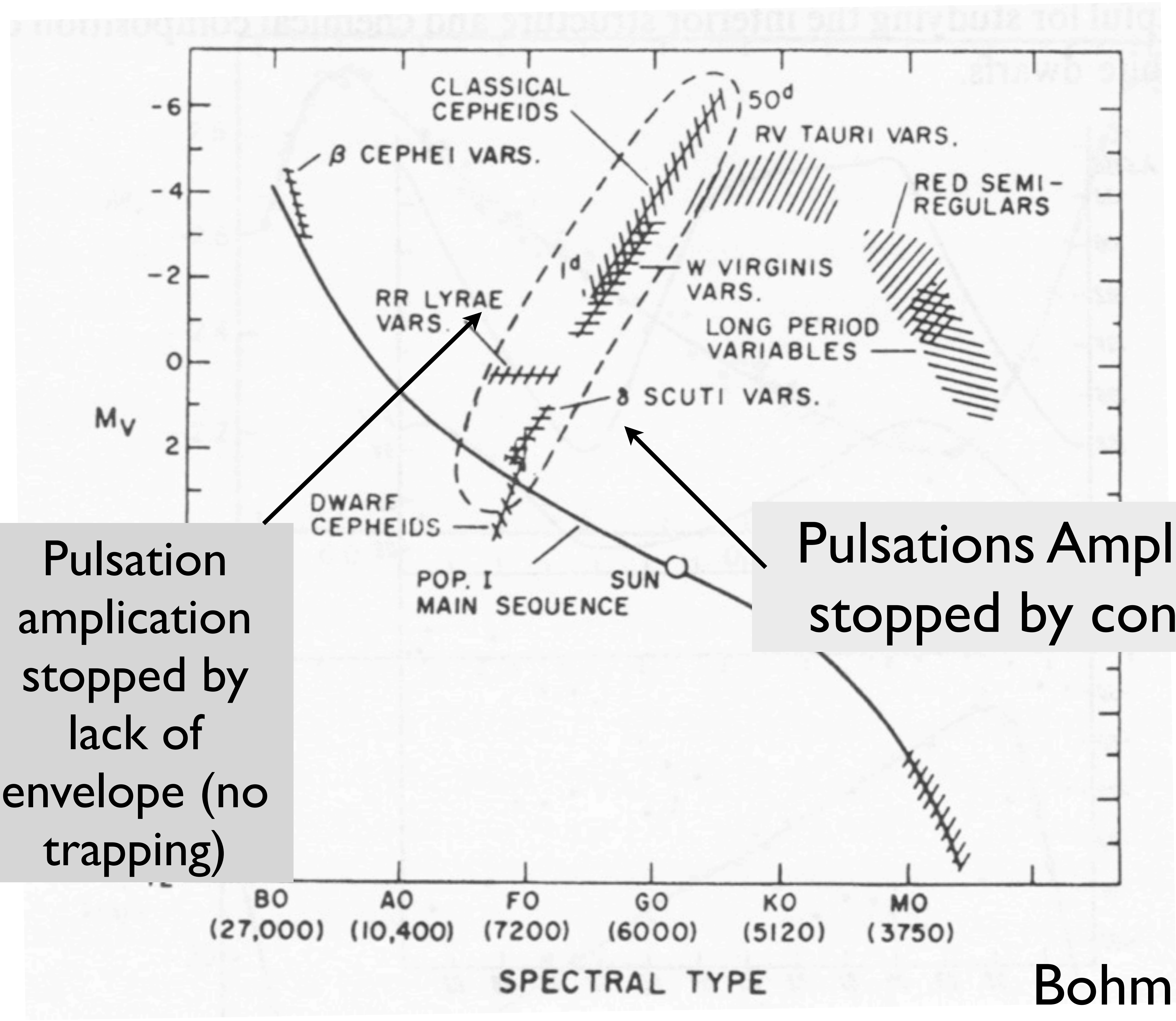
1st Harmonic

Bohm-Vintense

Some Stars can only sustain first harmonic



Bohm-Vintense



Pulsation amplication stopped by lack of envelope (no trapping)

Pulsations Amplification stopped by convection

Bohm-Vintense

Using Pulsations to Determine Stellar Masses with Kepler

The period relationship gives:

$$\left(\frac{\Delta\nu}{\Delta\nu_{\odot}}\right) \simeq \left(\frac{M}{\mathrm{M}_{\odot}}\right)^{0.5} \left(\frac{R}{\mathrm{R}_{\odot}}\right)^{-1.5},$$

The maximum power has been found to equal:

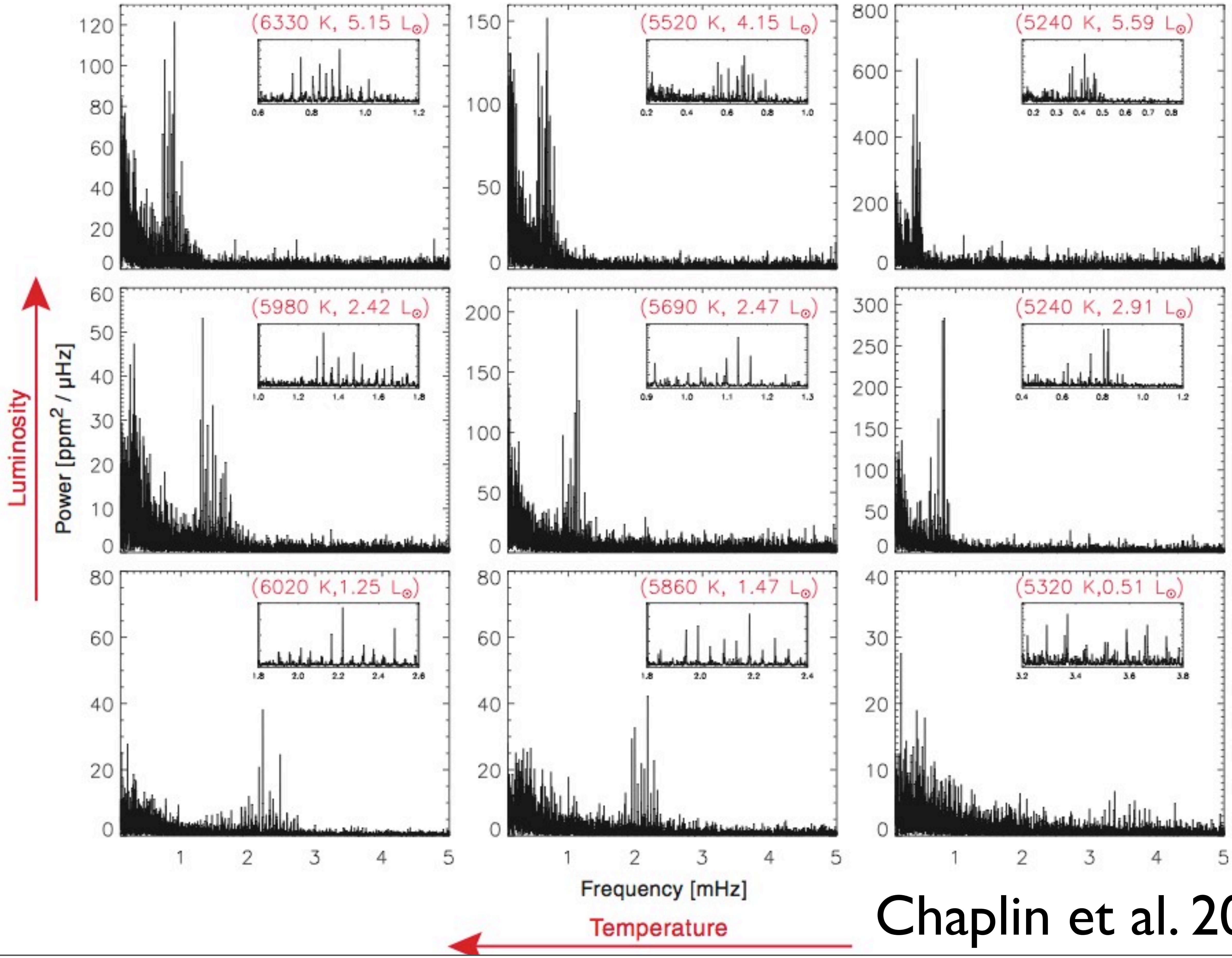
$$\left(\frac{\nu_{\max}}{\nu_{\max,\odot}}\right) \simeq \left(\frac{M}{\mathrm{M}_{\odot}}\right) \left(\frac{R}{\mathrm{R}_{\odot}}\right)^{-2} \left(\frac{T_{\mathrm{eff}}}{\mathrm{T}_{\mathrm{eff},\odot}}\right)^{-0.5}.$$

Giving:

$$\left(\frac{R}{\mathrm{R}_{\odot}}\right) \simeq \left(\frac{\nu_{\max}}{\nu_{\max,\odot}}\right) \left(\frac{\Delta\nu}{\Delta\nu_{\odot}}\right)^{-2} \left(\frac{T_{\mathrm{eff}}}{\mathrm{T}_{\mathrm{eff},\odot}}\right)^{0.5},$$

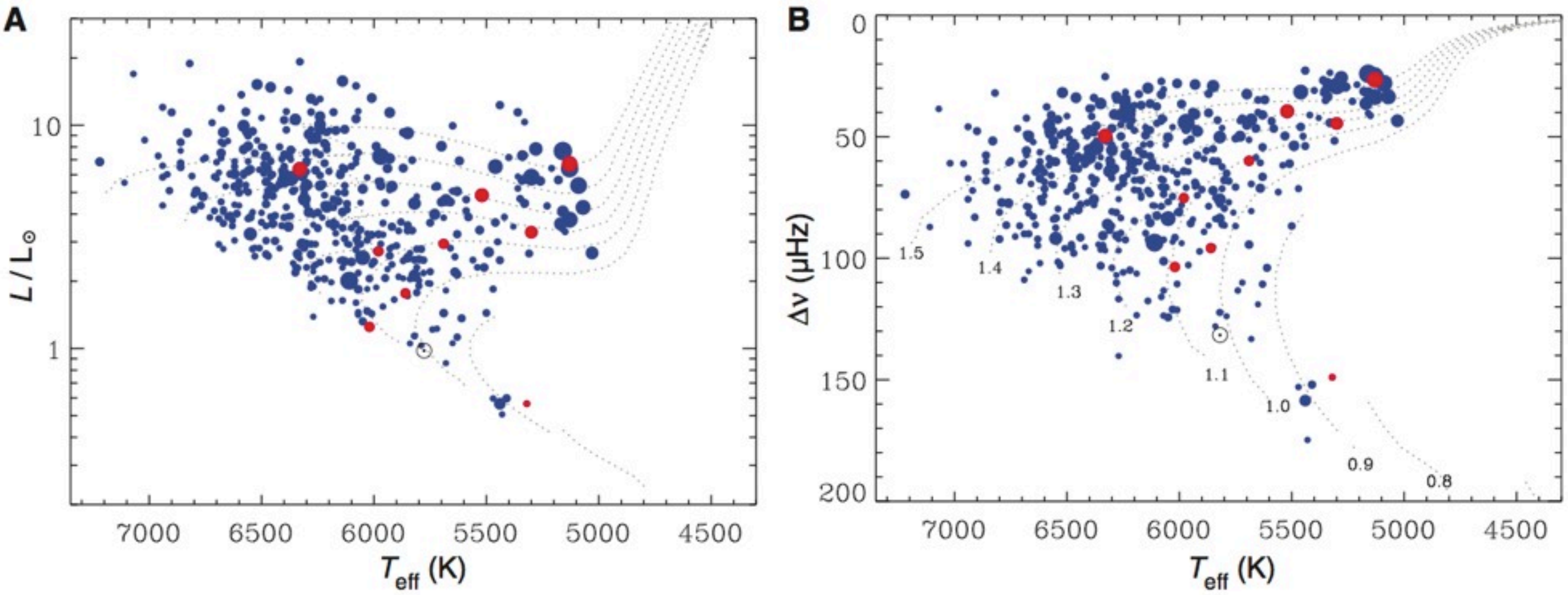
$$\left(\frac{M}{\mathrm{M}_{\odot}}\right) \simeq \left(\frac{\nu_{\max}}{\nu_{\max,\odot}}\right)^{3} \left(\frac{\Delta\nu}{\Delta\nu_{\odot}}\right)^{-4} \left(\frac{T_{\mathrm{eff}}}{\mathrm{T}_{\mathrm{eff},\odot}}\right)^{1.5}.$$

Pulsation Spectra for Dwarfs and Subgiants



Chaplin et al. 2011

Pulsations of ~solar mass dwarfs and subgiants in Kepler field: red dots are sources in previous field. Dotted lines are mass tracks.



Δν is spacing between harmonics, equal to fundamental frequency. Increased mass leads to lower Δν.

Chaplin et al. 2011

Summary

Stars often show pulsations and oscillations. Famous examples are Cepheus and RR Lyrae variables, were large pulsations lead to substantial change in brightness.

These pulsations are a diagnostic of the internal properties of the stars. A common pulsation is an acoustic standing wave in the star, driven by convection. The period scales as the $1/(\text{density})^{0.5}$

Adiabatic oscillations are damped and do not create large changes in luminosity.

Cepheids and RR Lyrae stars are driven by pulsationals instabilities which cause deviations from adiabatic behavior in the gas. They are found in a narrow instability strip in the HR diagram.

Pulsations in subgiants measured by Kepler can give direct measurements of mass and radius.