

Lecture 3

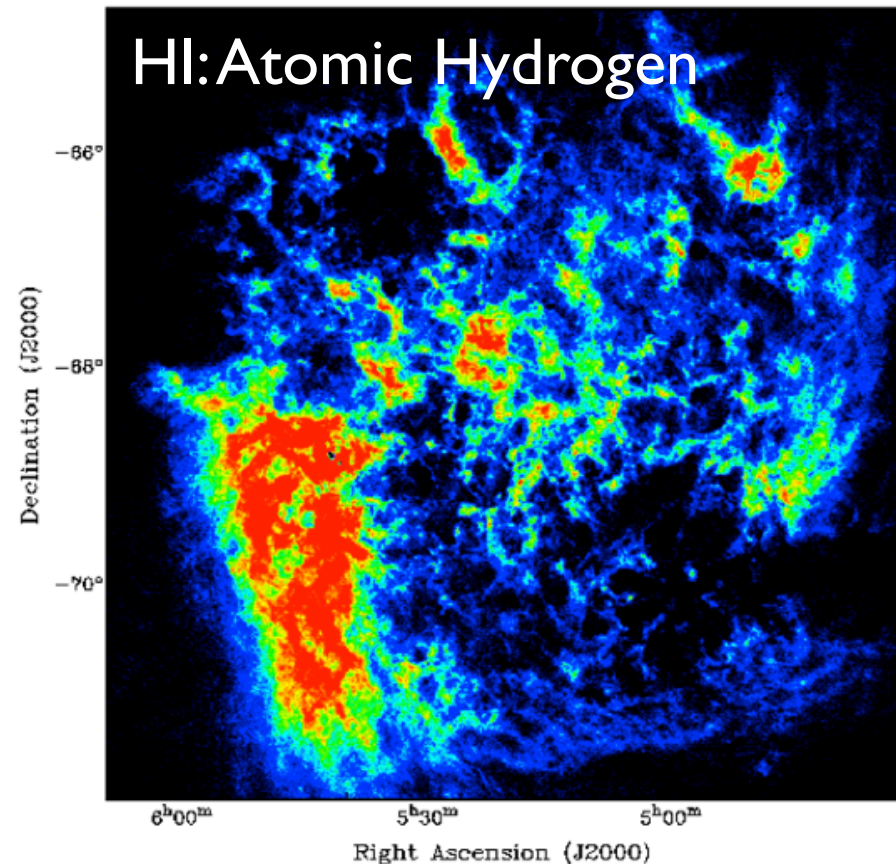
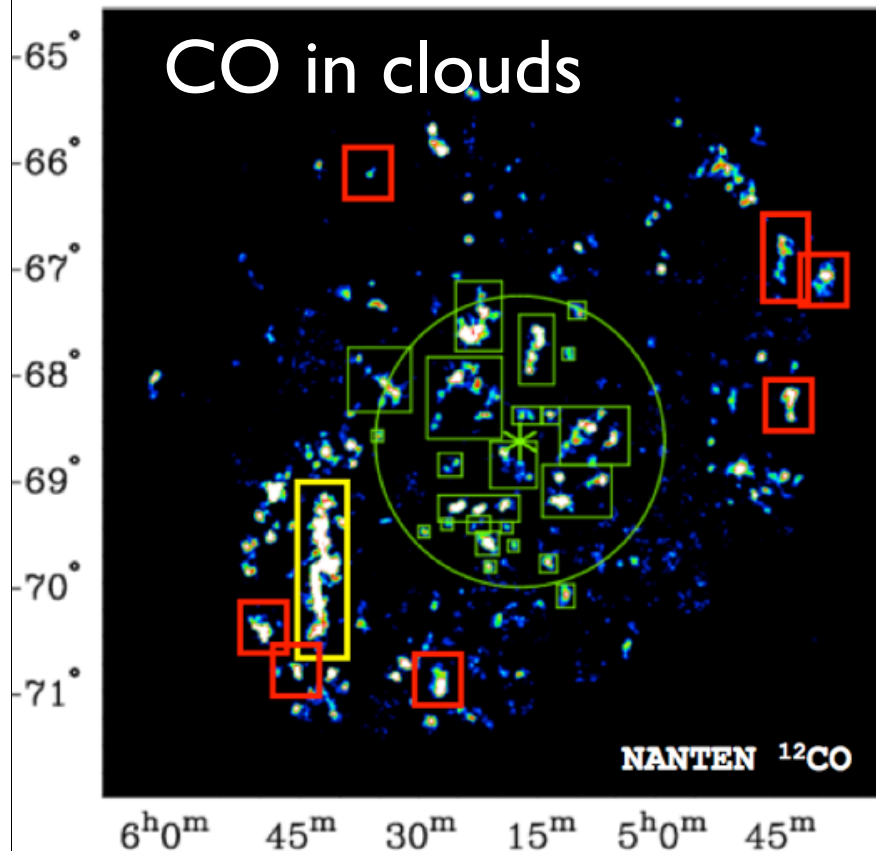
Properties and Evolution of Molecular Clouds

Spitzer space telescope image of Snake
molecular cloud (IRDC G11.11-0.11)

CO and HI in the LMC

From slide from Annie Hughes

Review



$M_{\text{mol}} \sim 4-7 \times 10^7 M_{\odot}$ (Fukui ea. 1999)

Mopra: $\theta=35''$ & $\Delta v=0.1 \text{ km s}^{-1}$

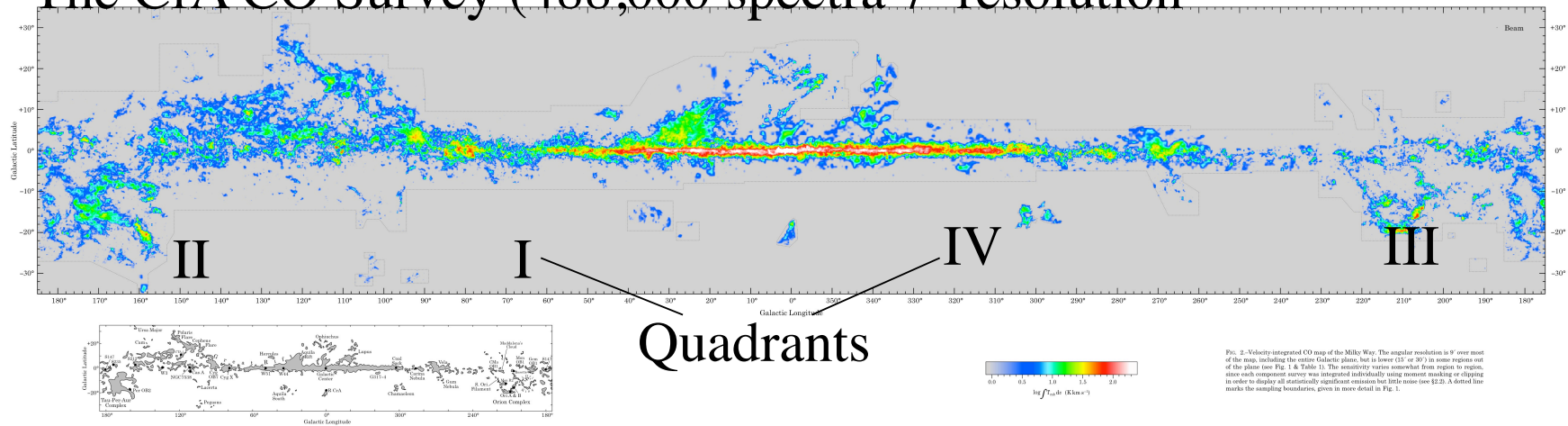
$M_{\text{HI}} \sim 4.8 \times 10^8 M_{\odot}$ (LSS ea. 2003)

ATCA+PKS: $\theta=60''$ & $\Delta v=1.7 \text{ km s}^{-1}$

http://www.atnf.csiro.au/research/LVmeeting/magsys_pres/ahughes_MagCloudsWorkshop.pdf

The Galaxy in CO

The CfA CO Survey (488,000 spectra 7' resolution)



<http://cfa-www.harvard.edu/mmw/MilkyWayinMolClouds.html>

2MASS Near-IR Survey

The Orion Giant Molecular Cloud Complex

Total mass
200,000 M_{sun}

CO map of Wilson et al. (2005) overlaid on image of Orion

Molecular Cloud Properties

Composition: H_2 , He, dust (1% mass), CO (10^{-4} by number),
and many other molecules with low abundances.

Sizes: 10-100 pc

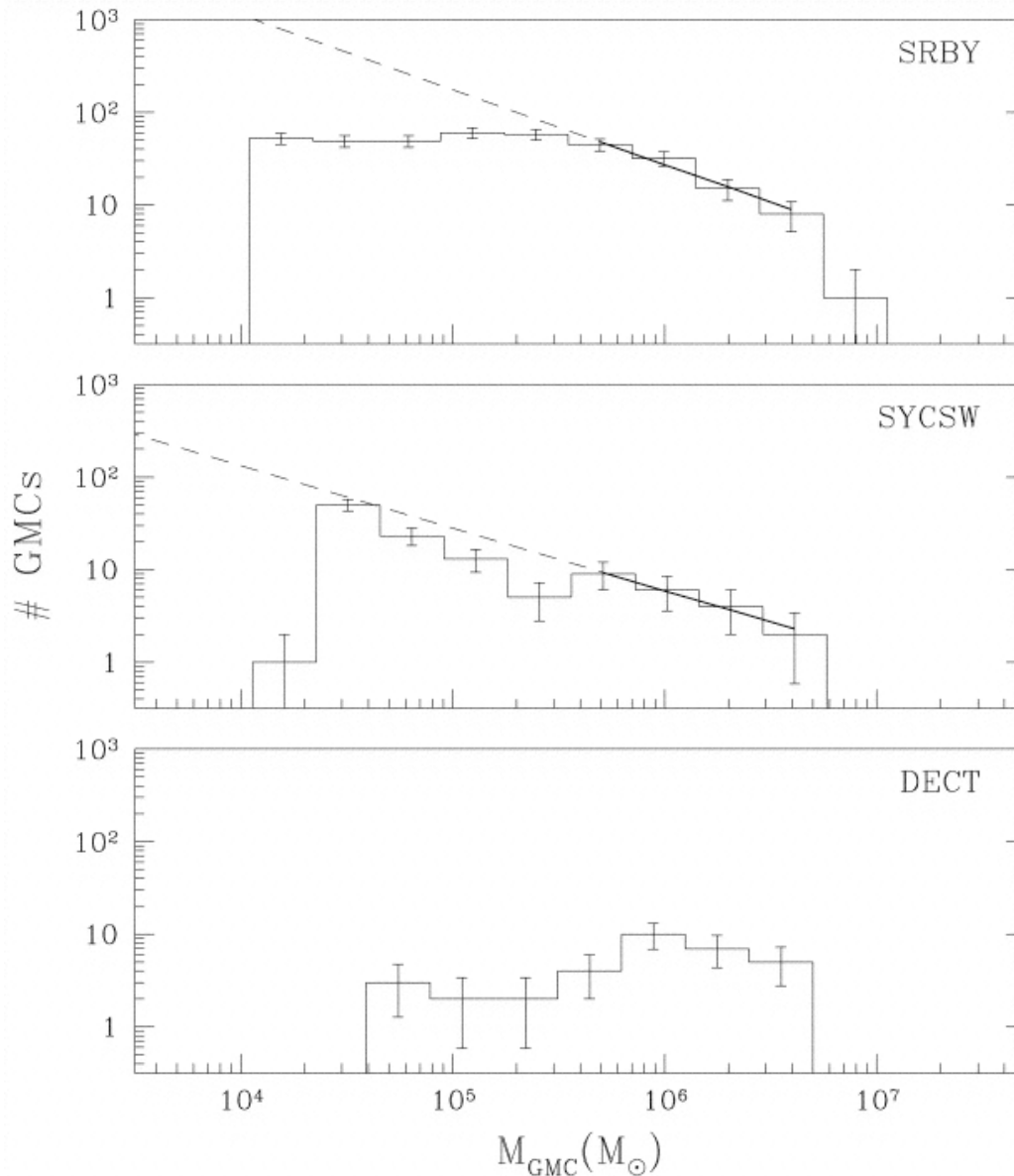
Masses: 10 to 10^6 M_{sun} . Most of the molecular gas mass in
galaxies is found in the more massive clouds.

Average Density: 100 cm^{-3}

Lifetimes: < 10 Myr - we will discuss this when we discuss the
lifetime problem.

Issue: it is hard to separate individual clouds - not well defined.

Cloud Mass Function



$$dN/d\log(m) = kM^{-0.81}$$
$$M_{\text{upper}} = 5.8 \times 10^6 M_{\text{sun}}$$

$$dN/d\log(m) = kM^{-0.67}$$
$$M_{\text{upper}} = 5.8 \times 10^6 M_{\text{sun}}$$

Mass is weighted toward the most massive clouds

Williams & McKee 1997
ApJ 476, 166.

Larson's Laws

In 1981, Richard Larson wrote a paper titled:

Turbulence and Star Formation in Molecular Clouds
(MNRAS 1984, 809)

Used tabulation of molecular cloud properties to discover to empirical laws based on the cloud length L (max. projected length)

Size-linewidth relationship: $\sigma_v \text{ (km s}^{-1}\text{)} = 1.10 L^{0.38} \text{ (pc)}$

more recent values give $\sigma_v = \text{const } L^{0.5}$

Density-radius relationship: $\langle n(\text{H}_2) \rangle \text{ (cm}^{-3}\text{)} = 3400 L^{-1.1} \text{ (pc)}$

more recent interpretation is that $N(\text{H}_2) = 1 \times 10^{22} \text{ cm}^{-2}$

Size-LineWidth Relationship

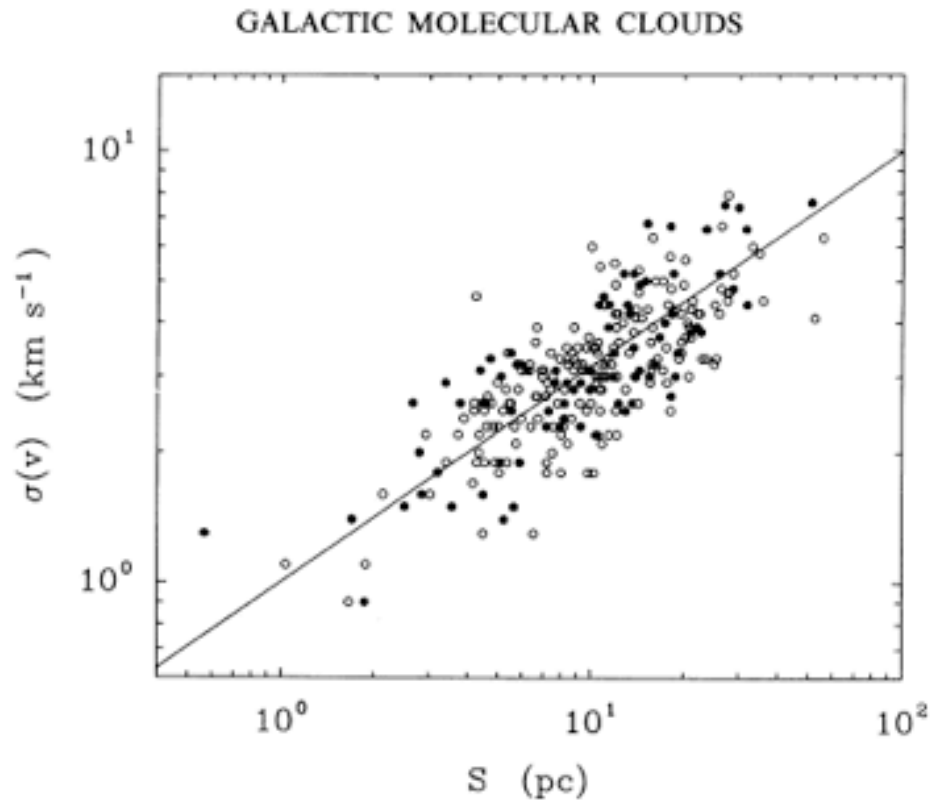


FIG. 1.—Molecular cloud velocity dispersion $\sigma(v)$ as a function of size S (defined in text) for 273 clouds in the Galaxy. The solid circles are calibrator clouds with known distances and the open circles are for clouds with the near-far distance ambiguity resolved by the method discussed in the text. The fitted line is $\sigma(v) = S^{0.5}$ km s⁻¹. For virial equilibrium the 0.5 power law requires clouds of constant average surface density.

Are Clouds Bound?

To ascertain if a cloud is bound, we can first check to see if the kinetic energy of the cloud, T , is less than the gravitational potential energy. The kinetic energy of the cloud is given by:

$$T = \frac{1}{2} M_{cloud} \langle v^2 \rangle \quad (1)$$

This can be related to the 1-D velocity dispersion by $v^2 = \sigma_x^2 + \sigma_y^2 + \sigma_z^2 = 3\sigma^2$ assuming that the velocity distribution is isotropic. The size vs linewidth relationship gives us $\sigma = kL^{0.5}$ which implies:

$$T = \frac{3}{2} M_{cloud} \langle \sigma^2 \rangle = k_1 ML \quad (2)$$

and the density vs radius relationship gives us $M = kL^2$ or

$$T = k_2 M^2 / L = k_3 U \quad (3)$$

where U is the gravitational potential energy which is $\propto GM^2/R$. Thus, Larson's laws combine to suggest that T is proportional to U . Larson found evidence that the ratio of U to T implied stars are in virial equilibrium.

Are Clouds Bound?

Does this imply that clouds are bound using more recent data? To test this, define a parameter of $\alpha = |U/T|$, where if $\alpha \geq 1$ then the cloud is bound and if $\alpha < 1$, the cloud is unbound. We assume that the star is a homogenous sphere, which is a lower limit to U:

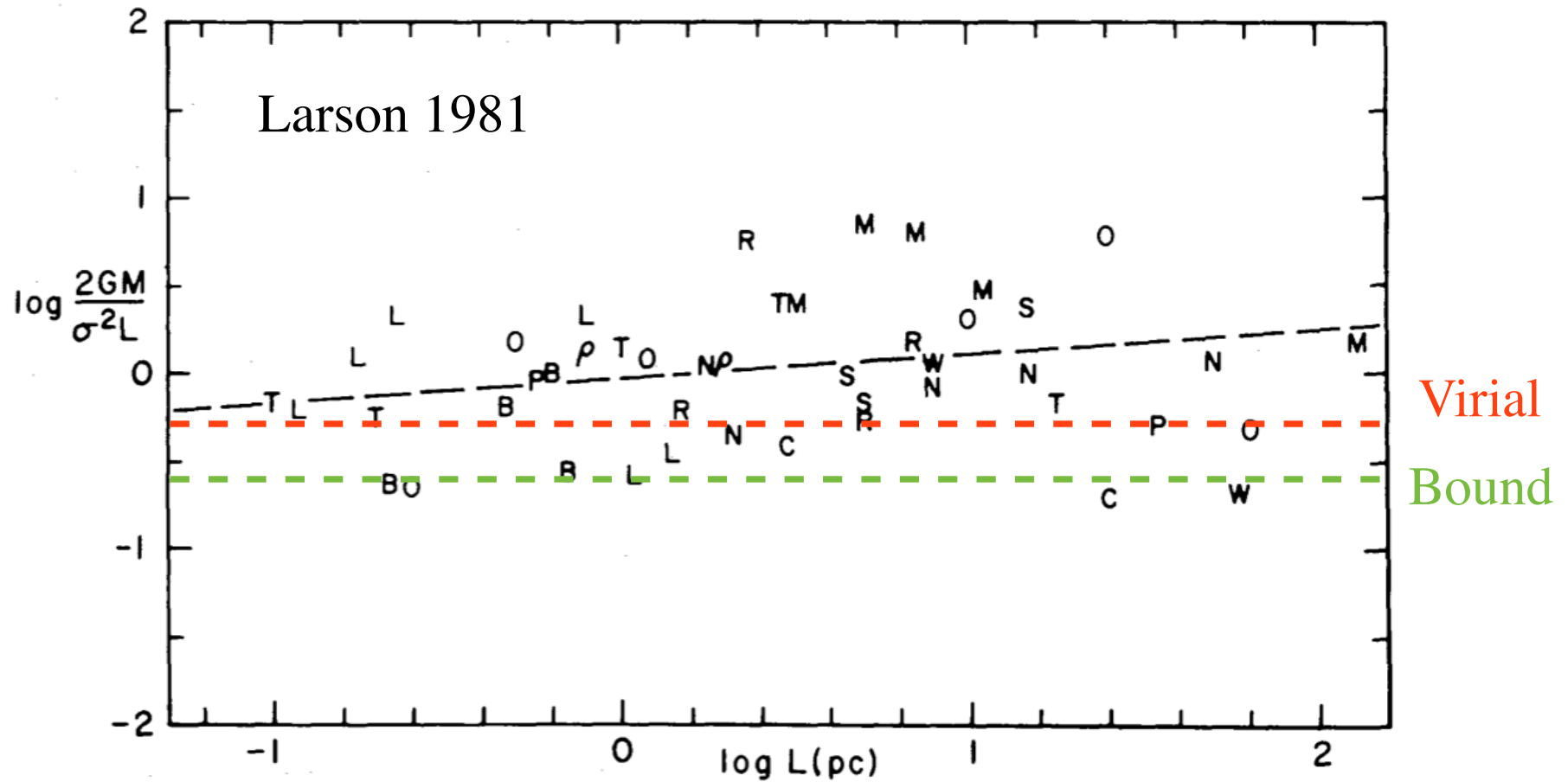
$$U = -\frac{3}{5} \frac{GM^2}{R} \quad (4)$$

then the ratio is given by:

$$\alpha \geq \frac{2GM}{5R\sigma^2} \quad (5)$$

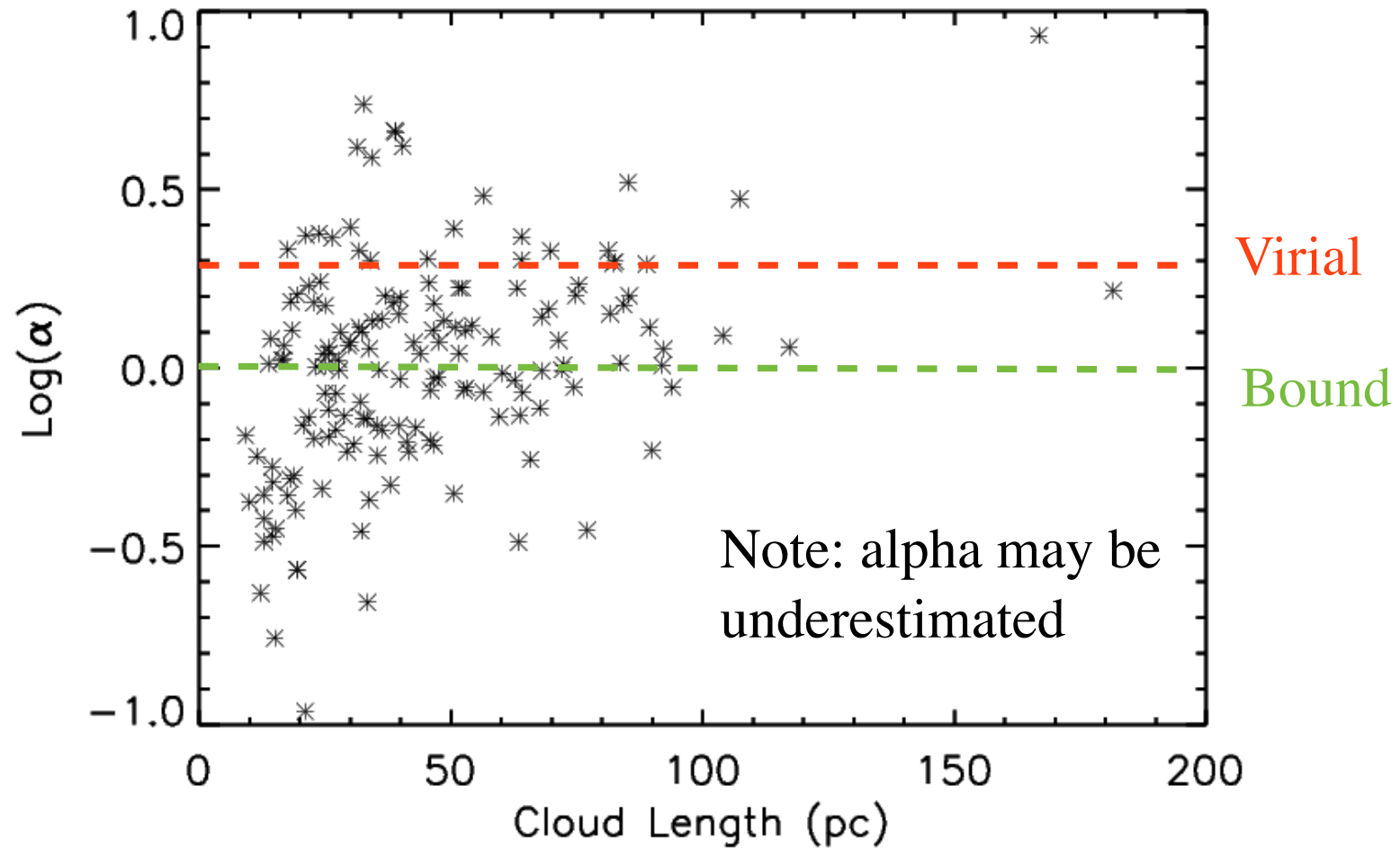
Virial equilibrium implies $\alpha = 2$ (potential energy is $-2T$.)

Are Clouds Bound?



Note: σ is 3-D velocity distribution

Using Galactic Ring Data



The Stability of Clouds: Jeans Instability

We start with the basic equations for fluid motions:

The continuity equation (conservation of mass):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (1)$$

where \mathbf{v} is a vector.

The 2nd equation is the momentum equation, which basically states $F = ma$:

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla P - \rho \nabla \phi \quad (2)$$

Note that the equation for hydrostatic equilibrium can be derived from Euler's in the case equation when \mathbf{v} is set to 0.

Finally, the gravitational potential ϕ is given by

$$\nabla^2 \phi = 4\pi G \rho \quad (3)$$

The Stability of Clouds: Jeans Instability

Consider an infinite, static medium with a density of ρ . Now, consider a small perturbation to this medium:

$$\mathbf{v} = \mathbf{v}_0 + \delta\mathbf{v}, \quad \rho = \rho_0 + \delta\rho, \quad \phi = \phi_0 + \delta\phi \quad (9)$$

If we substitute these values into the continuity equation we get:

$$\frac{\partial\rho}{\partial t} + \nabla \cdot (\rho\mathbf{v}) = 0 \quad (10)$$

goes to

$$\frac{\partial\delta\rho}{\partial t} + \delta\mathbf{v} \cdot \nabla(\rho_0) + \rho_0 \nabla \cdot \delta\mathbf{v} = 0 \quad (11)$$

where it is assumed $\mathbf{v}_0 = 0$ and that all 2nd order terms, i.e. $\delta\rho\delta v$, go to 0. For the momentum equation we get:

The Stability of Clouds: Jeans Instability

For the momentum equation we get:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla P - \nabla \phi \quad (12)$$

goes to

$$\frac{\partial \delta \mathbf{v}}{\partial t} = -\frac{c_s^2}{\rho} \nabla \delta \rho - \nabla \delta \phi \quad (13)$$

where the $(\mathbf{v} \cdot \nabla) \mathbf{v}$ term is 2nd order and vanishes and c_s is the sound speed for an ideal, isothermal gas which is equal to

$$c_s = \sqrt{\frac{kT}{\mu m_H}} \quad (14)$$

(where $P = c_s^2 \rho$ is the ideal gas law) and we have assumed that the unperturbed gas is in hydrostatic equilibrium, i.e.

$$\frac{c_s^2}{\rho_0} \nabla \rho_0 = -\nabla \phi_0 \quad (15)$$

The Stability of Clouds: Jeans Instability

Now we combine equations 6 and 8 by taking the derivative of equation 6 with respect to time:

$$\frac{\partial^2 \delta \rho}{\partial t^2} + \frac{\partial \delta \mathbf{v}}{\partial t} \cdot \nabla(\rho_0) + \rho_0 \nabla \cdot \frac{\partial \delta \mathbf{v}}{\partial t} = 0 \quad (16)$$

which if we assume ρ_0 is constant (and thus $\nabla \rho_0 = 0$) we get

$$\frac{\partial^2 \delta \rho}{\partial t^2} = -\rho_0 \nabla \cdot \frac{\partial \delta \mathbf{v}}{\partial t} \quad (17)$$

and then substituting equation 8 into the new equation:

$$\frac{\partial^2 \delta \rho}{\partial t^2} = c_s^2 \nabla^2 \delta \rho + \nabla^2 \delta \phi \quad (18)$$

Finally, using the Poisson equation, we arrive at our final equation:

$$\frac{\partial^2 \delta \rho}{\partial t^2} = \rho_0 \left(\frac{c_s^2}{\rho_0} \nabla^2 \delta \rho + 4\pi G \delta \rho \right) \quad (19)$$

The Stability of Clouds: Jeans Instability

Finally, using the Poisson equation, we arrive at our final equation:

$$\frac{\partial^2 \delta \rho}{\partial t^2} = \rho_0 \left(\frac{c_s^2}{\rho_0} \nabla^2 \delta \rho + 4\pi G \delta \rho \right) \quad (19)$$

Note that the equation:

$$\frac{\partial^2 \delta \rho}{\partial t^2} = \rho_0 \left(\frac{c_s^2}{\rho_0} \nabla^2 \delta \rho \right) \quad (20)$$

$$\frac{\partial^2 \delta \rho}{\partial t^2} = c_s^2 \nabla^2 \delta \rho \quad (21)$$

is the wave equation for a pressure wave with sound speed c_s .

The Stability of Clouds: Jeans Instability

Let's rewrite this as 1-D equation (we can assume 1-D perturbations).

$$\frac{\partial^2 \delta \rho}{\partial t^2} = \rho_0 \left(\frac{c_s^2}{\rho_0} \frac{\partial^2 \delta \rho}{\partial x^2} + 4\pi G \delta \rho \right) \quad (22)$$

We adopt the following solution:

$$\delta \rho = \delta \rho_0 e^{i(\omega t - kx)} \quad (23)$$

If we plug this solution into the problem, then we then get the following dispersion equation:

$$\omega^2 = c_s^2(k^2 - k_j^2) \quad (24)$$

where

$$k_j^2 = 4\pi G \frac{\rho_0}{c_s^2} \quad (25)$$

The Stability of Clouds: Jeans Instability

$$k_j^2 = 4\pi G \frac{\rho_0}{c_s^2} \quad (25)$$

Thus, if $k > k_J$, w is real and we get a normal wave equation. However, if $k < k_J$, then the solution for w is imaginary. The resulting time dependence solution for the perturbation density is

$$\delta\rho = \frac{\delta\rho_0}{2} (e^{(|k^2 - k_J^2|)^{1/2}t} + e^{-(|k^2 - k_J^2|)^{1/2}t}) \quad (26)$$

which increases exponentially with time. In other words, gravity wins over pressure, and the perturbations are unstable. We can convert that into a length, the Jeans length

$$k_J = 2\pi/\lambda_J, \quad \lambda_J = \sqrt{\frac{\pi c_s^2}{G\rho_0}} \quad (27)$$

the Jeans length can be turned into a mass

$$m_j = \rho_0 \lambda_J^3 = \frac{c_s^3 \pi^{3/2}}{G^{3/2} \rho_0^{1/2}} = \left(\frac{\pi k T}{\mu m_H G} \right)^{3/2} \rho_0^{-1/2} \quad (28)$$

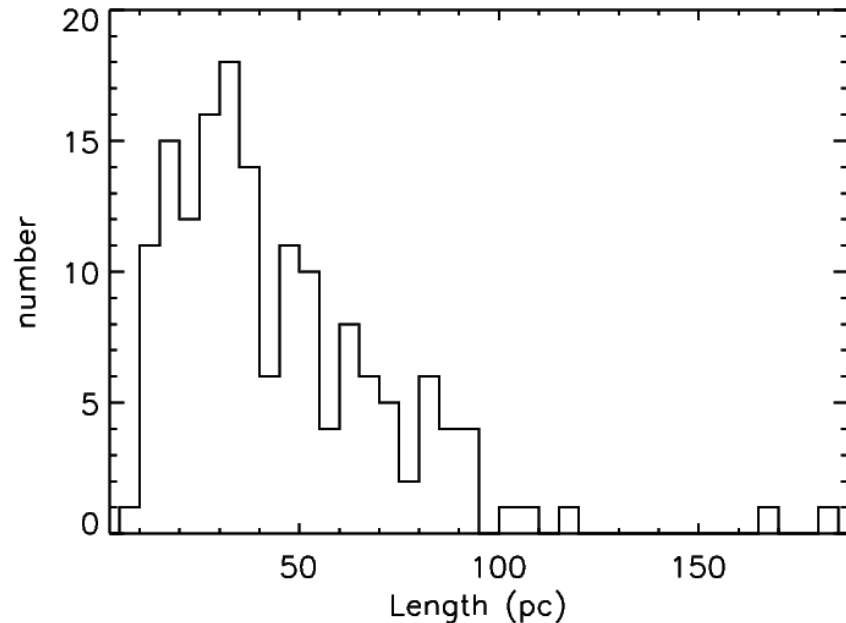
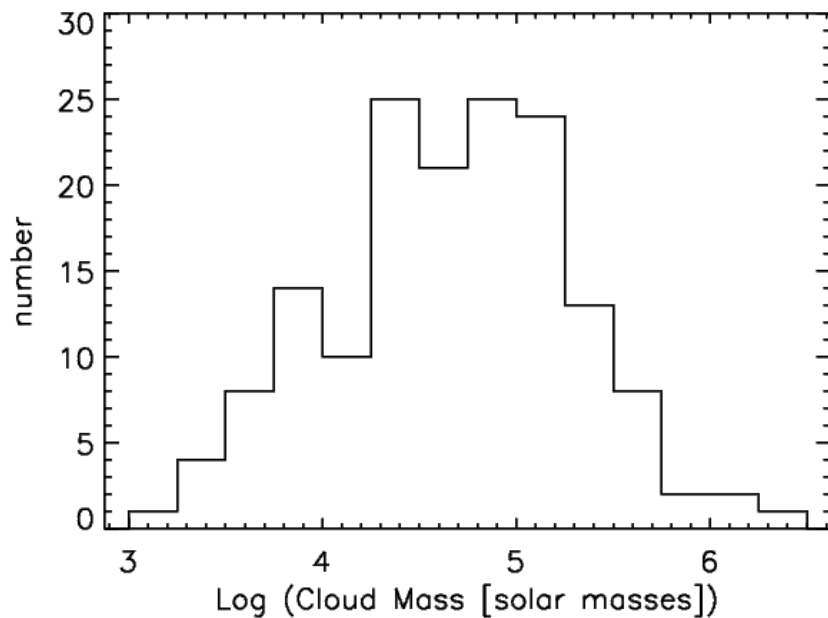
Are Clouds Stable to Fragmentation?

For $n(\text{H}_2) = 40 \text{ cm}^{-3}$ $\lambda_J = 6 \text{ pc}$ $M_J = 450 \text{ Msun}$ (cloud size)

For $n(\text{H}_2) = 100 \text{ cm}^{-3}$ $\lambda_J = 4. \text{ pc}$ $M_J = 350 \text{ Msun}$ (cloud size)

For $n(\text{H}_2) = 1000 \text{ cm}^{-3}$ $\lambda_J = 1.2 \text{ pc}$ $M_J = 100 \text{ Msun}$ (clump size)

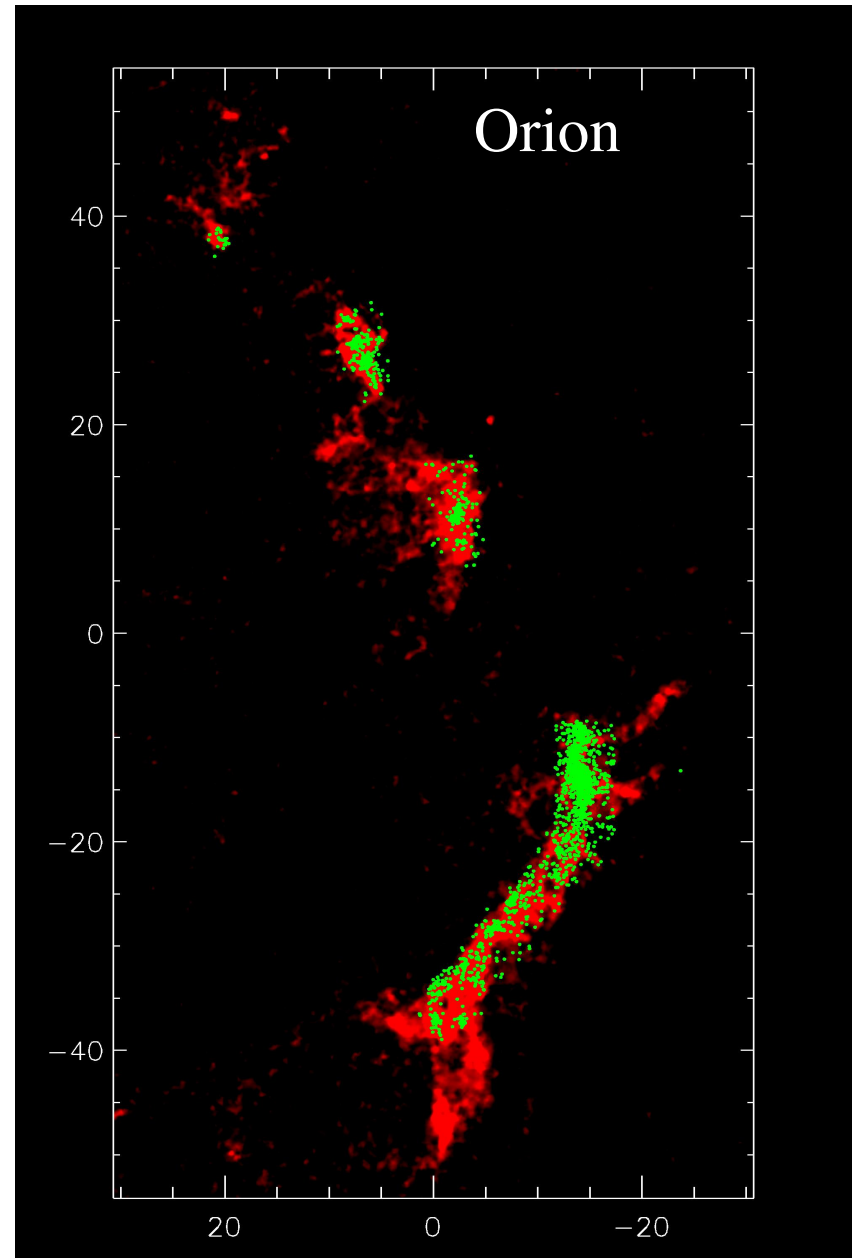
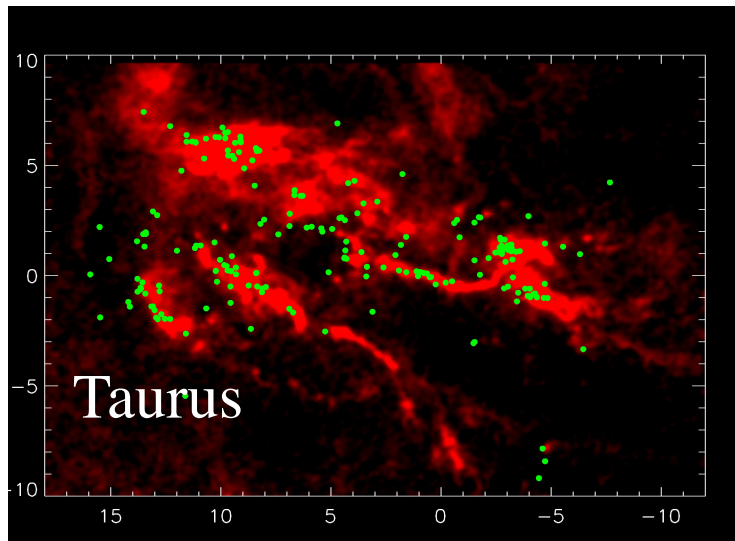
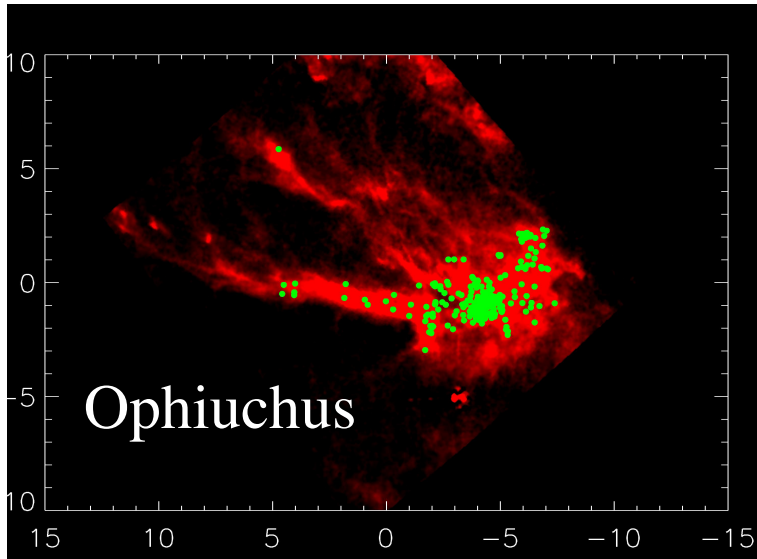
Cloud masses and lengths exceed Jeans length masses.



From clouds in the inner galaxy from Heyer et al. 2009

Molecular Clouds are Filamentary and Clumpy

Units in Parsecs



Star Formation Efficiency

The star formation efficiency of a molecular cloud, ϵ , is defined as the following:

$$\epsilon = \frac{M_{stars}}{M_{stars} + M_{gas}} \quad (30)$$

where M_{stars} is the mass in stars and M_{gas} is the mass in molecular mass. It is essentially the fraction of mass that is converted into stars. Since it is difficult to determine the mass of individual stars, M_{gas} is calculated usually as the number of stars times the average stellar mass of $0.3 M_{\odot}$, i.e. $M_{stars} = N_{stars} \times 0.3 M_{\odot}$.

Typical cloud efficiencies are a few percent for molecular clouds

Table 4
Efficiencies and Depletion Timescales

Cloud	$\frac{M_{*}}{(M(\text{cloud})+M_{*})}$	$M_{*}/M(\text{dense})$	$t_{\text{dep}}(\text{cloud})$ (Myr)	$t_{\text{dep}}(\text{dense})$ (Myr)	SFR _{ff}	Notes
Cha II	0.030		66	...	0.028	Alcalá et al. (2008)
Lupus ^a	0.054		35	...	0.050	Merín et al. (2008)
Perseus	0.038	0.69	50	2.9	0.049	S.-P. Lai et al. (2008, in preparation)
Serpens	0.053	1.2	35	1.6	0.036	Harvey et al. (2007a)
Ophiuchus	0.063	3.3	30	0.6	0.064	L. Allen et al. (2008, in preparation)
All Clouds ^b	0.048	1.2	40	1.8	0.040	...

Notes.

^aA sum over the three Lupus clouds of the values for each cloud.

^bFor all but SFR_{ff}, this number is calculated by adding all clouds with the relevant data together; for SFR_{ff}, it is the average over all clouds of the individual values.

From Evans et al. 2009

Lifetimes of Nearby Clouds

TABLE 1
STAR-FORMING REGIONS

Region	$\langle t \rangle^a$ (Myr)	Molecular Gas?	Ref. (age)
Coalsack	Yes	...
Orion Nebula	1	Yes	1
Taurus	2	Yes	1, 2, 3
Oph	1	Yes	1
Cha I, II	2	Yes	1
Lupus	2	Yes	1
MBM 12A	2	Yes	4
IC 348	1–3	Yes	1, 4, 5, 6
NGC 2264	3	Yes	1
Upper Sco	2–5	No	1, 6, 7
Sco OB2	5–15	No	8
TWA	~10	No	9
η Cha	~10	No	10

^a Average age in Myr.

REFERENCES.—(1) Palla & Stahler 2000. (2) Hartmann 2001. (3) White & Ghez 2001. (4) Luhman 2001. (5) Herbig 1998. (6) Preibisch & Zinnecker 1999. (7) Preibisch et al. 2001. (8) de Geus et al. 1989. (9) Webb et al. 1999. (10) Mamajek, Lawson, & Feigelson 1999.

The Lifetimes of Clouds

← INCREASING CLUSTER AGE →

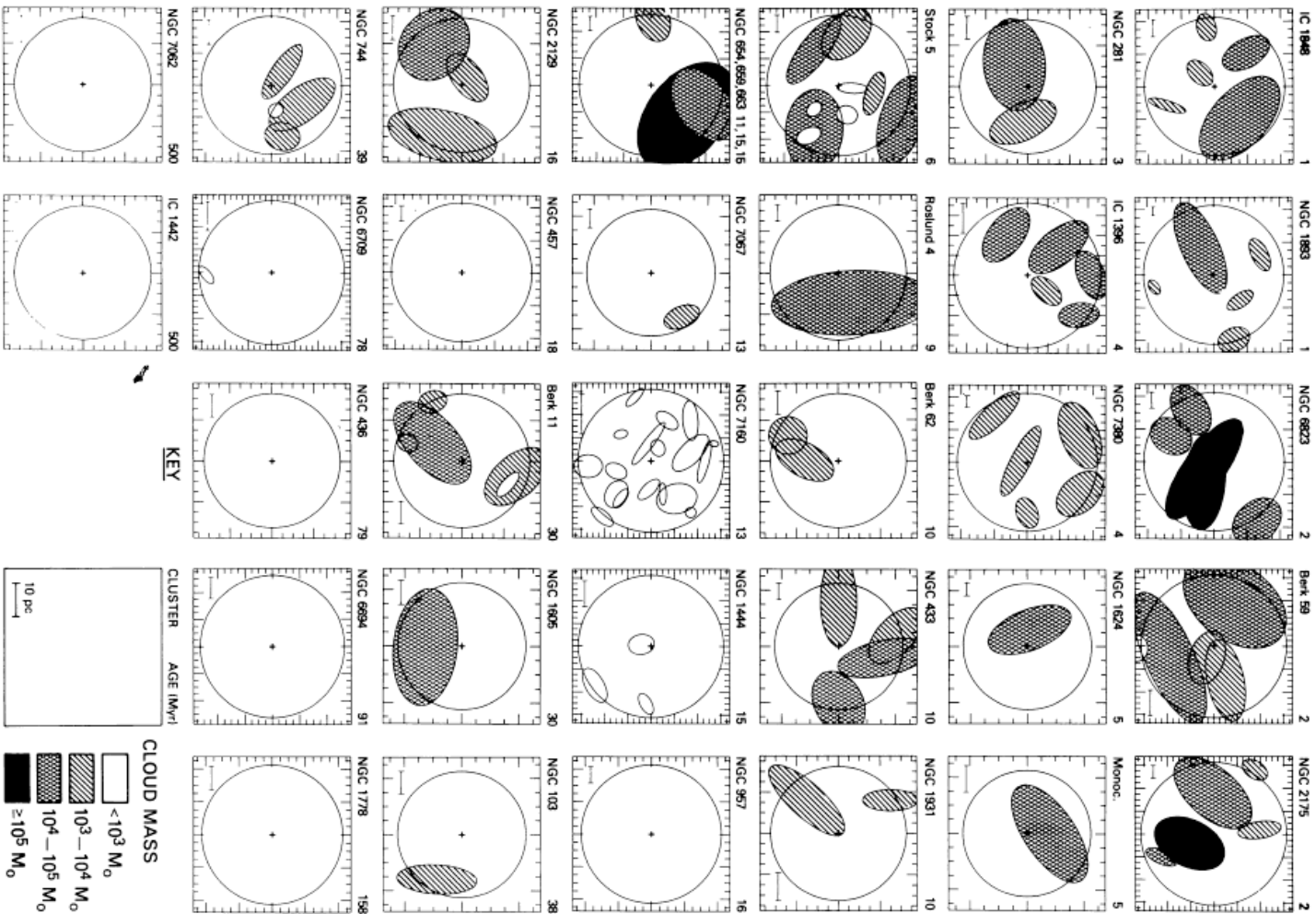


Fig. 43.— Montage of schematic diagrams showing cataloged molecular clouds associated or possibly associated with open clusters sorted by age with younger clusters at the top of the page and older ones at the bottom. Darker shading is used to symbolize more massive clouds.

798

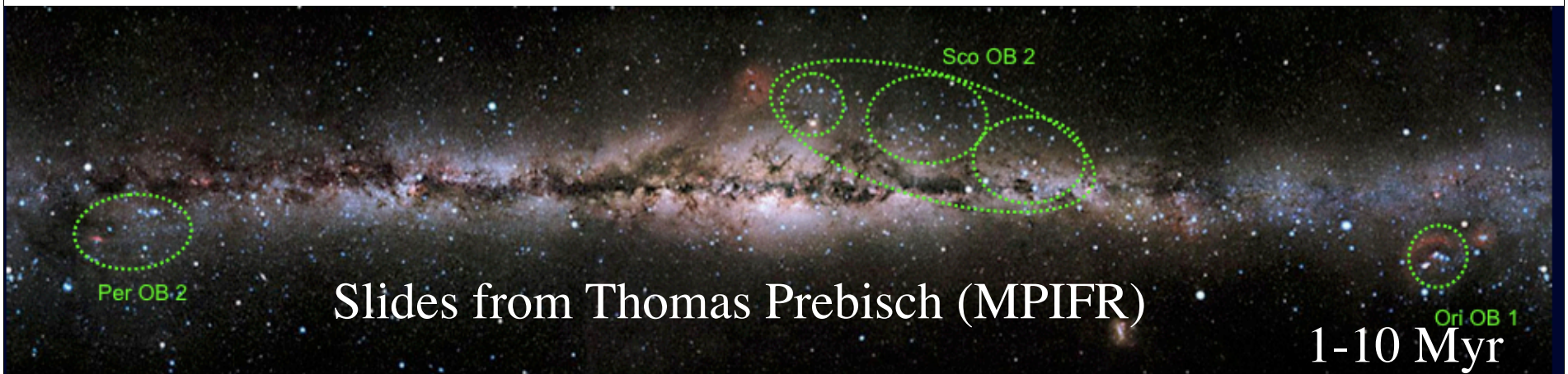
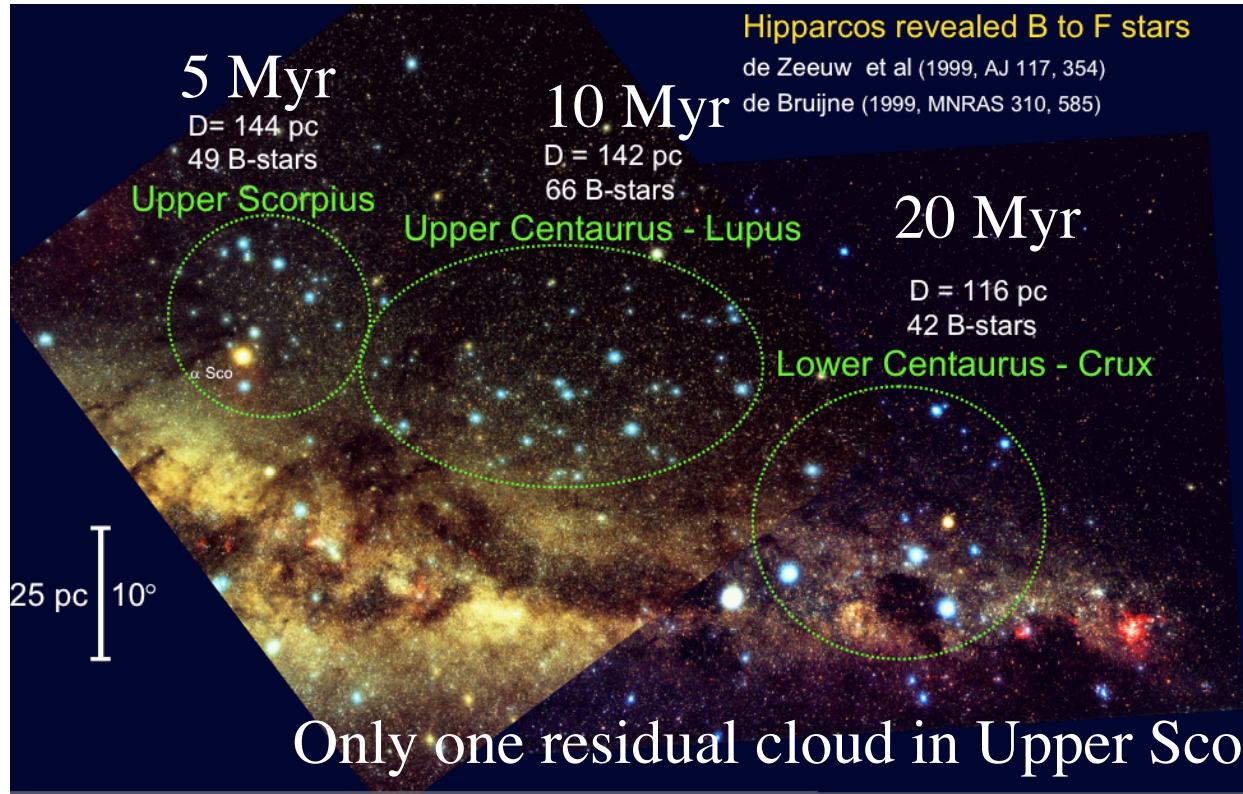
OB Associations

Unbound groups of OB stars plus many (usually undetected) low mass stars.

Near associations include Sco OB2 and Orion OB1.

Associations have subgroups of different ages.

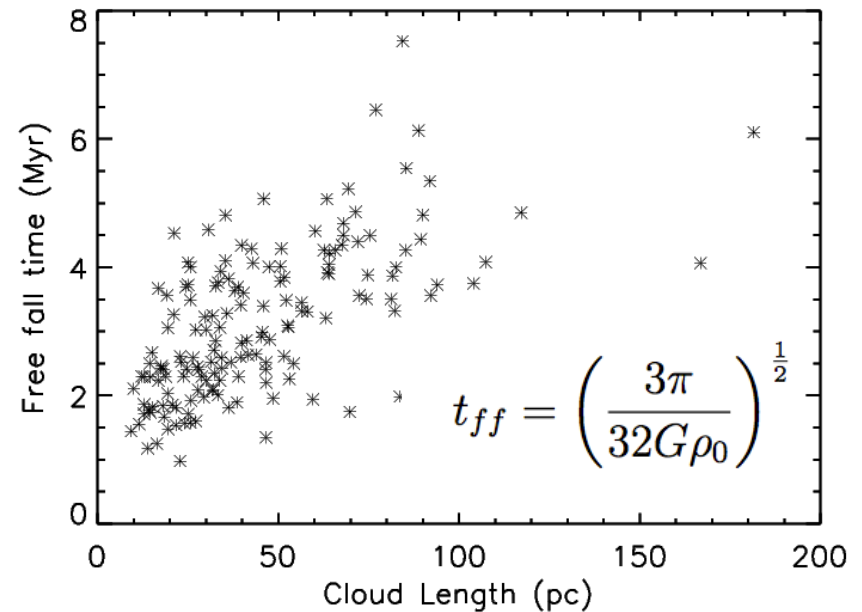
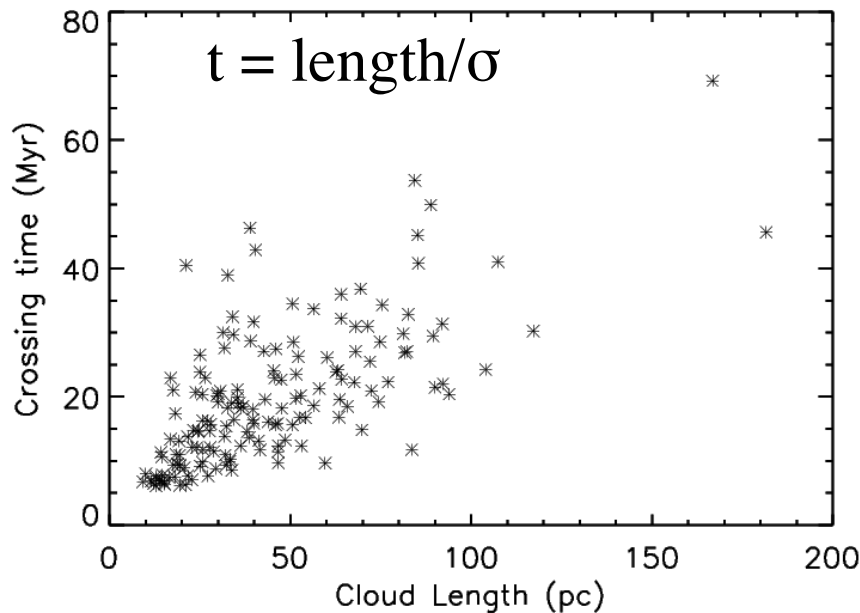
Subgroups with ages > 5 Myr typically don't have residual gas.



Timescales

- Star formation lifetime < 5 Myr

Star formation timescale comparable to cloud free-fall time and larger than crossing time.



A Proposed Picture for Star Formation in the Nearby Galaxy

The Formation of Molecular Hydrogen

The rate of H₂ formation in the gas phase is very low.

H₂ can form on the surfaces of dust grains (from Stahler & Palla):

Collision rate with grains: $1/t_{coll} = n_{HI} \sigma_d V_{th}$

H moves around grains through quantum tunneling, may find defect in lattice and stick. When another H comes along, the two atoms bind. Some of that energy kicks the H₂ off the grain.

Formation rate: $R_{H_2} = 1/2 \gamma_H n_d / t_{coll}$
 $= 1/2 \gamma_H n_d n_{HI} \sigma_d V_{th}$

γ_H is the sticking probability, n_d is density of grains, n_{HI} is density of hydrogen, σ_d is cross section of grains, V_{th} = thermal velocity of gas

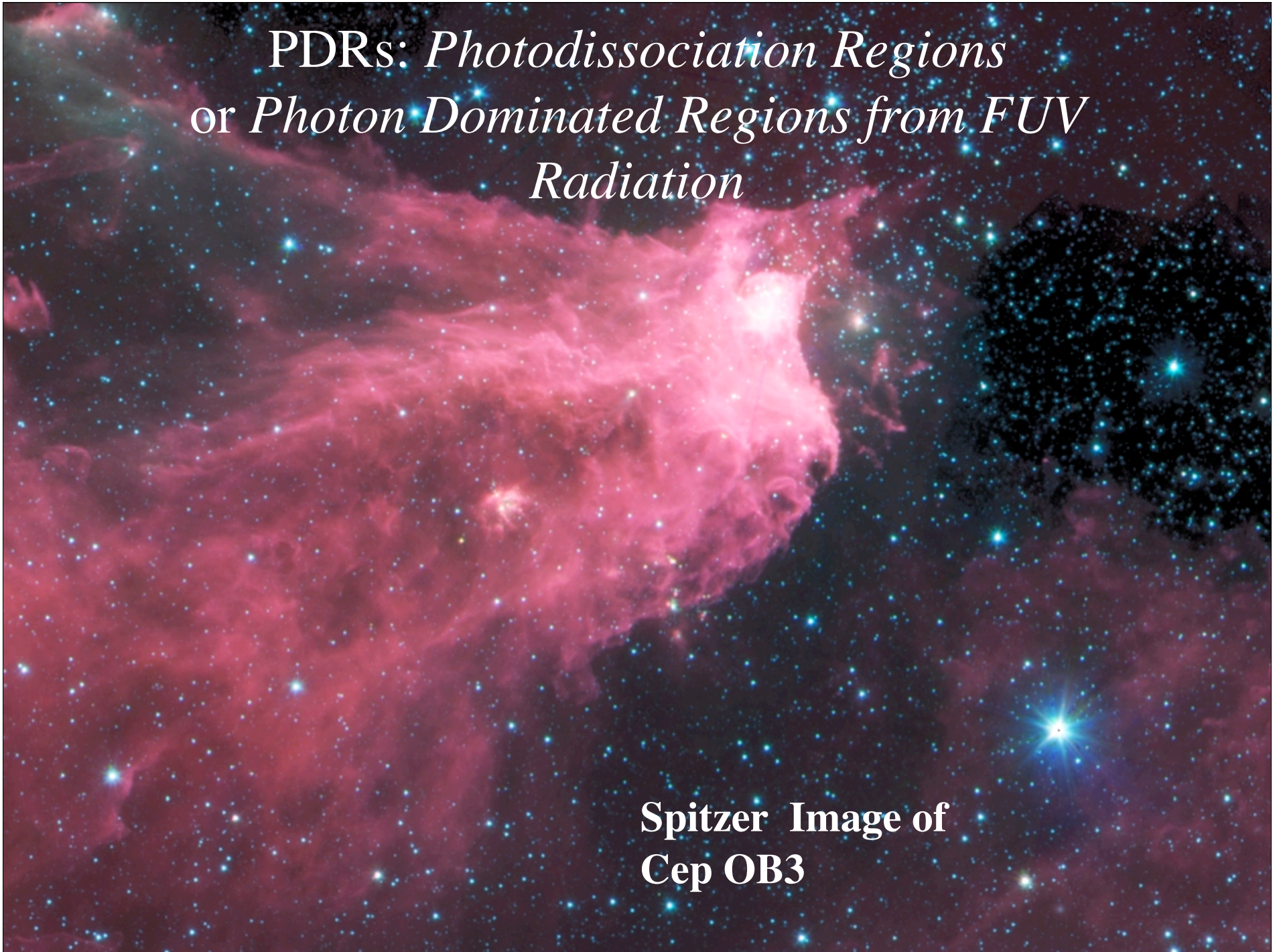
Dissociation of Molecular Hydrogen

1. Absorption of UV photons with energies of 4.48-13.6 eV
2. The photons raise molecule to excited electronic state.
3. This energy can be exchanged to vibrational and rotational modes.
4. A fraction of these excited molecules will dissociate into two H.

To prevent dissociation, molecules must be shielded from UV radiation from external starlight. This is done by dust and self-shielding of the molecule.

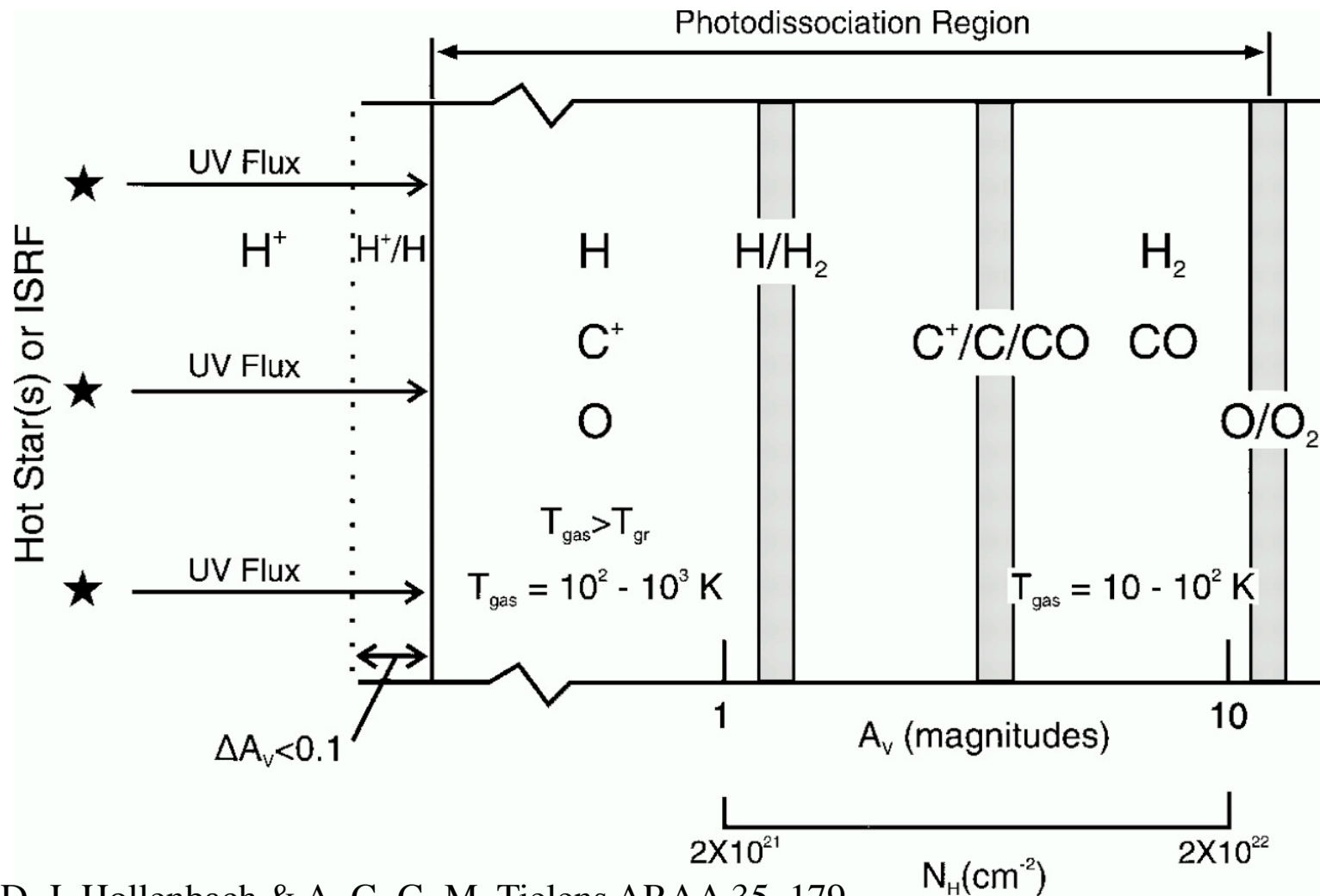
*PDRs: Photodissociation Regions
or Photon Dominated Regions from FUV
Radiation*

**Spitzer Image of
Cep OB3**



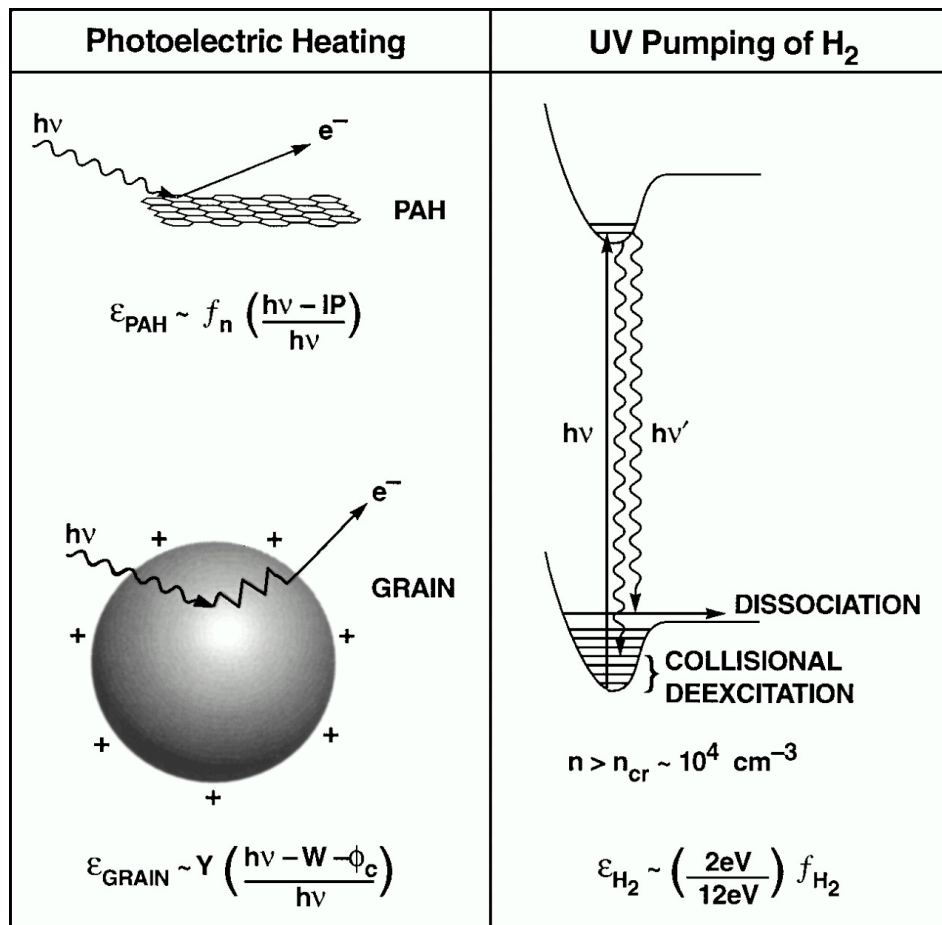
PDRs

Photons with $13.6 \text{ eV} > h\nu > 4.48 \text{ eV}$ cannot ionize H_2 , but can dissociate H_2

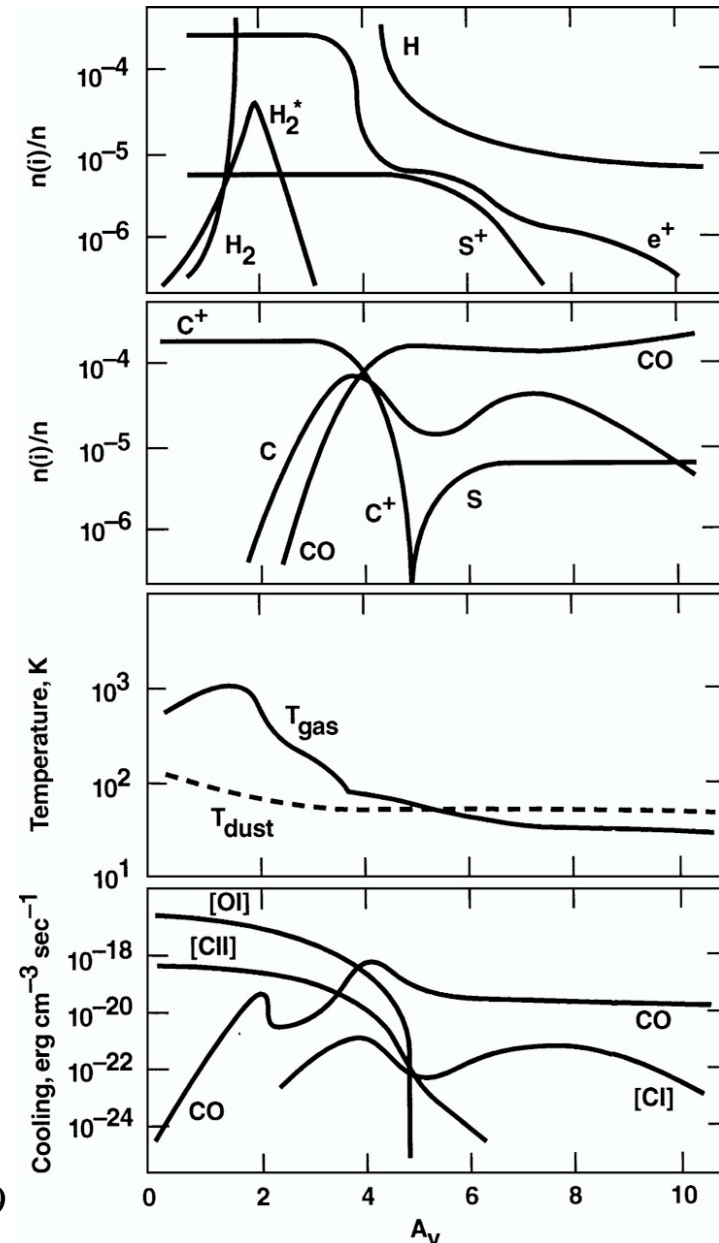


Heating and Cooling in PDRs

Temperature determined by balance of heating and cooling rates



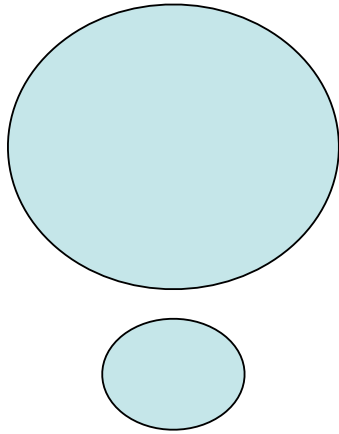
D. J. Hollenbach & A. G. G. M. Tielens ARAA 35, 179



Collapse vs Compression

How do we create a cloud with $N(\text{H}_2) = 2 \times 10^{21} \text{ cm}^{-3}$ ($2 A_V$)

3d collapse of spherical cloud

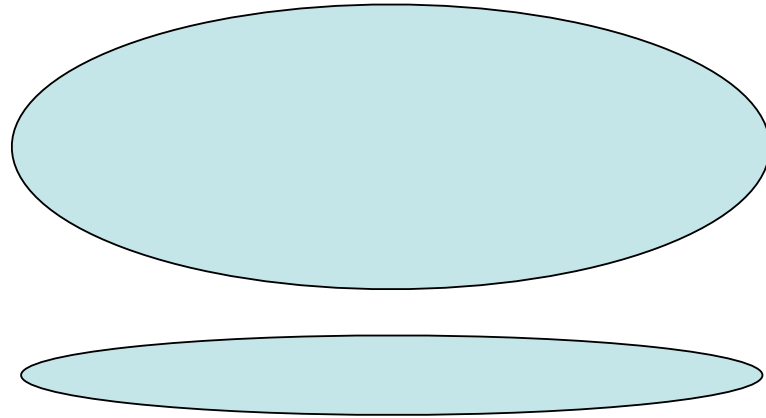


$$\tau = \kappa \rho 2R$$

$$\rho = M / (4/3\pi R^3)$$

$$\tau = [\kappa 3 / (4\pi)] R^{-2}$$

1d compression of elongated cloud



$$\tau = \kappa \rho \Delta x$$

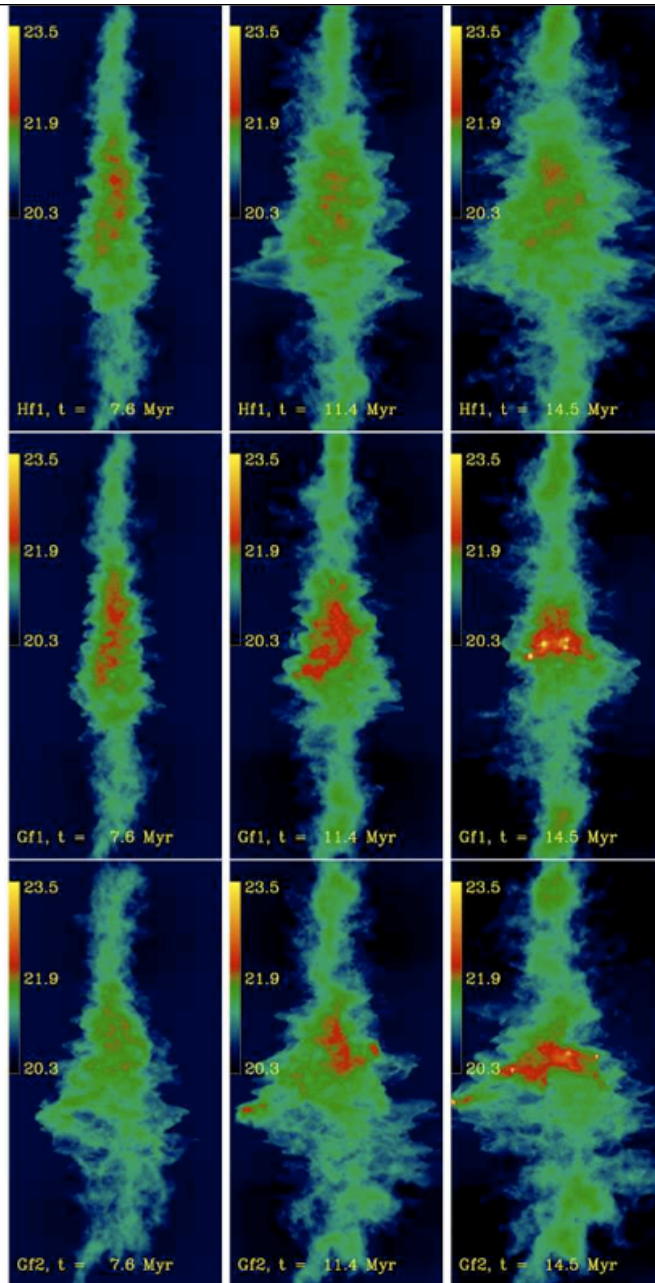
$$\rho = M / (AL)$$

$$\tau = kM/A$$

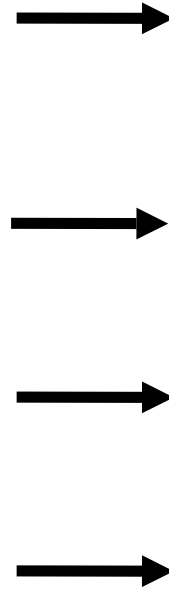
The time to collapse from an HI cloud with density of 10 cm^{-3}

Would take 6 Myr to sweep up from 10 cm^{-3} with a 10 kms^{-1} motion

Converging Flows

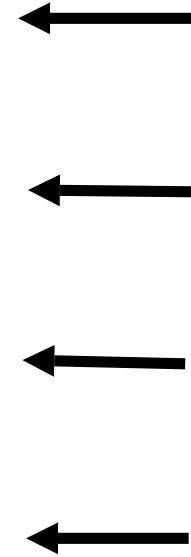


HI flow (from bubble as example)



\rightarrow
 H_2

HI flow (from bubble as example)



Heitsch et al. In press

FIG. 2.— Time sequence of logarithmic column density maps for models Hf1 (top), Gf1 (center) and Gf2 (bottom), seen perpendicular to the inflow direction. The full computational domain is shown, measuring 22×44 pc.

Formation of Clumps (Heitsch et al. 2008 in press)

heitsch et al.

3

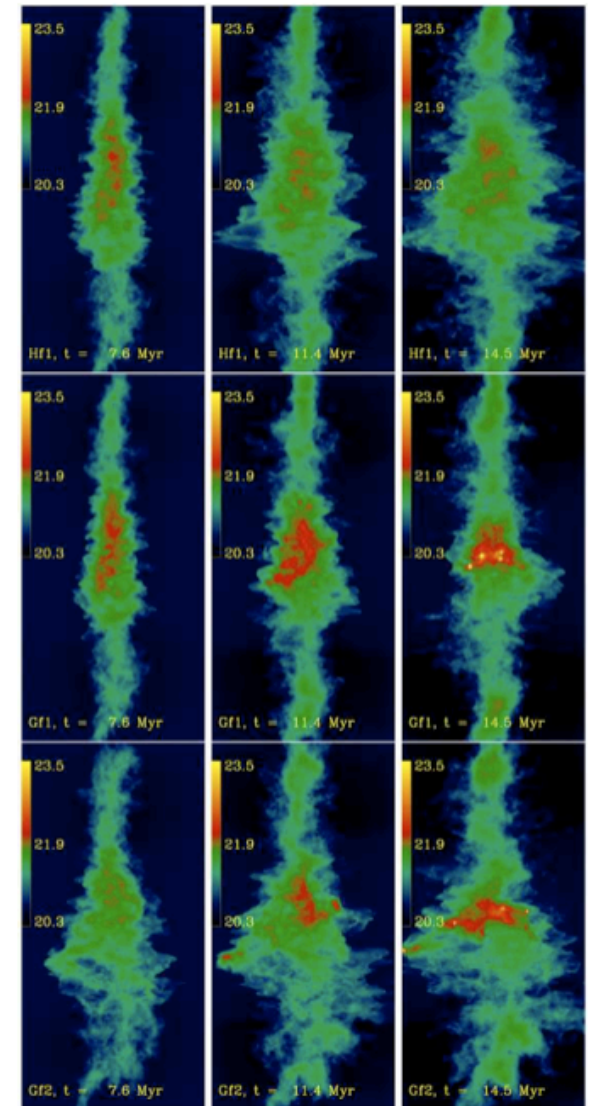
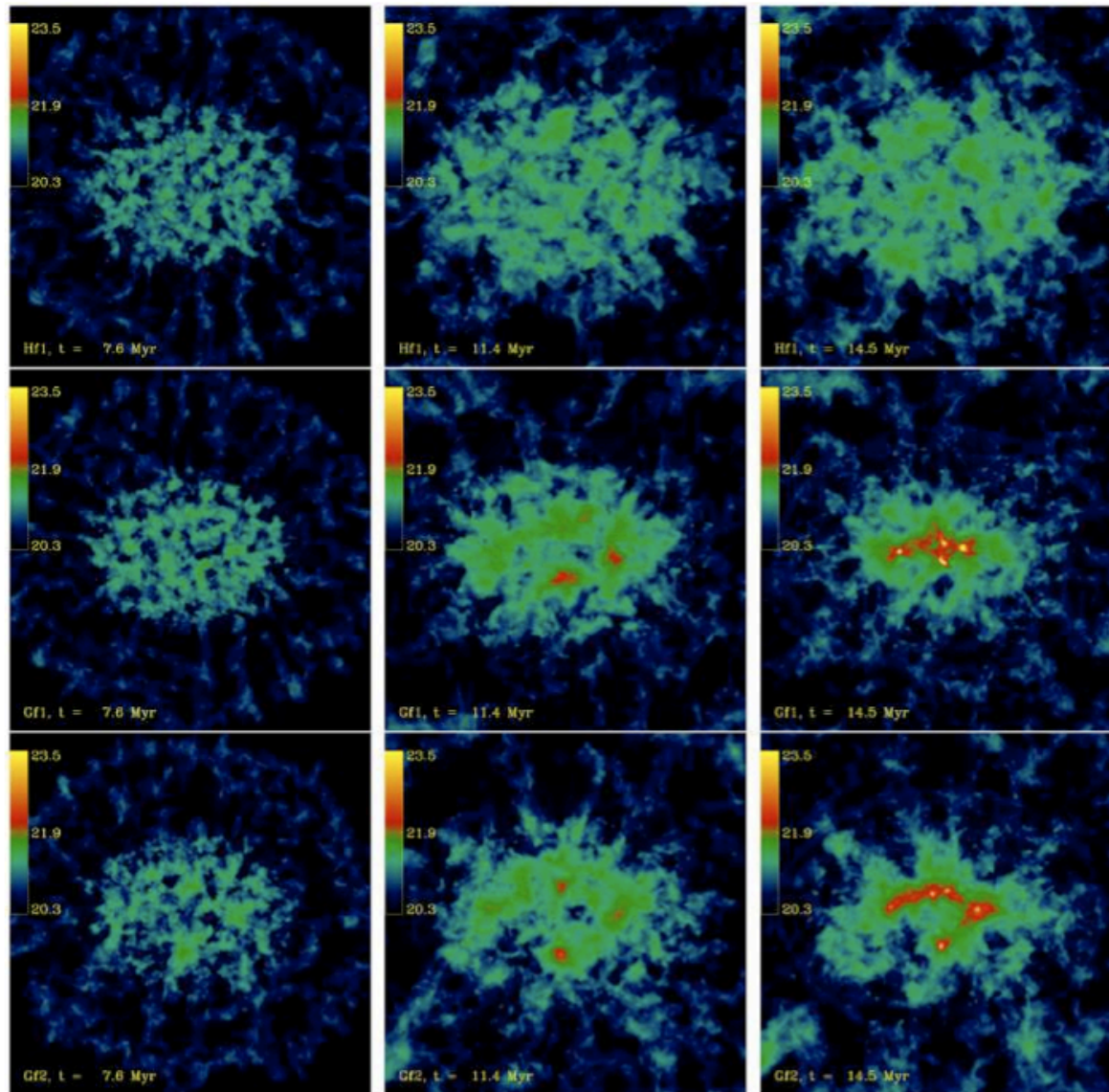
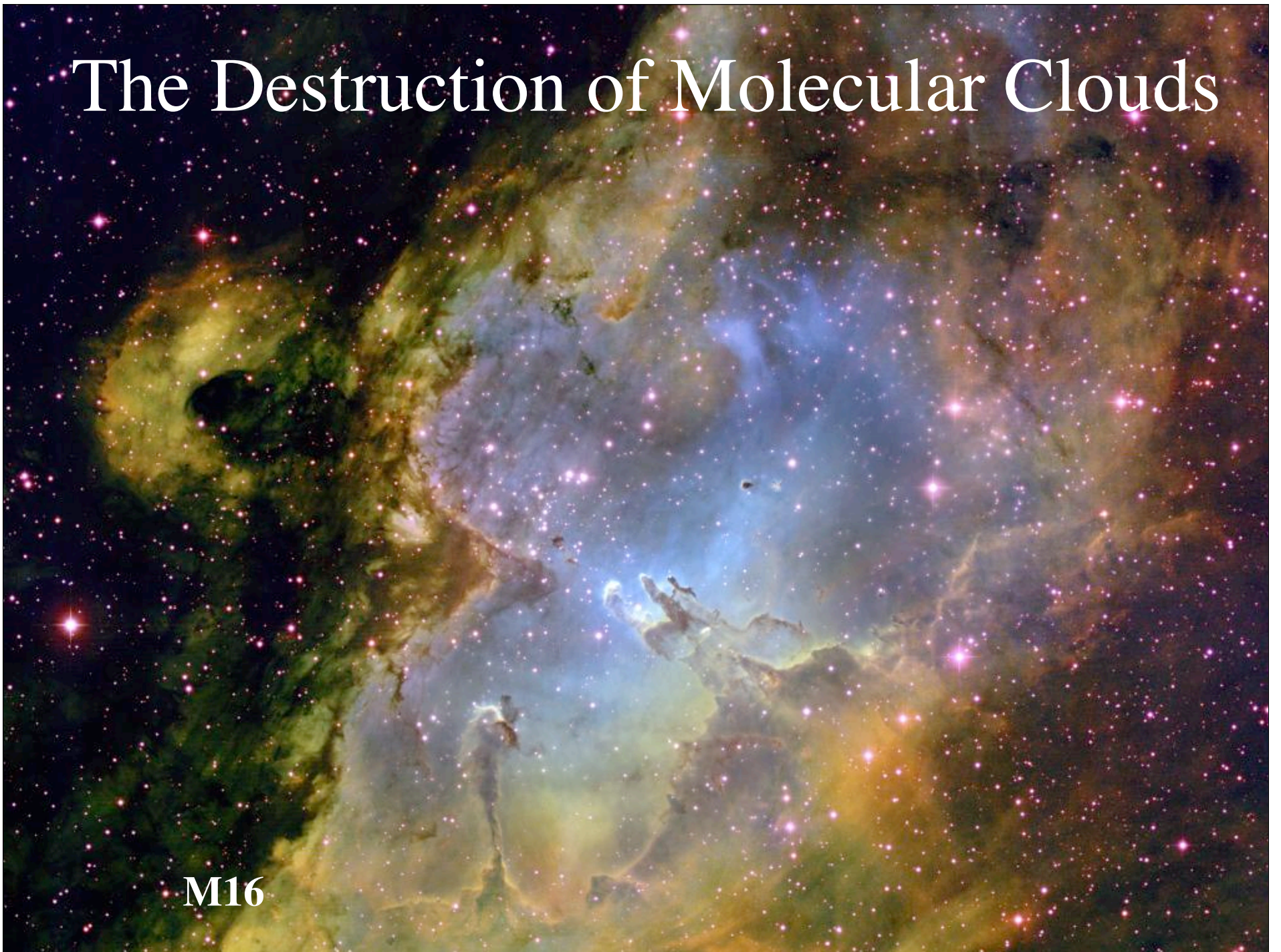


FIG. 2.— Time sequence of logarithmic column density maps for models Hf1 (top), Gf1 (center) and Gf2 (bottom), seen perpendicular to the inflow direction. The full computational domain is shown, measuring 22×44 pc.

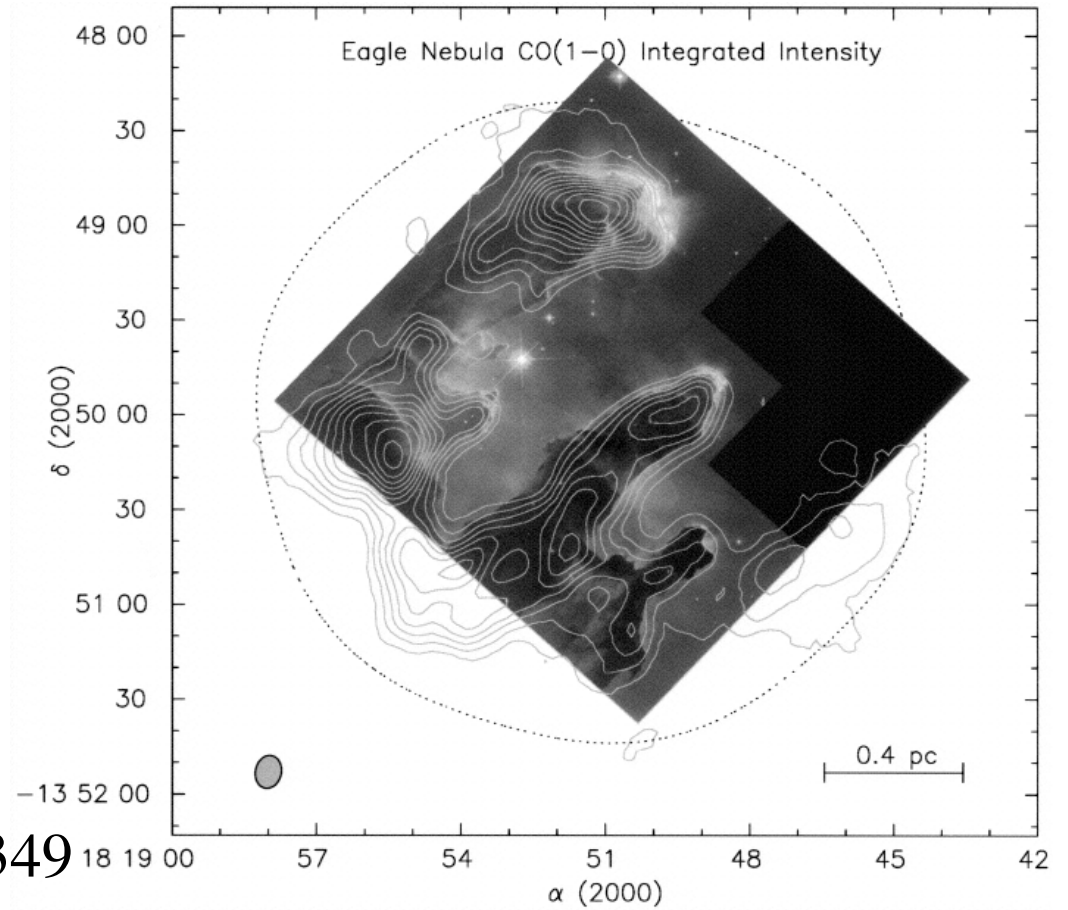
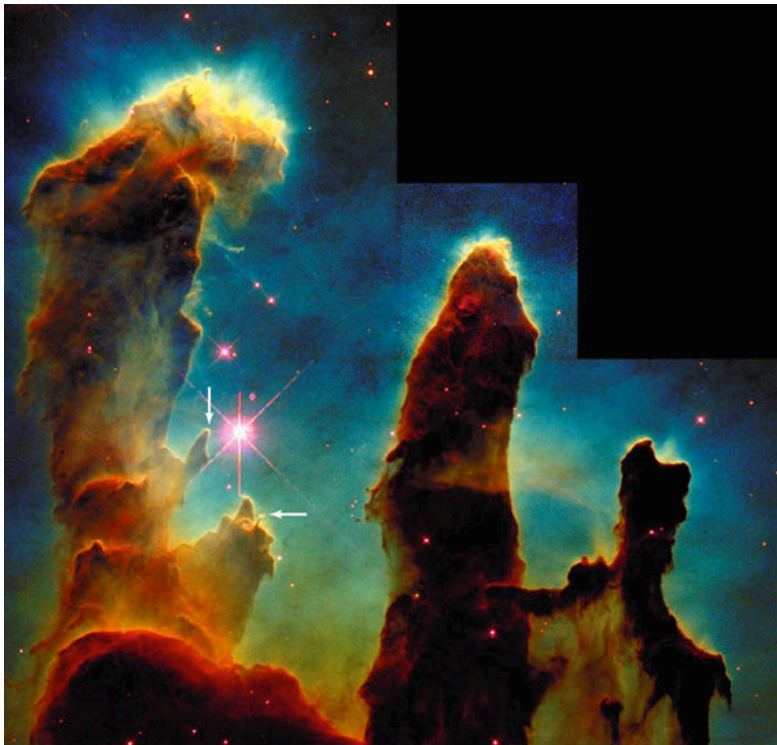
FIG. 1.— Time sequence of logarithmic column density maps for models Hf1 (top), Gf1 (center) and Gf2 (bottom), seen along the inflow direction. At $t = 7.6$ Myr (left column) the full domain (44 pc) is shown, while we restrict the field of view to the central 3/4 (33 pc) of the domain at later times, to highlight the small-scale dense structures forming.

The Destruction of Molecular Clouds

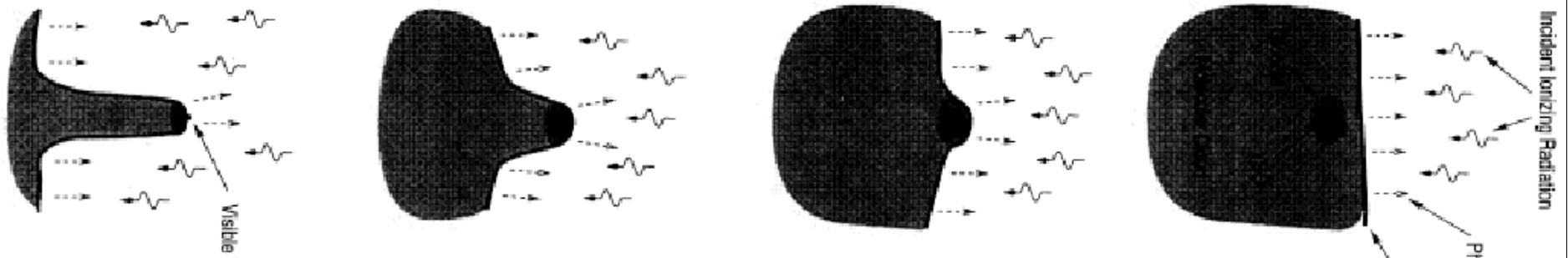
M16



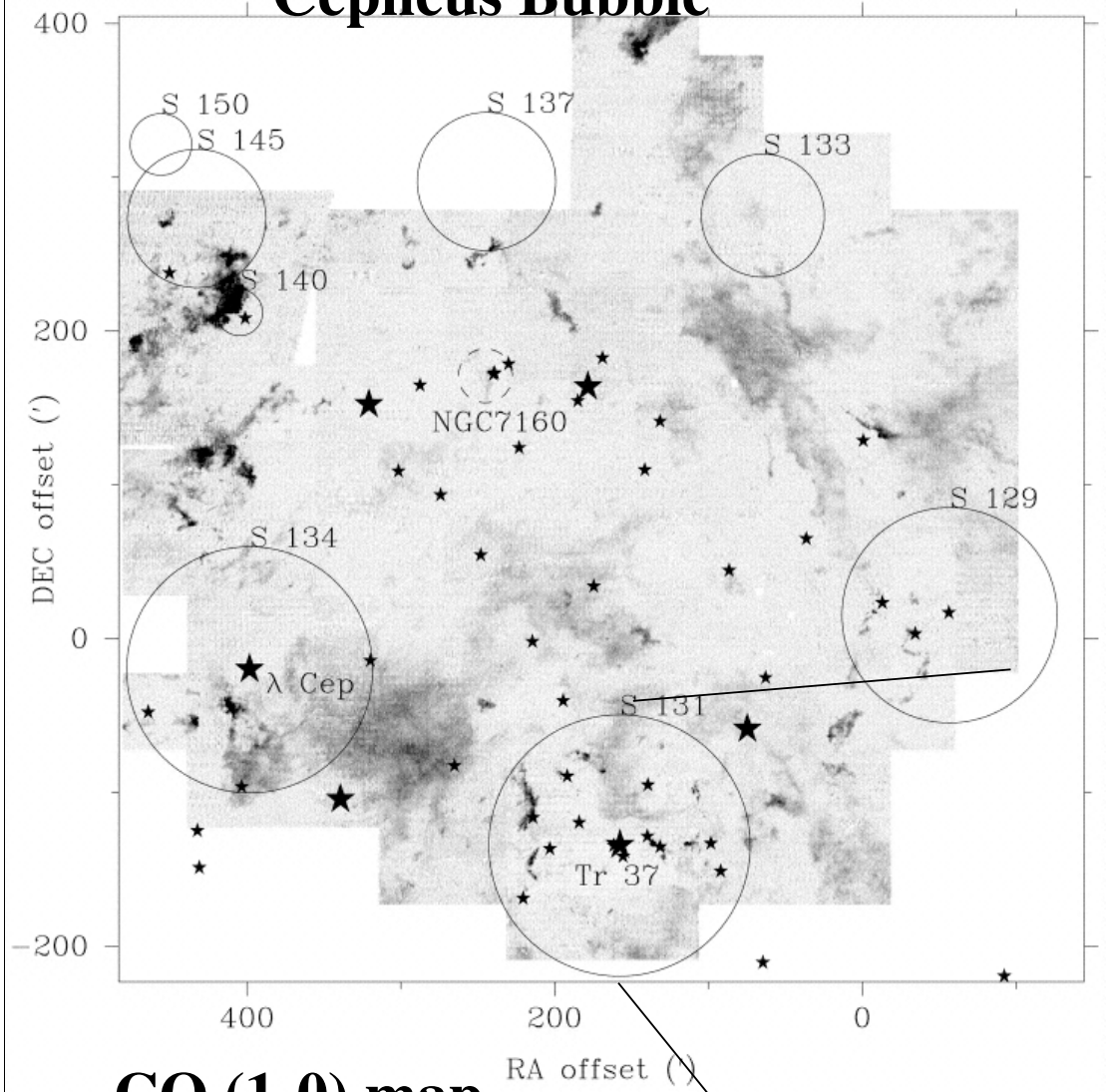
Photoevaporative Flows due to EUV Ionizing Radiation ($h\nu > 13.6 \text{ eV}$)



Hester et al. 1996 AJ 111, 2349



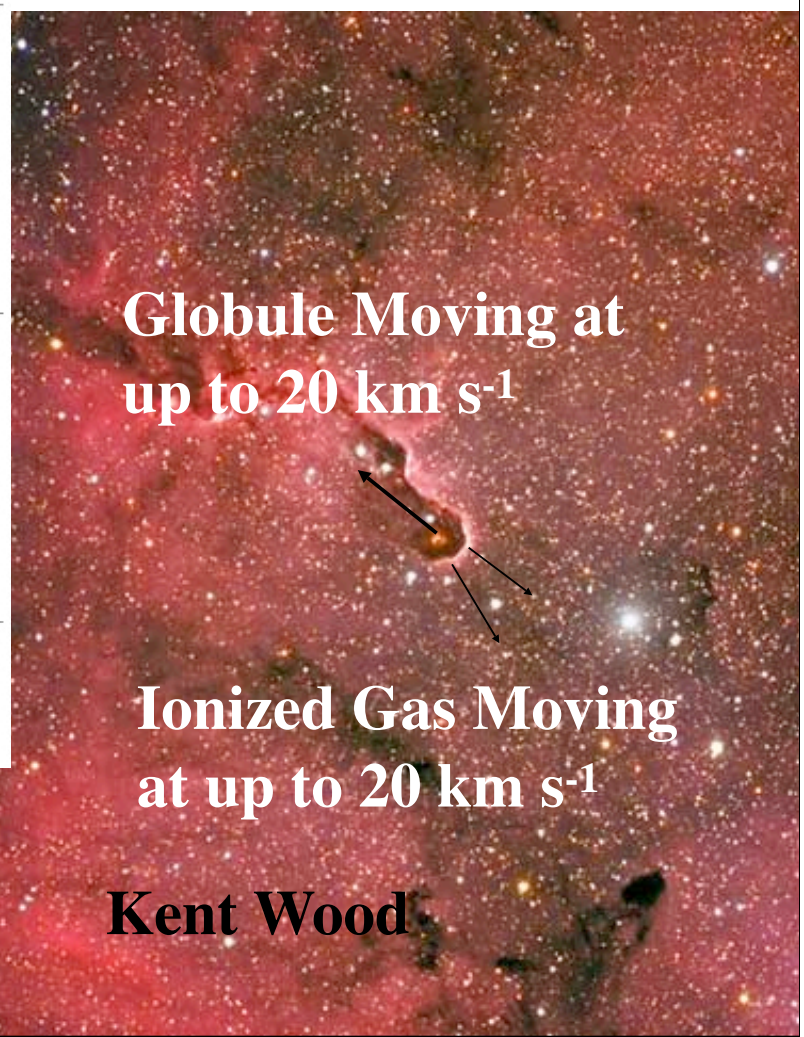
Cepheus Bubble



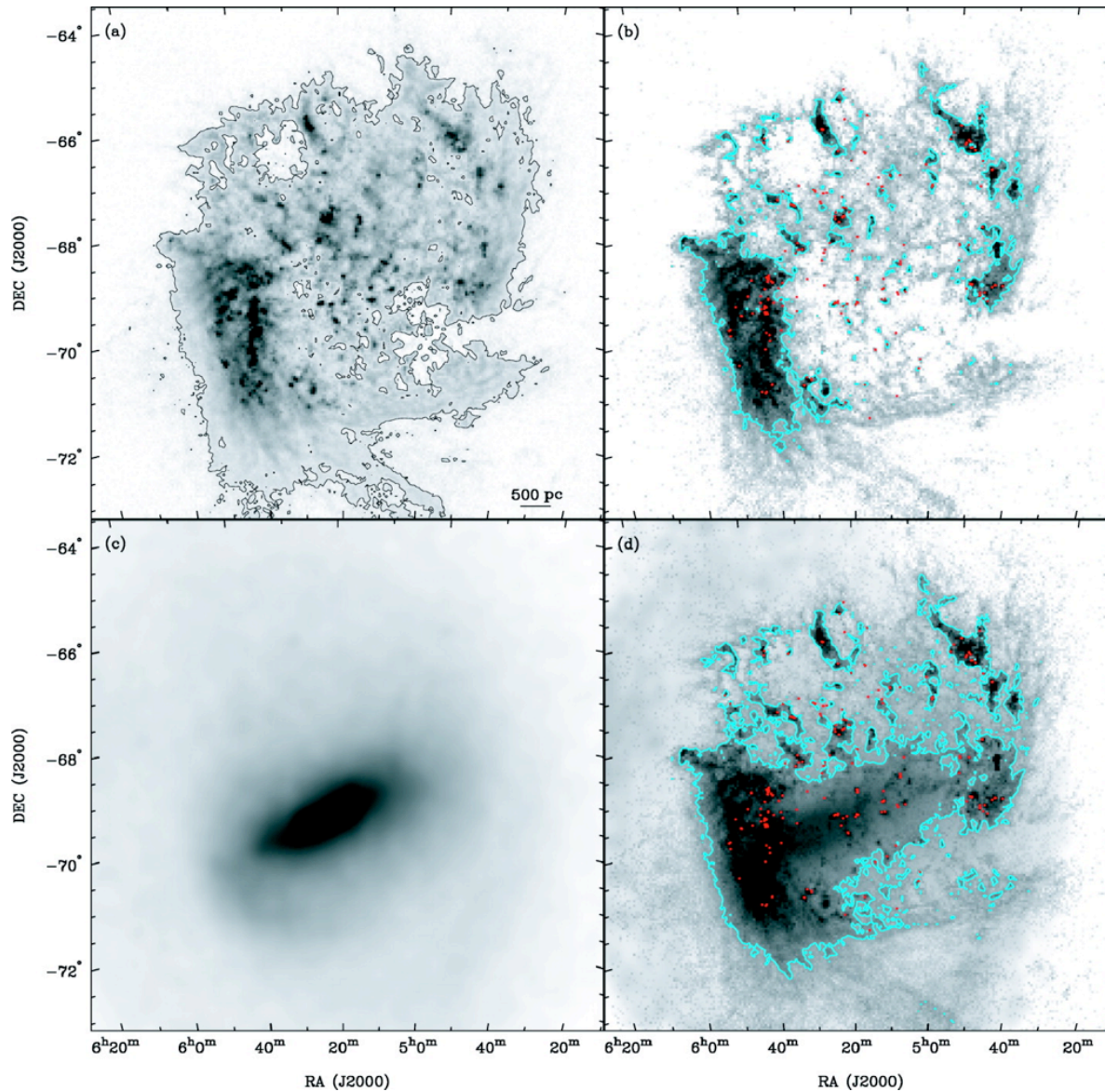
CO (1-0) map

Patel et al. 1998 ApJ 507, 241

The Rocket Effect



Gravity also plays an important role in forming clouds



$$Q_g \equiv \frac{\kappa C_g}{\pi G \Sigma_g} < 1,$$

Summary

Molecular clouds are typically massive objects with masses of $10^4 - 10^6$ solar masses

They tend to follow Larson's laws: $N(\text{H}_2) = 1 \times 10^{21} \text{ cm}^{-2}$, $\sigma = k R^{0.5}$

They tend to have approximately equal kinetic and potential energies.

They are unstable to fragmentation

Lifetimes are a few million years, comparable to free-fall time, less than crossing time.

Picture of star formation:

- Gas swept up into opaque sheets.
- Gravity forms filaments and clumps
- Gas fragments to form stars
- Stars (particularly massive stars) quickly dissipate the gas.

Sizescales

100 pc

< 0.1 pc

10 Myr

Cloud complexes OB associations

Timescales

Individual Clouds

Clusters and Clumps

1 Myr

Cores and Protostars

Supersonic Motions in Clouds

Sound speed in molecular clouds

$$c_s = (kT/m_{H_2})^{1/2}$$

$$c_s = 0.35 \text{ km s}^{-1}$$

Linewidths were a few km s⁻¹, thus motions are supersonic