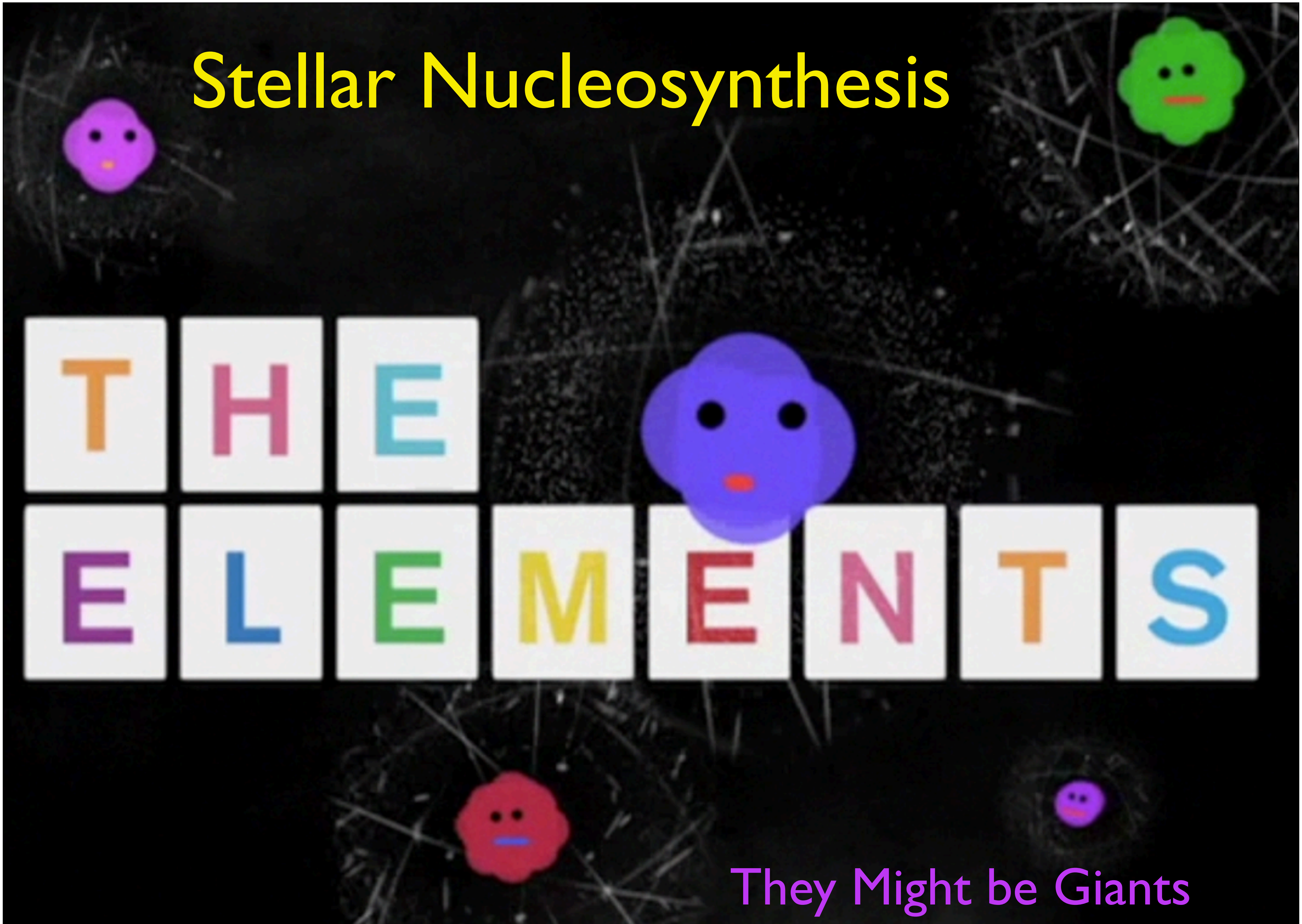
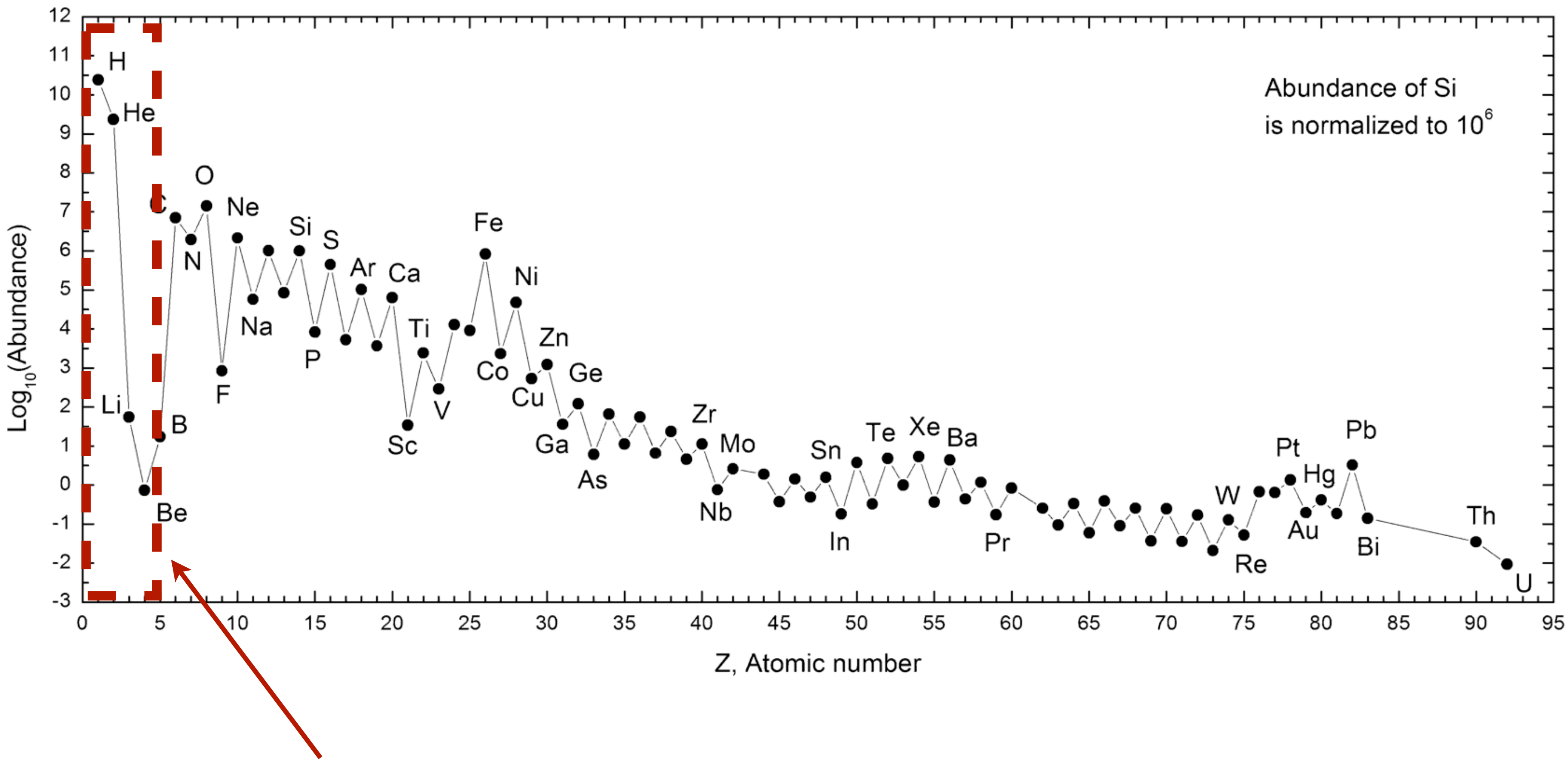


http://www.youtube.com/watch?v=Uy0m7jnyv6U

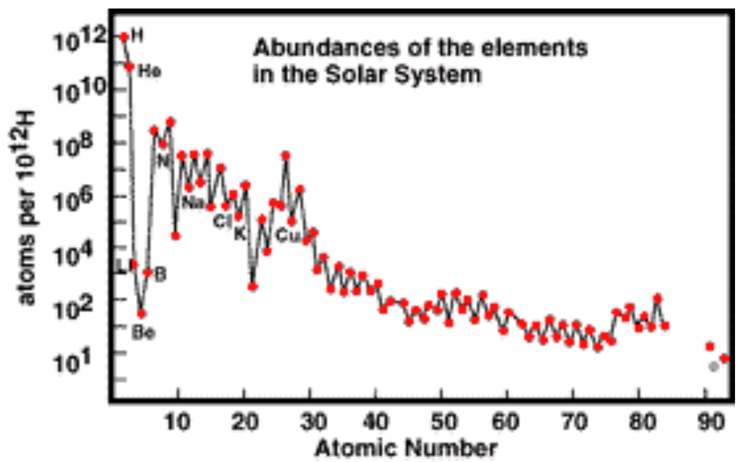
Elemental Abundances in the Solar System

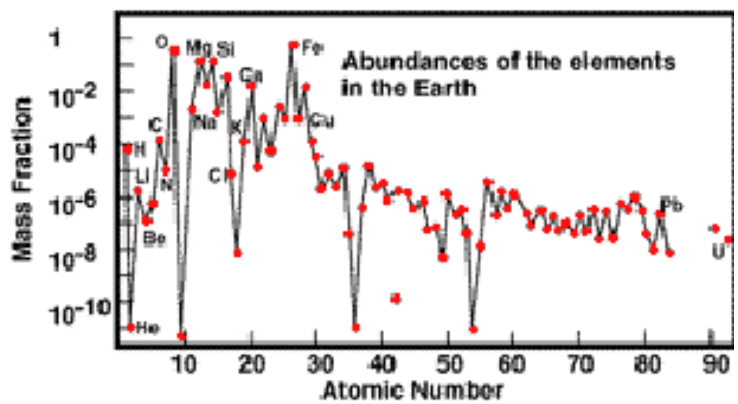


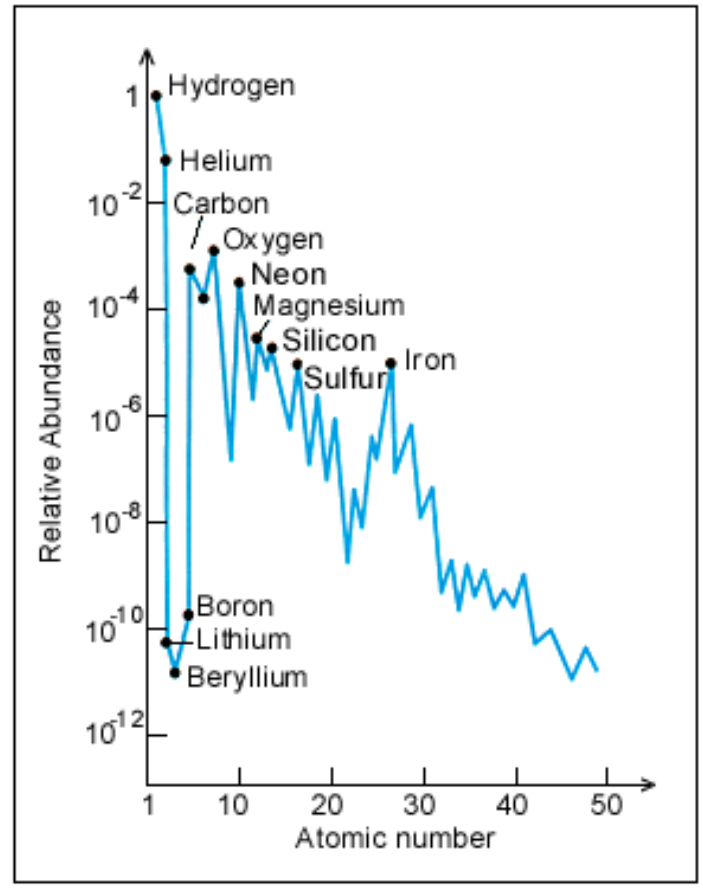
Synthesized in the Big Bang

We need to consider the creation of all the other elements in stars.

Abundances of nuclei







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Review: Nuclear Fusion in Stars

Consider the nuclear reaction:

$$I(A_i, Z_i) + J(A_J, A_J) \rightarrow K(A_K, Z_K) + L(A_L, Z_L) \quad (1)$$

The energy released from this reaction is

$$Q_{ijk} = (M_I + M_J - M_K - M_L)c^2 \quad (2)$$

we can rewrite this equation as:

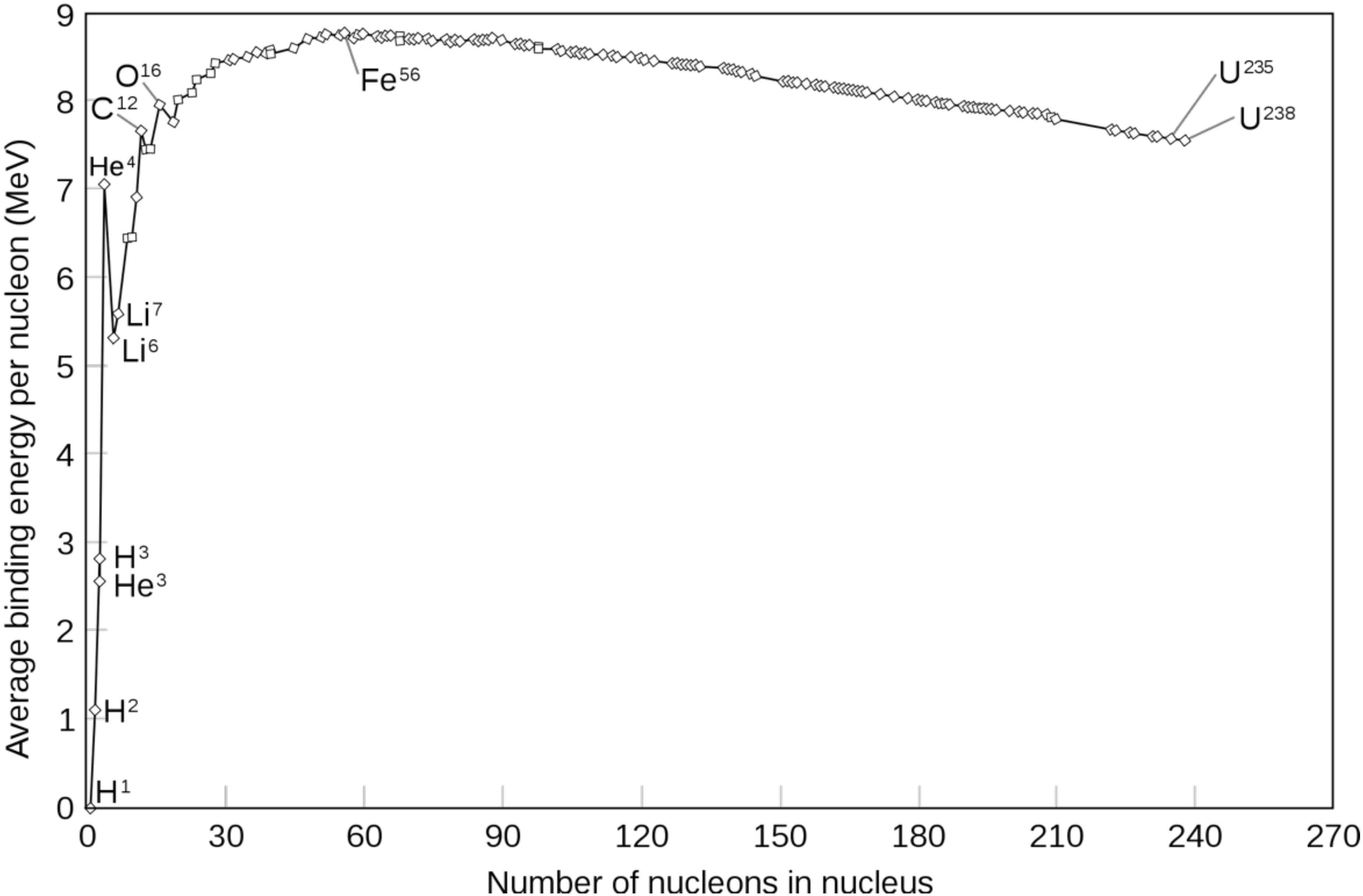
$$Q_{ijk} = [(M_I - A_i m_H) +)(M_J - A_J m_H) - (M_K - A_K m_H) - (M_L - A_L m_H)]c^2 \quad (3)$$

because $(A_i + A_j - A_K - A_L)m_H c^2 = 0$. We then define

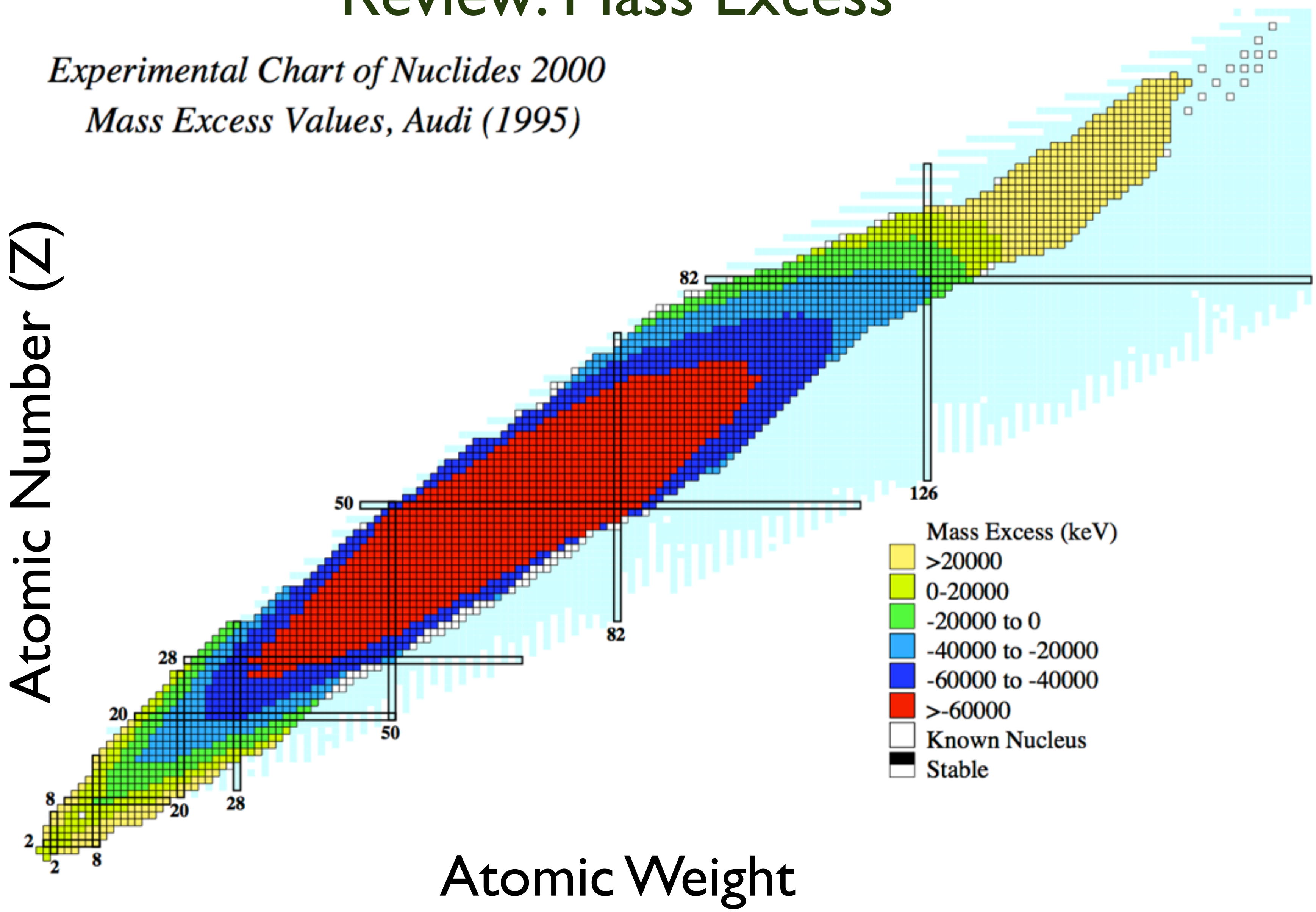
$$\Delta M_I = (M_I - A_i m_H)c^2 \quad (4)$$

as the mass excess, and can be found in tabulations. The binding energy per nucleon is then $\Delta M_I / A_I$.

Review: Binding Energy of Nuclei



Review: Mass Excess



http://ie.lbl.gov/systematics/chart_qme.pdf

Rate of Reactions

The rate of reactions can written can be written as:

$$Rate = R_{ijk} n_i n_j \tag{5}$$

where R_{ijk} is the velocity times the cross section integrated over the velocity range

$$R_{ijk} = \int \sigma_{ijk} f(v) \tag{6}$$

where is the $f(v)$ is the maxwellian distribution for the gas temperature, $v = v_i - v_j$ and reduced mass:

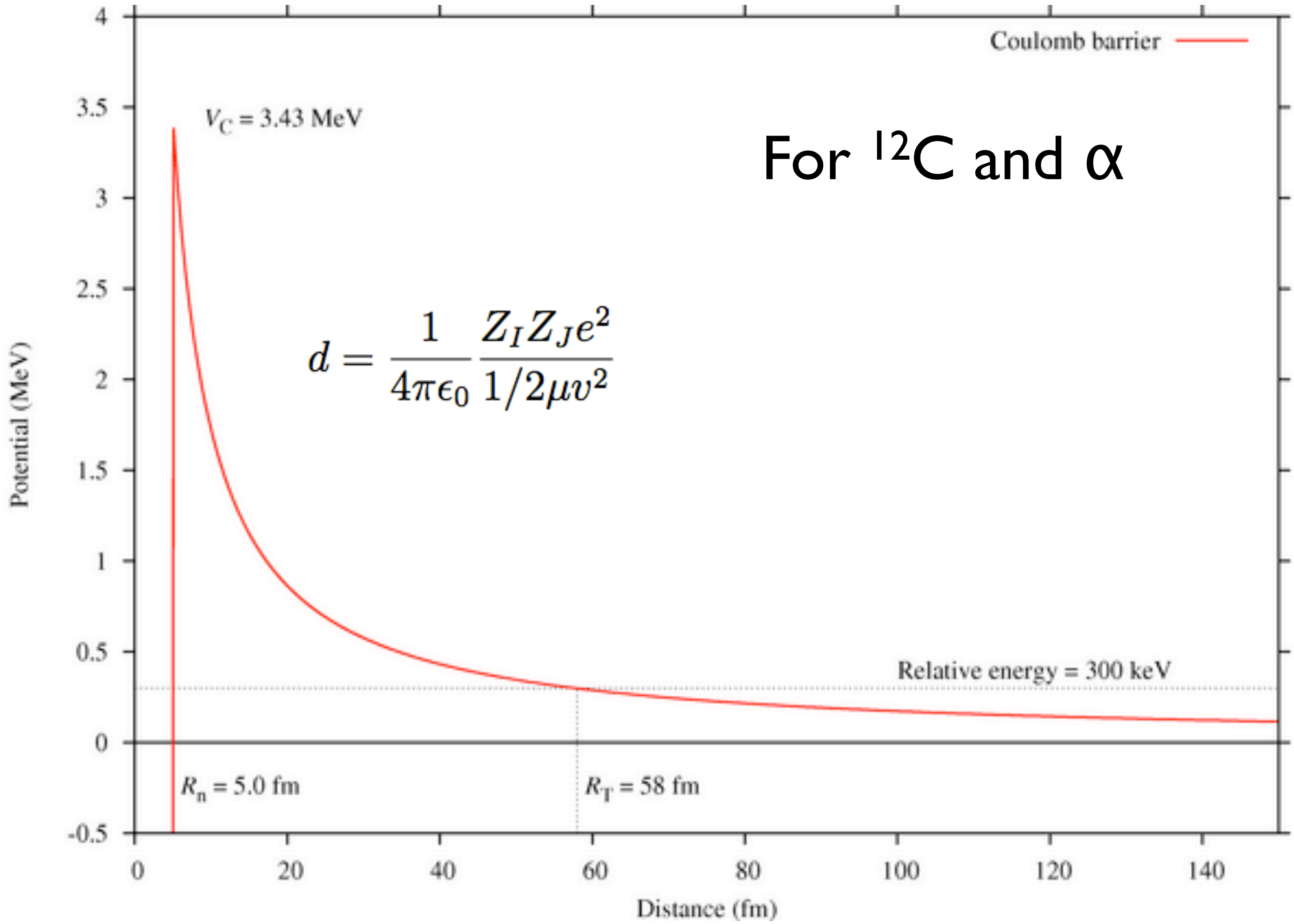
$$\mu = m_H \frac{A_I A_J}{A_I + A_J} \tag{7}$$

The energy produced per gram of material is then

$$q = \frac{\rho}{m_H^2} \sum \frac{1}{1 + \delta_{ij}} \frac{X_i}{A_i} \frac{X_J}{A_J} R_{ijk} Q_{ijk} \tag{8}$$

where Q_{ijk} is the net energy released.

The Coloumb Barrier



For ^{12}C and α

$$d = \frac{1}{4\pi\epsilon_0}\frac{Z_I Z_J e^2}{1/2\mu v^2}$$

$1 \text{ fm} = 10^{-15} \text{ m}$

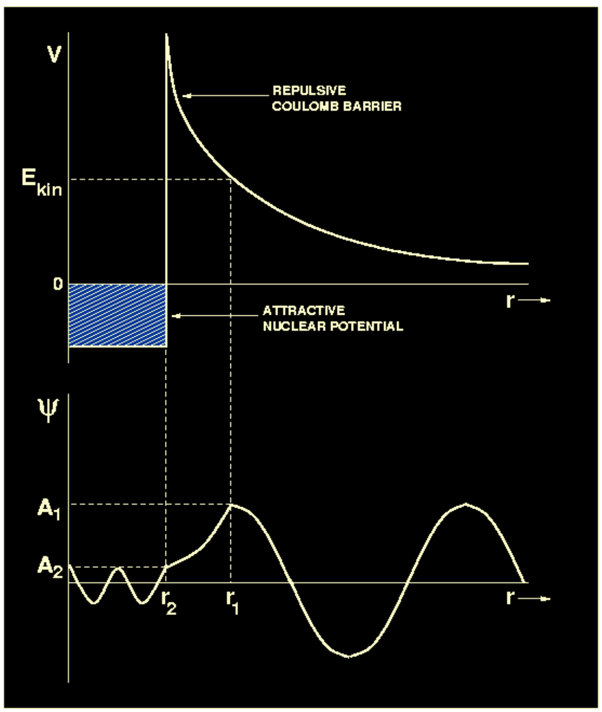
http://nu.phys.laurentian.ca/~fleurot/fusionrate/

Occurrence of fusion reactions

Now will discuss the conditions under which fusion can occur – and whether such conditions exist in stellar interiors

Nuclei interact through four forces of physics – only electromagnetic and strong nuclear important here

Two positively charged nuclei must overcome coulomb barrier (long range force $\propto 1/r^2$), to reach separation distances where strong force dominates (10^{-15} m, typical size of nucleus)



Schematic plots

V (potential energy) vs nuclei separation distance

Wave function representing penetration of a potential barrier by nucleus with kinetic energy of approach E_{kin} (below barrier height).

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Quantum Tunneling

As derived in your Quantum Mechanics courses, there is a finite probability for a particle to penetrate the Coulomb barrier as if "tunnel" existed.

Quantum effect discovered by George Gamow (1928) in connection with radioactivity.

Penetration probability (calculated by Gamow) is given as:

$$e^{\frac{-\pi Z_1 Z_2 e^2}{\varepsilon_0 h v}}$$

Hence this increases with *v* (particle velocity), but we know *v* will be Maxwellian distribution for ideal gas. Hence fusion probability is product

$$prob(fusion) \propto e^{\frac{-\pi Z_1 Z_2 e^2}{\varepsilon_0 h v}} e^{-\frac{mv^2}{2kT}}$$

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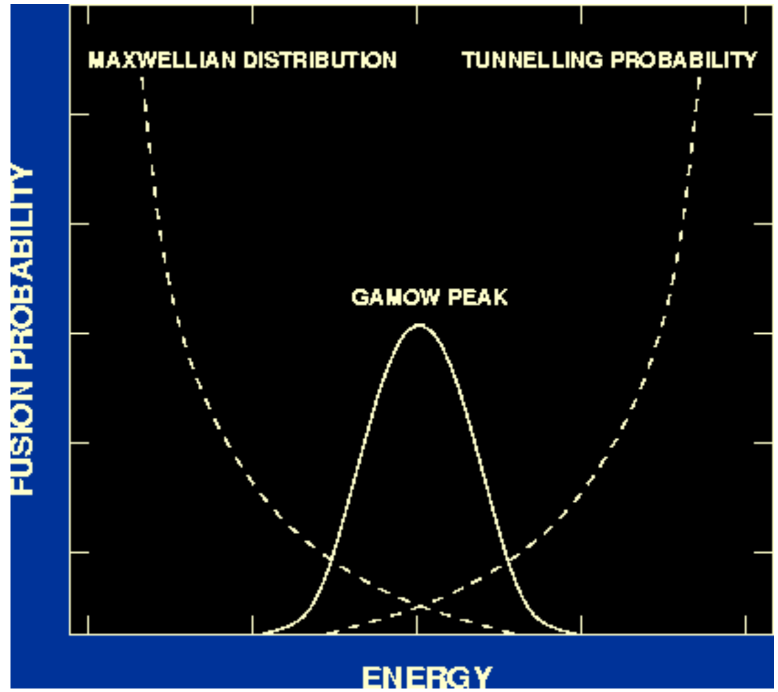
The Gamow peak

Schematically this is plotted, and the fusion most likely occurs in the energy window defined as the Gamow Peak.

The Gamow peak is the product of the Maxwellian distribution and tunnelling probability. The area under the Gamow peak determines the reaction rate.



The higher the electric charges of the interacting nuclei, the greater the repulsive force, hence the higher the E_{kin} and T before reactions occur.

Highly charged nuclei are obviously the more massive, so reactions between light elements occur at lower T than reactions between heavy elements.

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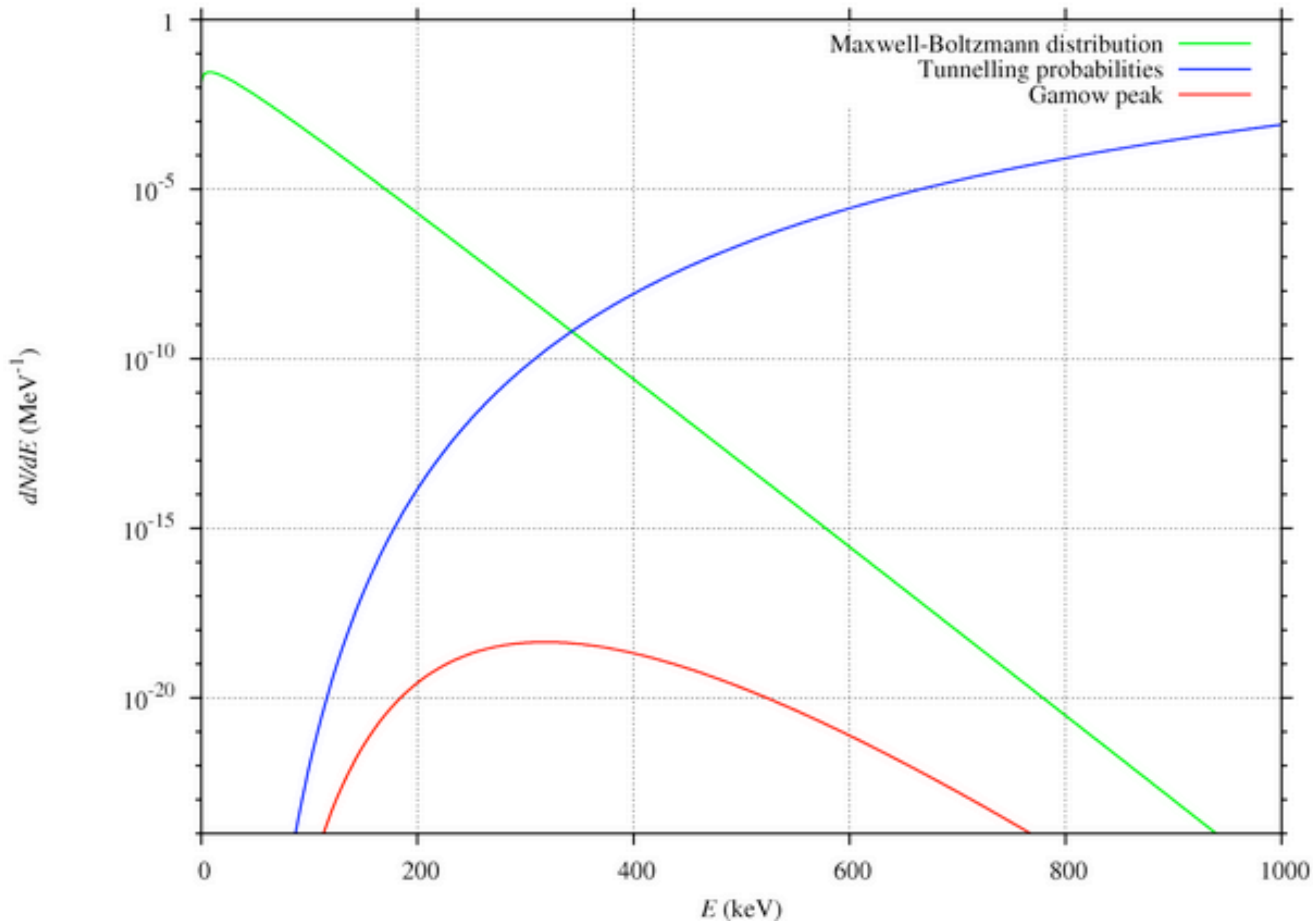
The Gamow Peak



Resonances

298 PRINCIPLES OF STELLAR EVOLUTION AND NUCLEOSYNTHESIS

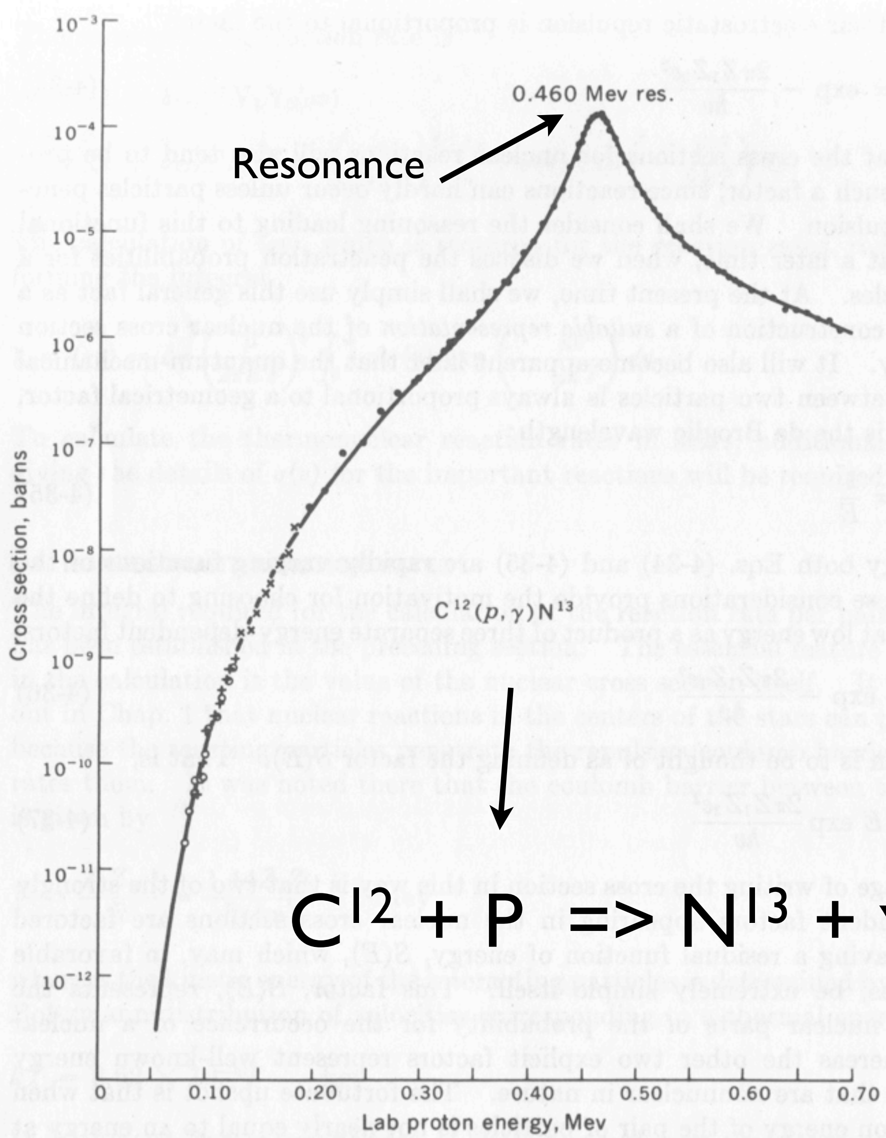


Fig. 4-4 The measured cross section for the reaction $C^{12}(p,\gamma)N^{13}$ as a function of laboratory proton energy. A four-parameter theoretical curve has been fitted to the experimental points. An extrapolation to $E_p = 0.025$ Mev, which is an interesting energy for this reaction in astrophysics, appears treacherous. (*Courtesy of W. A. Fowler and J. L. Vogl.*)

Calculation of Gamow peak assumes that the process is far from a resonances. Resonances can increase the cross section of a reaction significantly and consequently increase the reaction rate.

From Clayton: Principles of Stellar Evolution and Nucleosynthesis

Hydrogen and helium burning

The most important series of fusion reactions are those converting H to He (H-burning).
As we shall see this dominates ~90% of lifetime of nearly all stars.

Fusion of 4 protons to give one ^{4}He is completely negligible

Reaction proceeds through steps – involving close encounter of 2 particles

We will consider the main ones: the **PP-chain** and the **CNO cycle**

The PP Chain

The PP chain has three main branches called the PPI, PPII and PPIII chains.

PPI Chain	PPII Chain	PPIII Chain
1 $p + p \rightarrow d + e^+ + \nu_e$	3' $^3He + {}^4He \rightarrow {}^7Be + \gamma$	4'' $^7Be + p \rightarrow {}^8B + \gamma$
2 $d + p \rightarrow {}^3He + \gamma$	4' $^7Be + e^- \rightarrow {}^7Li + \nu_e$	5'' $^8B \rightarrow {}^8Be + e^+ + \nu_e$
3 $^3He + {}^3He \rightarrow {}^4He + 2p$	5' $^7Li + p \rightarrow {}^4He + {}^4He$	6'' $^8Be \rightarrow 2{}^4He$

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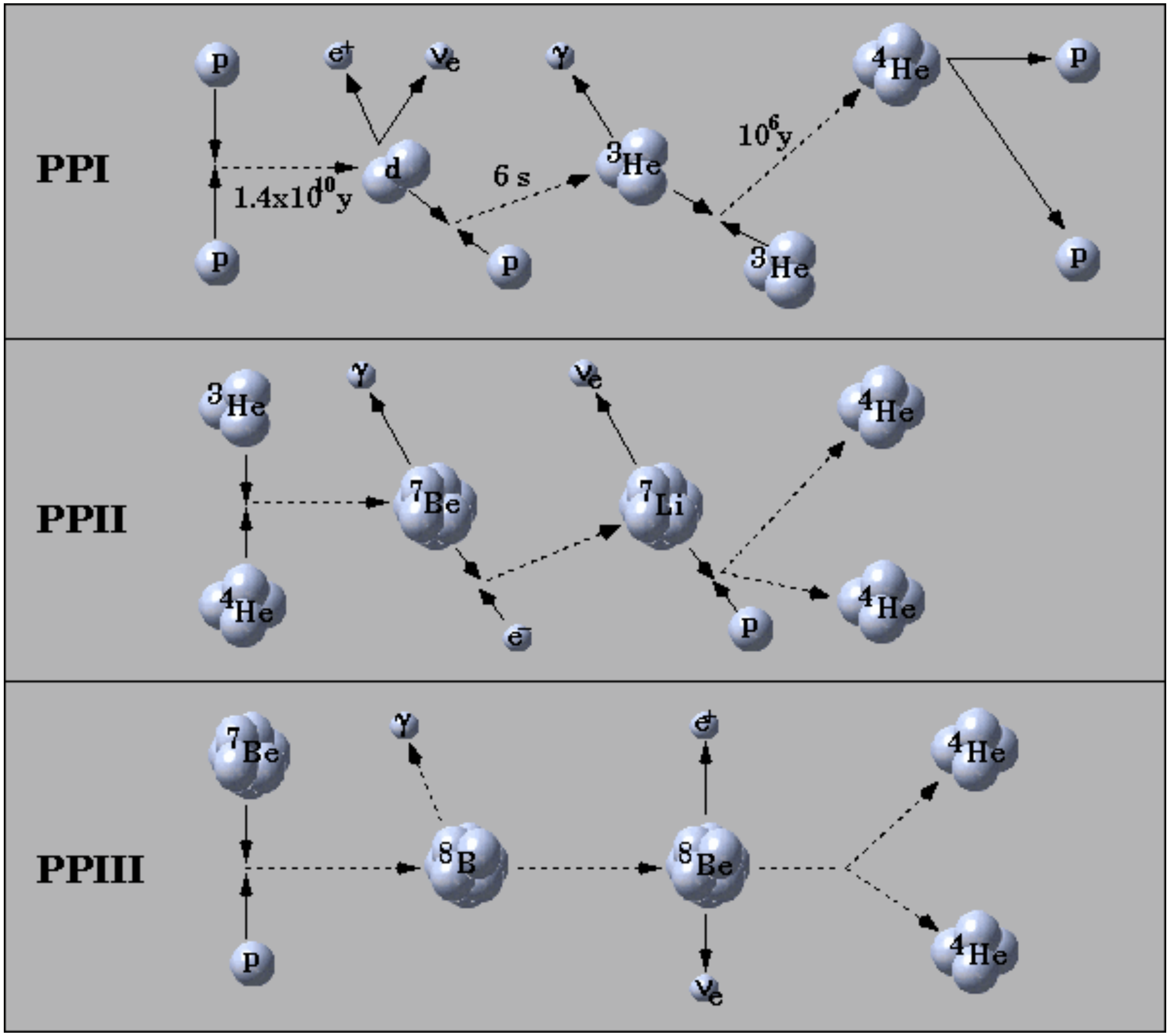

Relative importance of PPI and PPII chains (*branching ratios*) depend on conditions of H-burning (T, ρ , abundances). The transition from PPI to PPII occurs at temperatures in excess of 1.3×10^7 K.

Above 3×10^7 K the PPIII chain dominates over the other two, but another process takes over in this case.

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The Three P-P Chains

Fig. 5-7 The alternative PP chains. When He^3 is destroyed by the capture of an alpha particle, the chain is completed either through PPII or PPIII, depending upon the fate of the Be^7 nucleus.

Proton-proton chain

PP I

$$H^1 + H^1 \rightarrow D^2 + e^+ + \nu$$

$$D^2 + H^1 \rightarrow He^3 + \gamma$$

$$He^3 + He^3 \rightarrow He^4 + 2H^1$$

or

PP II

$$He^3 + He^4 \rightarrow Be^7 + \gamma$$

$$Be^7 + e^- \rightarrow Li^7 + \nu$$

$$Li^7 + H^1 \rightarrow He^4 + He^4$$

or

PP III

$$Be^7 + H^1 \rightarrow B^8 + \gamma$$

$$B^8 \rightarrow Be^8 + e^+ + \nu$$

$$Be^8 \rightarrow 2He^4$$

From Clayton: Principles of Stellar Evolution and Nucleosynthesis

When does a given P-P chain dominate?

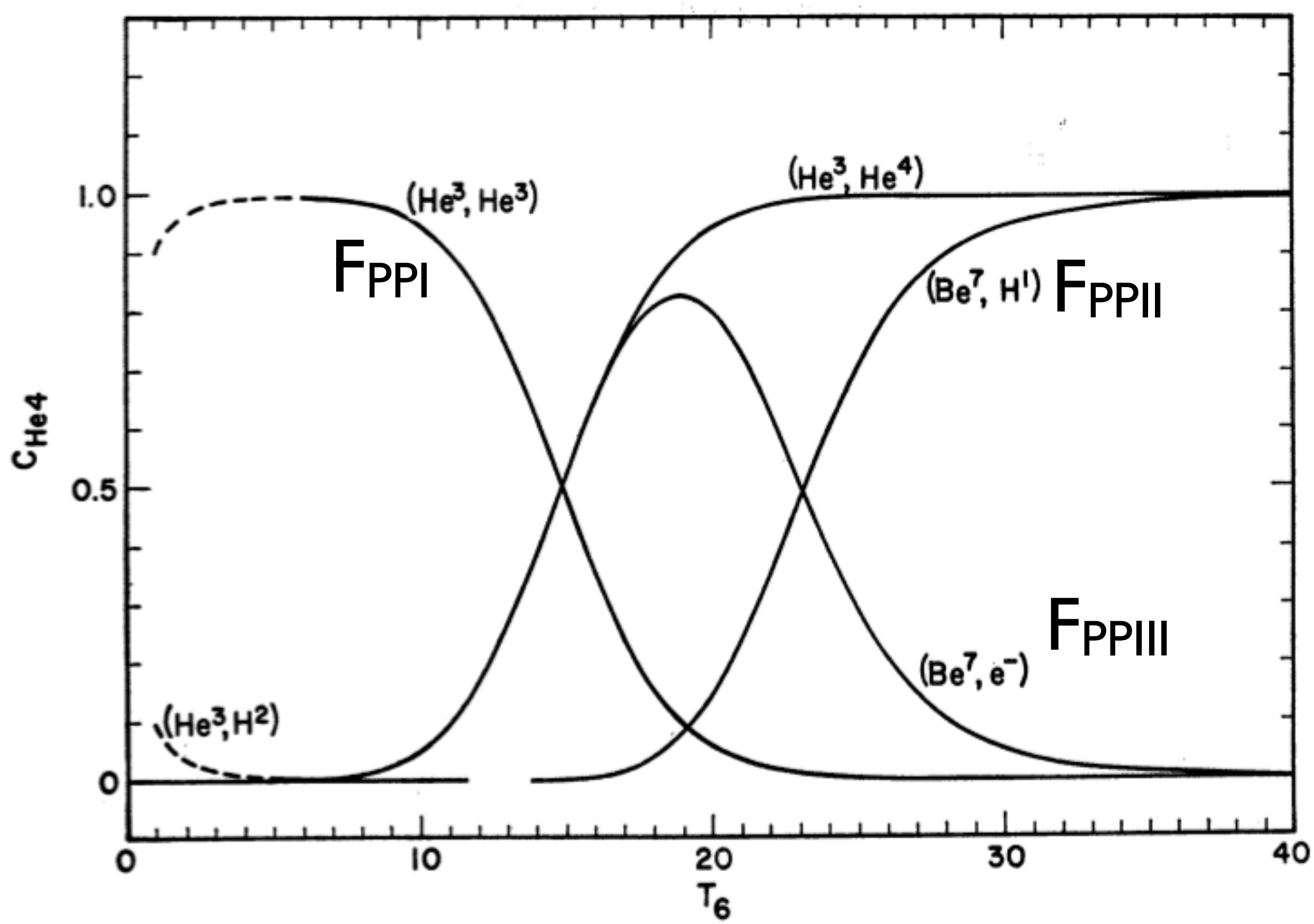

FIG. 8.—The fractions of He^4 produced by various terminations are shown as a function of T_6 for $X_H = X_{He} = 0.5$ and $\rho = 100$ gm cm^{-3}. The He^4-production fractions were calculated assuming an equilibrium abundance for He^3 and neglecting possible Li^4 terminations. The dashed curves have the same significance as in Fig. 2.

From Clayton: Principles of Stellar Evolution and Nucleosynthesis

Energy production and neutrino emission

Energy released in the formation of an α particle by fusion of four protons. Is essentially given by the difference of the mass excesses of four protons and one α particle.

$$Q_{p-p} = \left[4\Delta M(^{1}H) - \Delta M(^{4}He)\right]c^{2} = 26.7 \text{ MeV}$$

Since any reaction branch that completes this must turn 2 protons in 2 neutrons, two neutrinos are also emitted, which carry energy away from the reaction site.

It is these neutrinos that *directly* confirm the occurrence of nuclear reactions in the interior of the Sun. No other *direct* observational test of nuclear reactions is possible. The mean neutrino energy flux is ~0.26MeV for d creation (PPI/II) and ~7.2MeV for B decay (PPIII). But as PPIII is negligible, the energy released for each He nucleus assembled is ~26MeV (or 6×10^{14} JKg^{-1})

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The CNO Cycle

At birth stars contain a small (2%) mix of heavy elements, some of the most abundant of which are carbon, oxygen and nitrogen (CNO). These nuclei may induce a chain of H-burning reactions in which they act as catalysts.

The process is known as the CNO Cycle. There are alternative names that you may come across :

The CNO bi-cycle

The CNOF cycle

The CN and NO cycles

The CN and NO bi-cycles

In this course we will just refer to it all as the CNO cycle – and discuss the branches, but not specifically label them.

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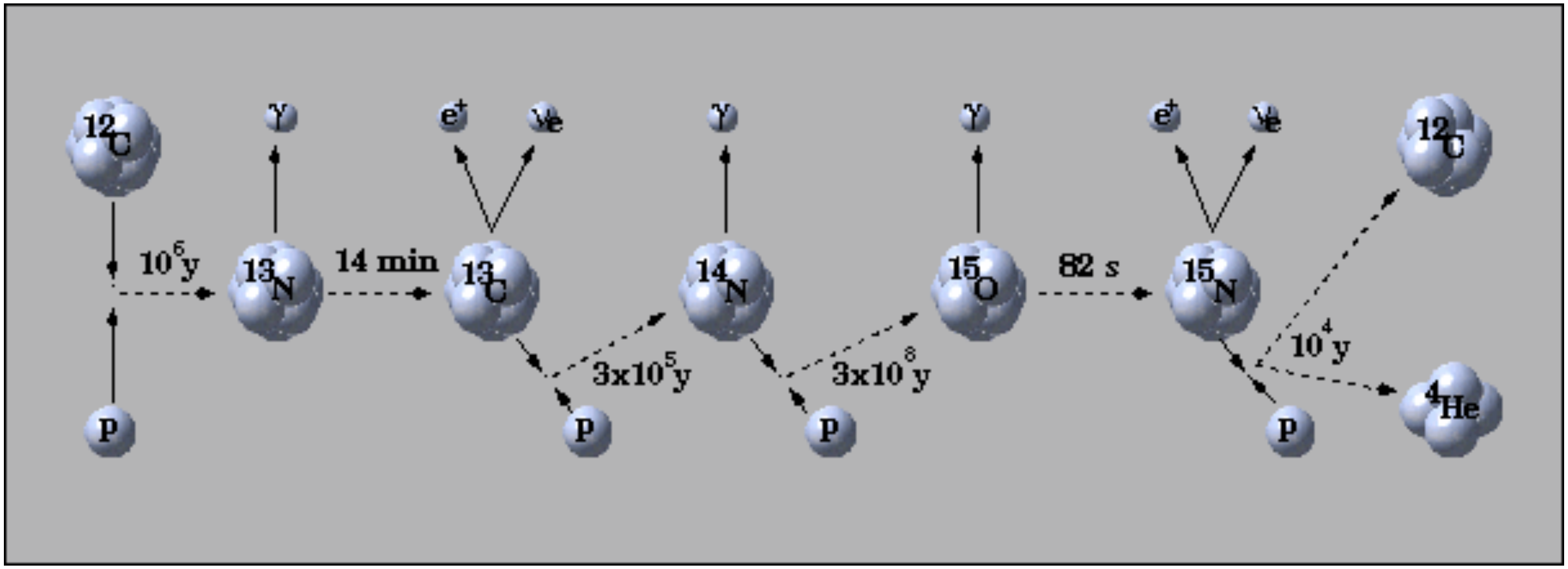
The main branch



1 $^{12}C + p \rightarrow {}^{13}N + \gamma$

2 $^{13}N \rightarrow {}^{13}C + e^{+} + \nu_e$

3 $^{13}C + p \rightarrow {}^{14}N + \gamma$

4 $^{14}N + p \rightarrow {}^{15}O + \gamma$

5 $^{15}O \rightarrow {}^{15}N + e^{+} + \nu_e$

6 $^{15}N + p \rightarrow {}^{12}C + {}^{4}He$

In the steady state case, the abundances of isotopes must take values such that the isotopes which react more slowly have higher abundance. The slowest reaction is p capture by ^{14}N . Hence most of ^{12}C is converted to ^{14}N.

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Temperature dependence of PP chain and CNO Cycle

The two processes have very different temperature dependences. The rate of energy production in each:

$$\varepsilon_{PP} = \varepsilon_0 \rho X_H^2 \left(\frac{T}{T_0} \right)^{4.6} \qquad \varepsilon_{CNO} = \varepsilon_0 \rho X_H X_{CNO} f_N \left(\frac{T}{25 \times 10^6} \right)^{16.7}$$

Equating this two gives the *T* at which they produce the same rate of energy production:

$$T \approx 1.7 \times 10^7 \left(\frac{X_H}{50 X_{CN}} \right)^{\frac{1}{12.1}} \text{ K}$$

Below this temperature the PP chain is most important, and above it the CNO Cycle dominates. This occurs in stars slightly more massive than the sun e.g. 1.2-1.5$M_{\odot}$.

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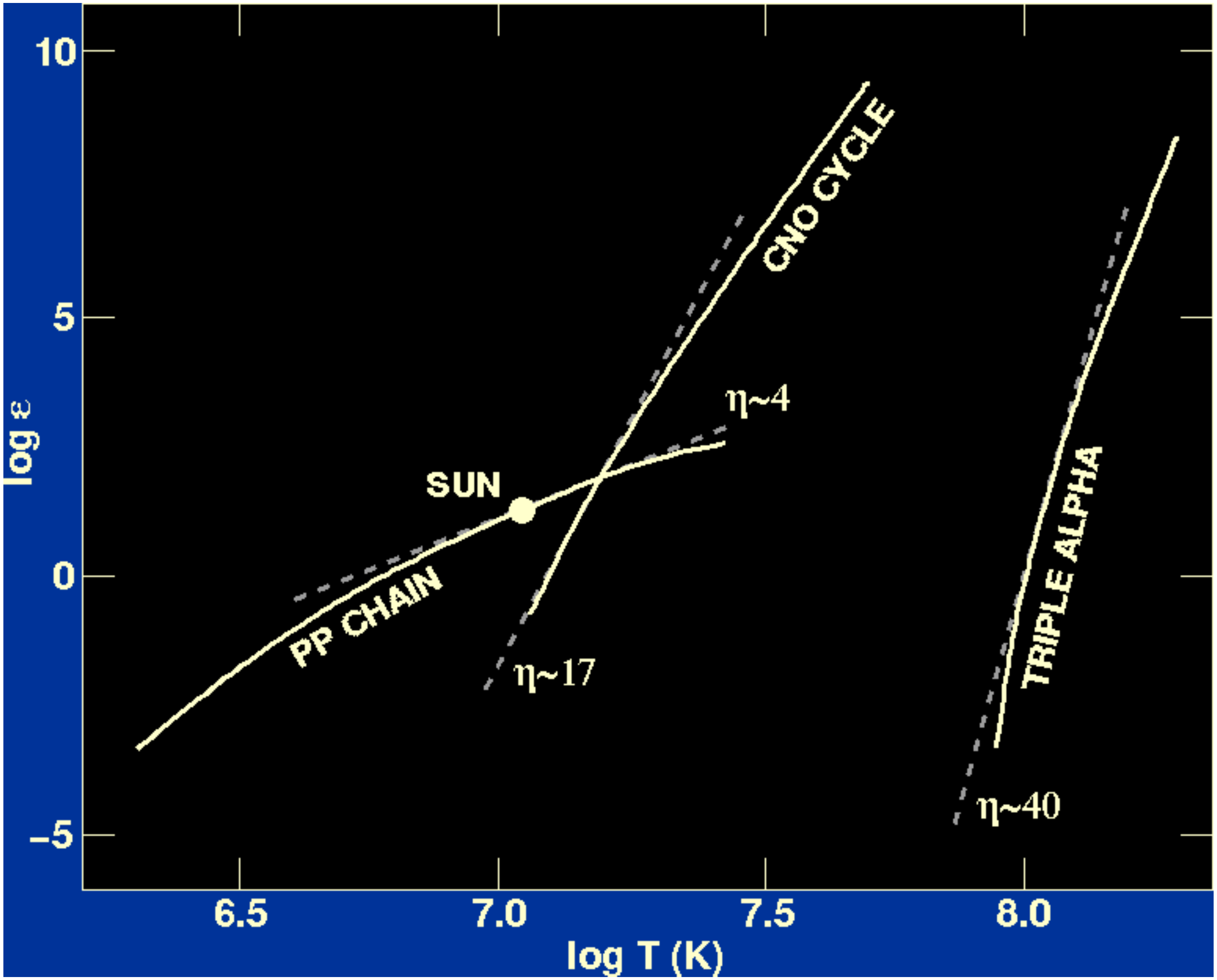

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Changes in Elemental Abundances During CNO

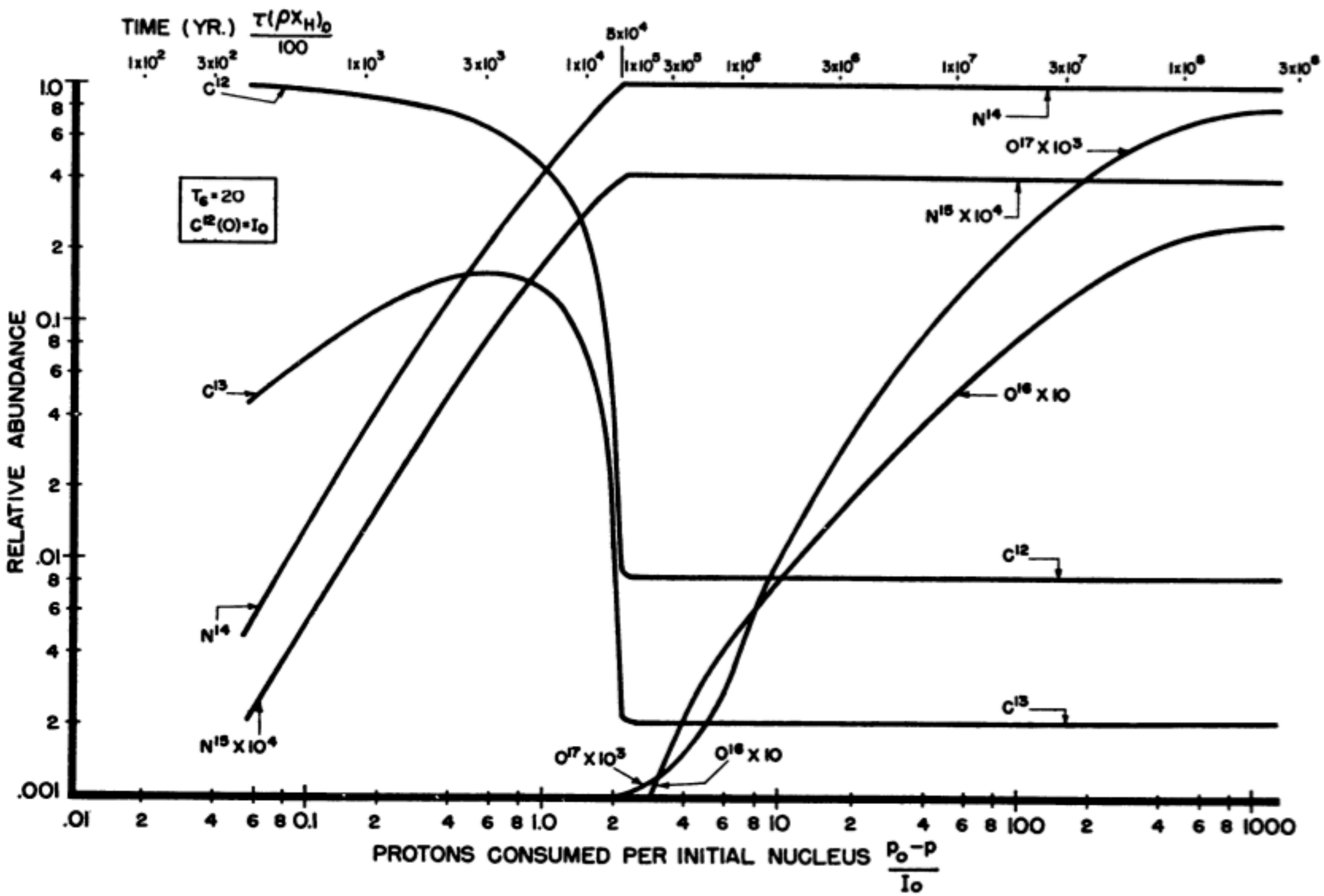
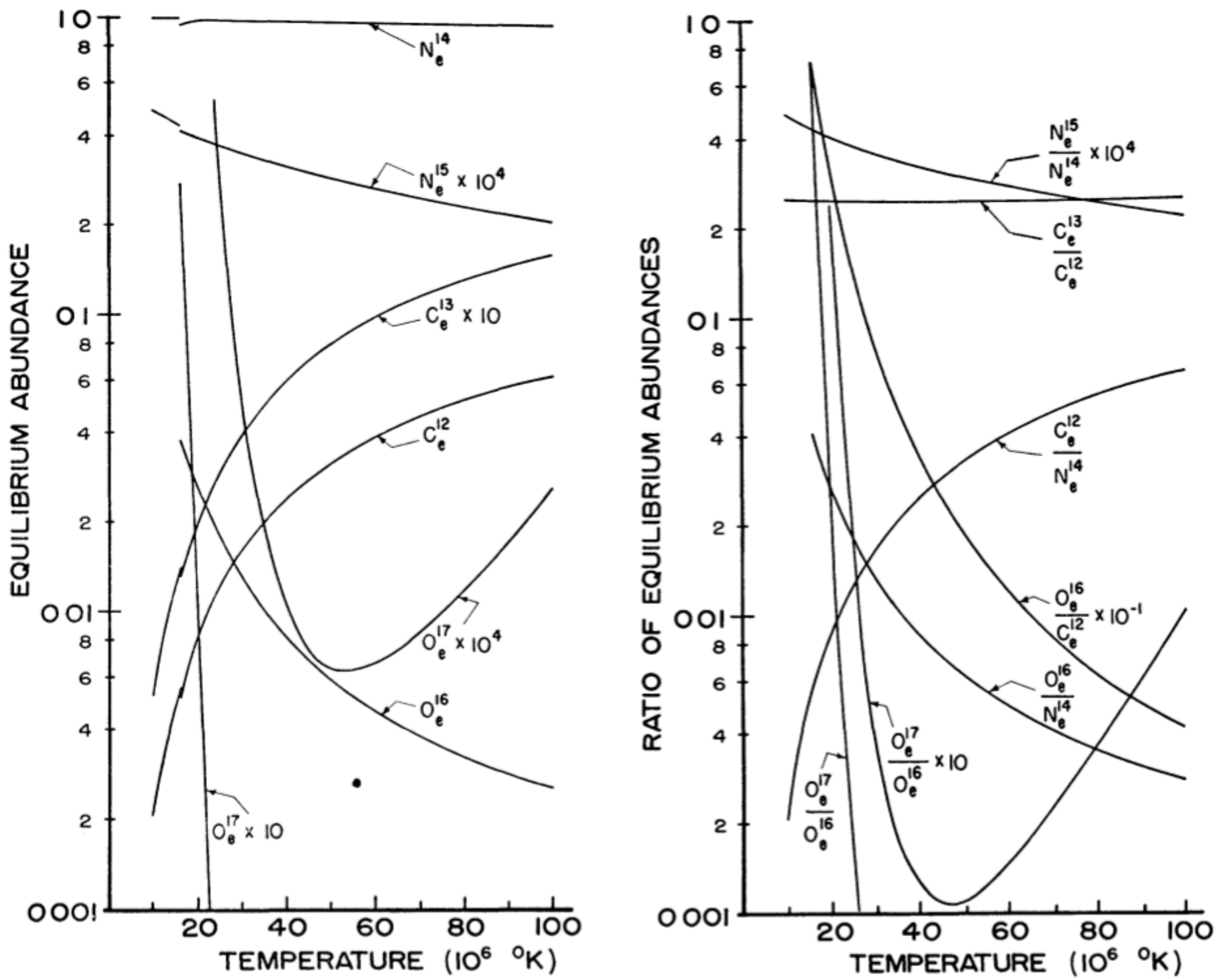


FIG. 1.—Relative abundances of the CNO nuclei as functions of protons consumed per initial nucleus, $T_6 = 20$, $C^{12}(0) = I_o$.

Caughlan 1962

Changes in Elemental Abundances During CNO



Caughlan & Fowler 1962

Helium Burning: the triple-α reaction.

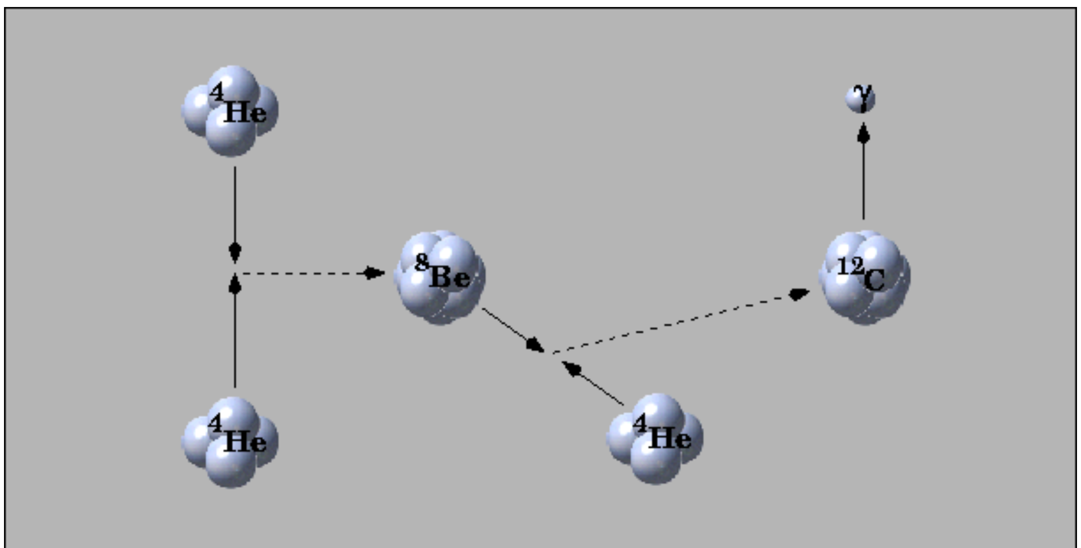
Simplest reaction in a helium gas should be the fusion of two helium nuclei. There is no stable configuration with *A=8.* For example the beryllium isotope ^{8}Be has a lifetime of only 2.6×10^{-16} s

$$^{4}\mathrm{He} + {}^{4}\mathrm{He} \rightarrow {}^{8}\mathrm{Be}$$

But a third helium nucleus can be added to ^{8}Be before decay, forming ^{12}C by the "triple-alpha" reaction

$$^{4}\mathrm{He} + {}^{4}\mathrm{He} \rightarrow {}^{8}\mathrm{Be}$$

$$^{8}\mathrm{Be} + {}^{4}\mathrm{He} \rightarrow {}^{12}\mathrm{C} + \gamma$$



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Helium Burning: the triple-α reaction.

Fred Hoyle (1952-54) suggested this small probability of α-capture by short lived ^{8}Be would be greatly enhanced if the C nucleus had an energy level close to the combined energies of the reacting ^{8}Be and ^{4}He nuclei. The reaction would be a faster "resonant" reaction.

This resonant energy level of ^{12}C was not experimentally known at the time. Hoyle's prediction led to nuclear experiment at Caltech, and resonant level discovered.

Thus helium burning proceeds in a 2-stage reaction, and energy released is

$$Q_{3\alpha} = \left[3\Delta M(^{4}He) - \Delta M(^{12}C)\right]c^{2} = 7.275\text{MeV}$$

In terms of energy generated per unit mass ≡ 5.8×10^{13} JKg^{-1} (I.e. 1/10 of energy generated by H-burning). But the *T* dependence is astounding:

$$\varepsilon_{3\alpha} \propto \rho^{2} T^{40}$$

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Carbon and oxygen burning

Carbon burning (fusion of 2 C nuclei) requires temperatures above 5×10^{8} K, and oxygen burning in excess of 10^{9} K.

Interactions of C and O nuclei are negligible – as at the intermediate temperatures required by the coulomb barrier the C nuclei are quickly destroyed by interacting with themselves

$$
\begin{aligned}
{}^{12}C + {}^{12}C &\rightarrow {}^{24}Mg + \gamma \\
&\rightarrow {}^{23}Mg + n \\
&\rightarrow {}^{23}Na + p \\
&\rightarrow {}^{20}Ne + \alpha \\
&\rightarrow {}^{16}O + 2\alpha
\end{aligned}
$$

$$
\begin{aligned}
{}^{16}O + {}^{16}O &\rightarrow {}^{32}S + \gamma \\
&\rightarrow {}^{31}S + n \\
&\rightarrow {}^{31}P + p \\
&\rightarrow {}^{28}Si + \alpha \\
&\rightarrow {}^{24}Mg + 2\alpha
\end{aligned}
$$

The branching ratios for these reactions are temperature dependent probabilities.

$${}^{12}C + {}^{12}C \rightarrow \sim 13\text{MeV } (\sim 5.2 \times 10^{13}\ \text{JKg}^{-1})$$

$${}^{16}O + {}^{16}O \rightarrow \sim 16\text{MeV } (\sim 4.8 \times 10^{13}\ \text{JKg}^{-1})$$

These reactions produce p, n, α, which are immediately captured by heavy nuclei, thus many isotopes created by secondary reactions.

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Silicon burning: nuclear statistical equilibrium

Two Si nuclei could fuse to create ^{56}Fe – the end of the fusion chain.

But now very high Coulomb barrier, at *T* above O burning, but below that required for Si burning, ***photodisintegration*** takes place

$$^{16}O + \alpha \leftrightarrow {}^{20}Ne + \gamma$$

This produces Ne at $T \sim 10^9$K but reverses above 1.5×10^9 K.

Si disintegration occurs around 3×10^9 K, and the light particles emitted are recaptured by other Si nuclei.

Although the reactions tend to a state of equilibrium, a leakage occurs towards the stable iron group nuclei (Fe, Co, Ni), which resist photodisintegration up to 7×10^9 K.

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Major nuclear burning processes

Common feature is release of energy by consumption of nuclear fuel. Rates of energy release vary enormously. Nuclear processes can also absorb energy from radiation field, we shall see consequences can by catastrophic.

Nuclear Fuel	Process	$T_{threshold}$ 10^6K	Products	Energy per nucleon (Mev)
H	PP	~4	He	6.55
H	CNO	15	He	6.25
He	3α	100	C,O	0.61
C	C+C	600	O,Ne,Ma,Mg	0.54
O	O+O	1000	Mg,S,P,Si	~0.3
Si	Nuc eq.	3000	Co,Fe,Ni	<0.18

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The *s*-process and *r*-process

Interaction between nuclei and free neutrons (neutron capture) – the neutrons are produced during C, O and Si burning.

Neutrons capture by heavy nuclei is not limited by the Coulomb barrier – so could proceed at relatively low temperatures. The obstacle is the scarcity of free neutrons. If enough neutrons available, chain of reactions possible:

$$I(A, Z) + n \rightarrow I_1(A+1, Z)$$

$$I_1(A+1, Z) + n \rightarrow I_2(A+2, Z)$$

$$I_2(A+2, Z) + n \rightarrow I_3(A+3, Z) \quad \ldots\text{etc}$$

If a radioactive isotope is formed it will undergo β-decay, creating new element.

$$I_N(A+N, Z) \rightarrow J(A+N, Z+1) + e^- +$$

If new element stable, it will resume neutron capture, otherwise my undergo series of β-decays

$$J(A+N, Z+1) \rightarrow K(A+N, Z+2) + e^- + \nu$$

$$K(A+N, Z+2) \rightarrow L(A+N, Z+3) + e^- + \nu$$

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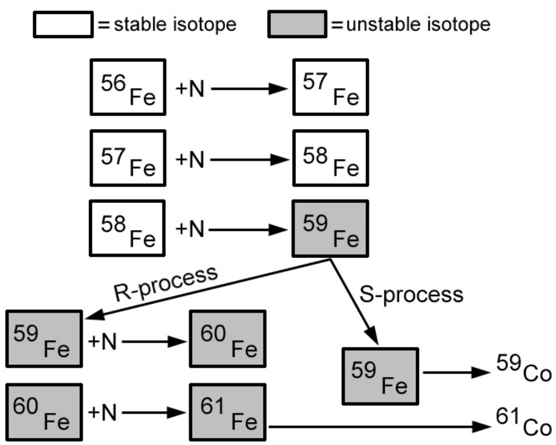
In the process two types of reactions and two types of nuclei are involved:
Neutron captures and β-decays ; stable and unstable nuclei

Stable nuclei may undergo only neutron captures, unstable ones my undergo both, with the outcome depending on the timescales for the two processes.
What can we say about the timescales of these processes ?

Hence neutron capture reactions may proceed more ***slowly*** or more ***rapidly*** than the competing β-decays. The different chains of reactions and products are called the ***s-process*** and ***r-process***.

Formation of Cobalt from Neutron Capture



R Process

Rapid neutron capture

$$^{65}Cu + 5n \Rightarrow {}^{70}Zn + \beta^{-} + \nu + Q$$

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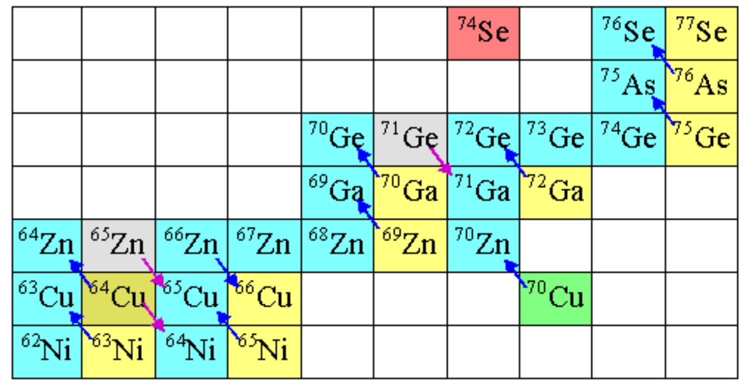
Some basic examples

R Process

Rapid neutron capture

$$^{65}Cu + 5n \Rightarrow {}^{70}Zn + \beta^- + \nu + Q$$

$\Rightarrow$



S Process

$$^{62}Ni + n \Rightarrow {}^{63}Ni + \gamma + Q$$

$$^{63}Ni \Rightarrow {}^{63}Cu + \beta^- + \nu + Q$$

$$^{63}Cu + n \Rightarrow {}^{64}Cu + \gamma + Q$$

Branched Decay

$$^{64}Cu \Rightarrow {}^{64}Zn + \beta^- + \nu + Q$$

$$^{64}Cu \Rightarrow {}^{64}Ni + \beta^+ + \nu + Q$$

Simple explanation of *r* and *s* process on:
ultraman.ssl.berkeley.edu/nucleosynthesis.html

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Summary

1. We described the basic accounting for nuclear reactions. This accounting includes:

   1.1. the mass excess (binding energy released when creating an element)

   1.2. the rate of interactions (depends on the density, mass fractions, and rate coefficient).

   1.4. This hides most of the physics.

2. Iron has highest binding energy

3. In the Sun, the P-P chain dominates

4. There are three P-P chains, the other two require He^3

5. At higher temperatures (> 10^6 K), the CNO cycle takes over (using existing C, N and O abundances in star). This rearranges the C, N, and O temperatures.

6. At 100 K, Carbon burning occurs.

7. Triple α requires unstable Be^8 as an intermediary state.

8. Silicon Burning (at 10^9 K) will be described in next lecture

9. We introduced the S and R processes. These work by Neutron capture: S-process (slow process) occurs when B decay faster than neutron capture, the R-process (rapid process) when neutron capture is much faster than B decay.