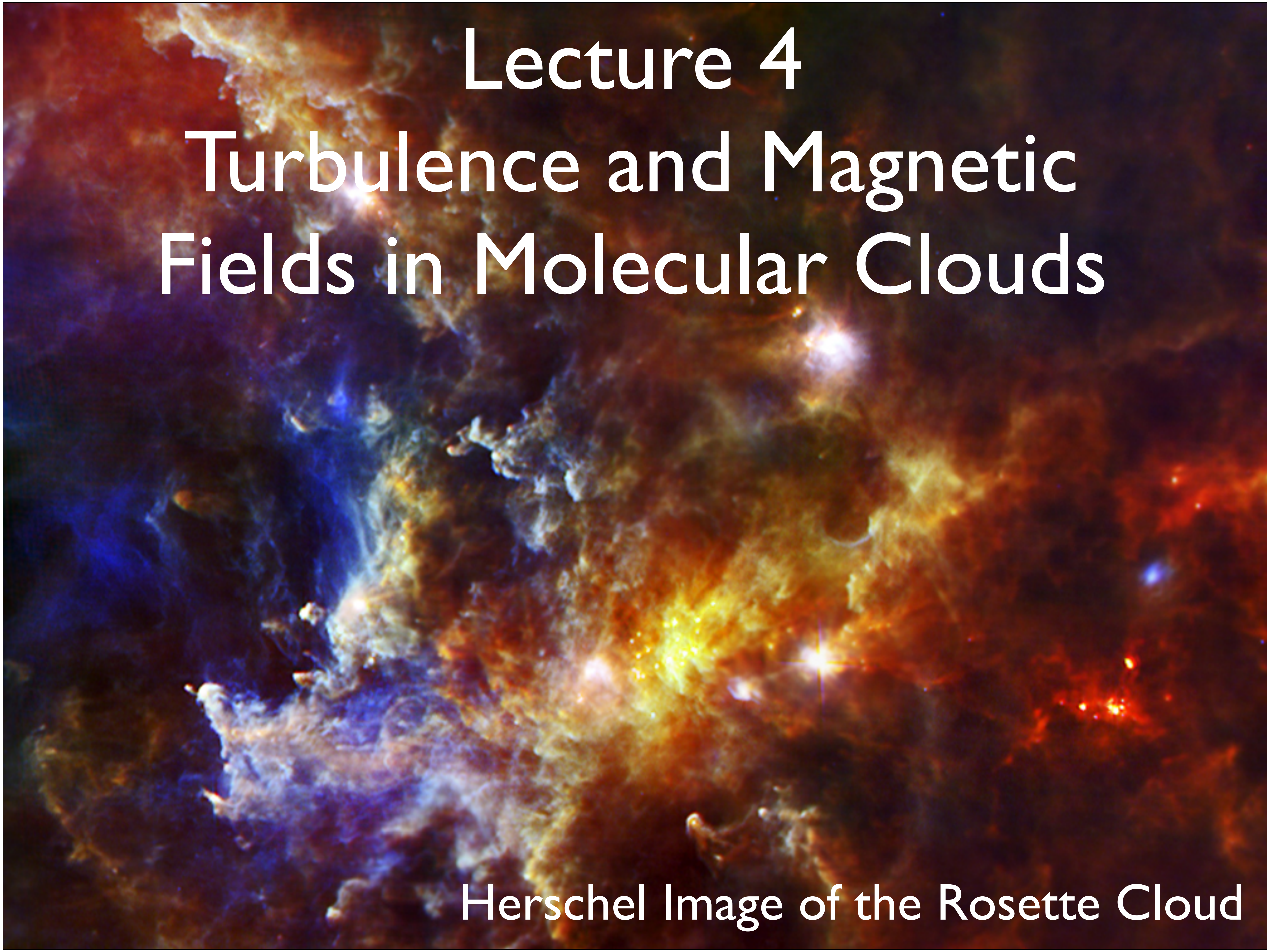
Lecture 4
Turbulence and Magnetic Fields in Molecular Clouds



Herschel Image of the Rosette Cloud

Molecular Cloud Properties

Composition: H_2, He, dust (1% mass), CO (10^{-4} by number), and many other molecules with low abundances.

Sizes: 10-100 pc

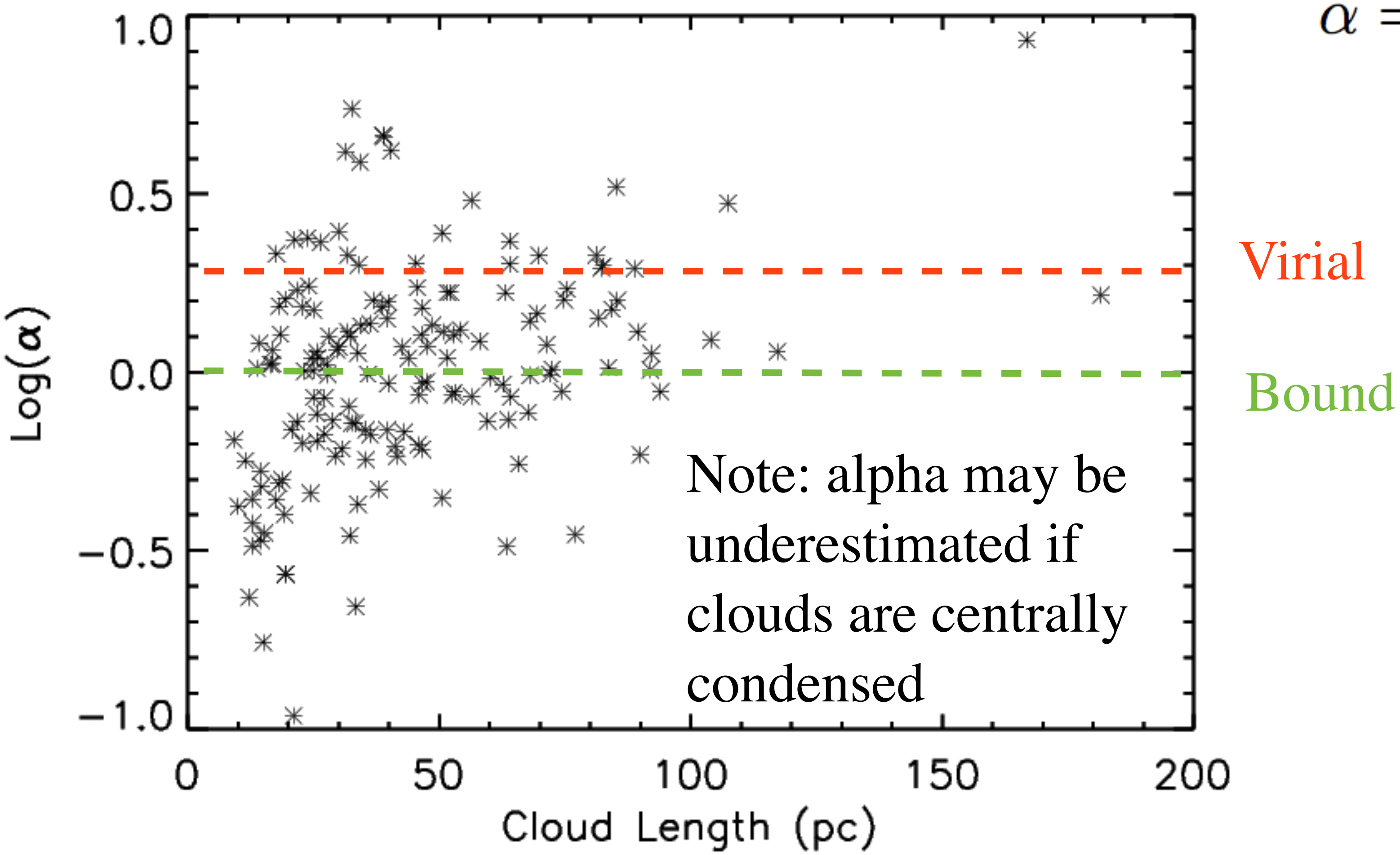
Masses: 10 to 10^6 Msun. Most of the molecular gas mass in galaxies is found in the more massive clouds.

Average Density: 100 cm^{-3}

Gas Temperature: 10-30 K

Review: Properties of Molecular Clouds

$$\alpha = \frac{2}{5}\frac{GM}{R\sigma^2}$$



Heyer et al. 2009

The Stability of Clouds: Jeans Instability

$$k_j^2 = 4\pi G \frac{\rho_0}{c_s^2} \tag{25}$$

Thus, if $k > k_J$, w is real and we get a normal wave equation. However, if $k < k_J$, then the solution for w is imaginary. The resulting time dependence solution for the perturbation density is

$$\delta\rho = \frac{\delta\rho_0}{2}(e^{(|k^2-k_J^2|)^{1/2}t} + e^{-(|k^2-k_J^2|)^{1/2}t}) \tag{26}$$

which increases exponentially with time. In other words, gravity wins over pressure, and the pertubations are unstable. We can convert that into a length, the Jeans length

$$k_J = 2\pi/\lambda_J, \ \lambda_J = \sqrt{\frac{\pi c_s^2}{G\rho_0}} \tag{27}$$

the Jeans length can be turned into a mass

$$m_j = \rho_0 \lambda_J^3 = \frac{c_s^3 \pi^{3/2}}{G^{3/2}\rho_0^{1/2}} = \left(\frac{\pi k T}{\mu m_H G}\right)^{3/2} \rho_0^{-1/2} \tag{28}$$

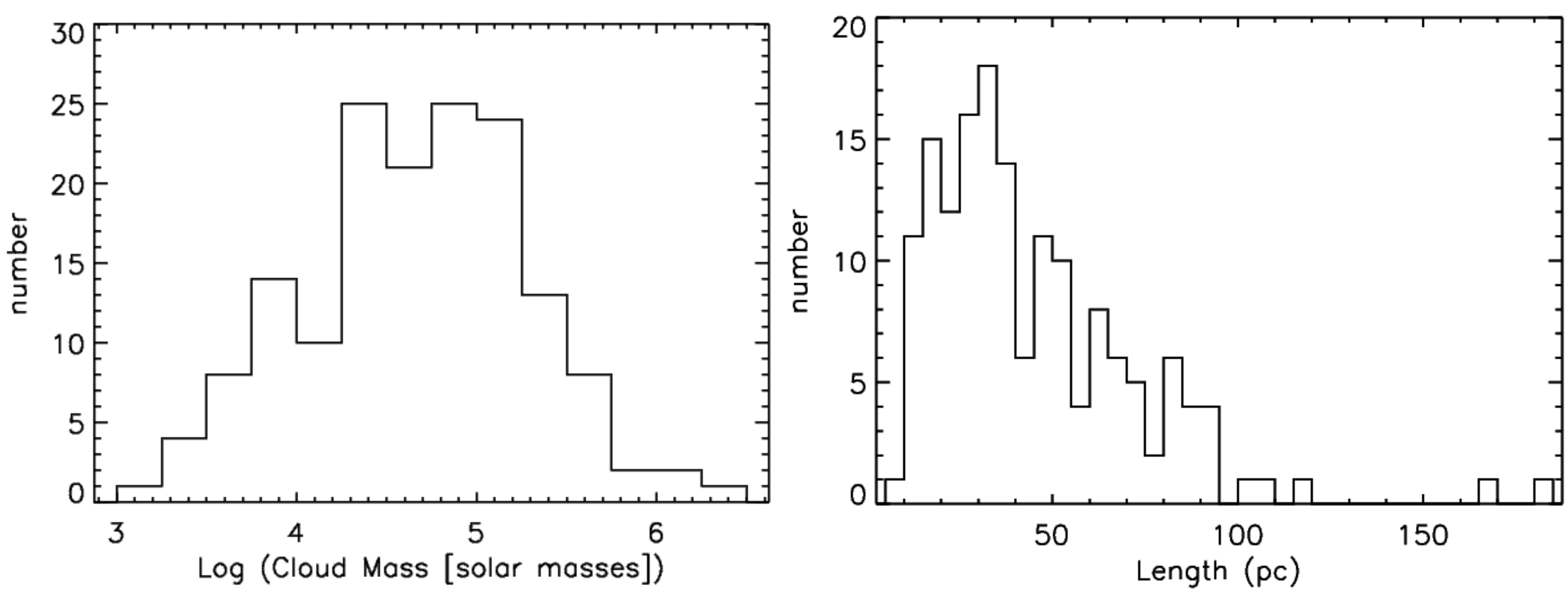
Are Clouds Stable to Fragmentation?

For $n(H_2) = 40\ cm^{-3}$ $\lambda_J = 6$ pc $M_J = 450$ Msun (cloud size)

For $n(H_2) = 100\ cm^{-3}$ $\lambda_J = 4.$ pc $M_J = 350$ Msun (cloud size)

For $n(H_2) = 1000\ cm^{-3}$ $\lambda_J = 1.2$ pc $M_J = 100$ Msun (clump size)

Cloud masses and lengths exceed Jeans length masses.



From clouds in the inner galaxy from Heyer et al. 2009

Star Formation Efficiency

The star formation efficiency of a molecular cloud, ϵ, is defined as the following:

$$\epsilon = \frac{M_{stars}}{M_{stars} + M_{gas}} \tag{30}$$

where M_{stars} is the mass in stars and M_{gas} is the mass in molecular mass. It is essentially the fraction of mass that is converted into stars. Since it is difficult to determine the mass of individual stars, M_{gas} is calculated usually as the number of stars times the average stellar mass of 0.3 $M_{\odot}$, i.e. $M_{stars} = N_{stars} \times 0.3\ M_{\odot}$.

Typical cloud efficiencies are a few percent for molecular clouds

Table 4
Efficiencies and Depletion Timescales

Cloud	$\frac{M_*}{(M(\text{cloud})+M_*)}$	$M_*/M(\text{dense})$	$t_{\text{dep}}(\text{cloud})$	$t_{\text{dep}}(\text{dense})$	SFR_{ff}	Notes
			(Myr)	(Myr)		
Cha II	0.030		66	...	0.028	Alcalá et al. (2008)
Lupus[a]	0.054		35	...	0.050	Merín et al. (2008)
Perseus	0.038	0.69	50	2.9	0.049	S.-P. Lai et al. (2008, in preparation)
Serpens	0.053	1.2	35	1.6	0.036	Harvey et al. (2007a)
Ophiuchus	0.063	3.3	30	0.6	0.064	L. Allen et al. (2008, in preparation)
All Clouds[b]	0.048	1.2	40	1.8	0.040	...

Notes.
[a] A sum over the three Lupus clouds of the values for each cloud.
[b] For all but SFR_{ff}, this number is calculated by adding all clouds with the relevant data together; for SFR_{ff}, it is the average over all clouds of the individual values.

From Evans et al. 2009

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From Evans et al. 2009

Lifetimes of Nearby Clouds

TABLE 1

STAR-FORMING REGIONS

Region	$\langle t \rangle$[a] (Myr)	Molecular Gas?	Ref. (age)
Coalsack	...	Yes	...
Orion Nebula	1	Yes	1
Taurus	2	Yes	1, 2, 3
Oph	1	Yes	1
Cha I, II	2	Yes	1
Lupus	2	Yes	1
MBM 12A	2	Yes	4
IC 348	1–3	Yes	1, 4, 5, 6
NGC 2264	3	Yes	1
Upper Sco	2–5	No	1, 6, 7
Sco OB2	5–15	No	8
TWA	$\sim$10	No	9
η Cha	$\sim$10	No	10

[a] Average age in Myr.

REFERENCES.—(1) Palla & Stahler 2000. (2) Hartmann 2001. (3) White & Ghez 2001. (4) Luhman 2001. (5) Herbig 1998. (6) Preibisch & Zinnecker 1999. (7) Preibisch et al. 2001. (8) de Geus et al. 1989. (9) Webb et al. 1999. (10) Mamajek, Lawson, & Feigelson 1999.

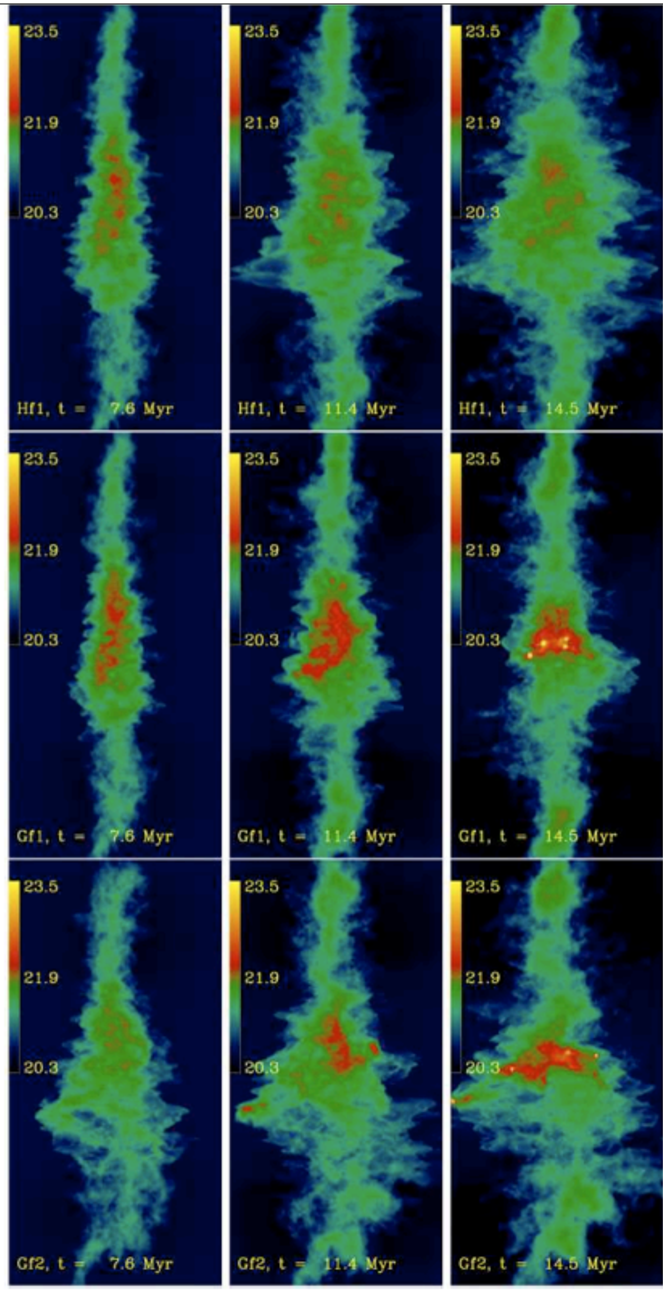
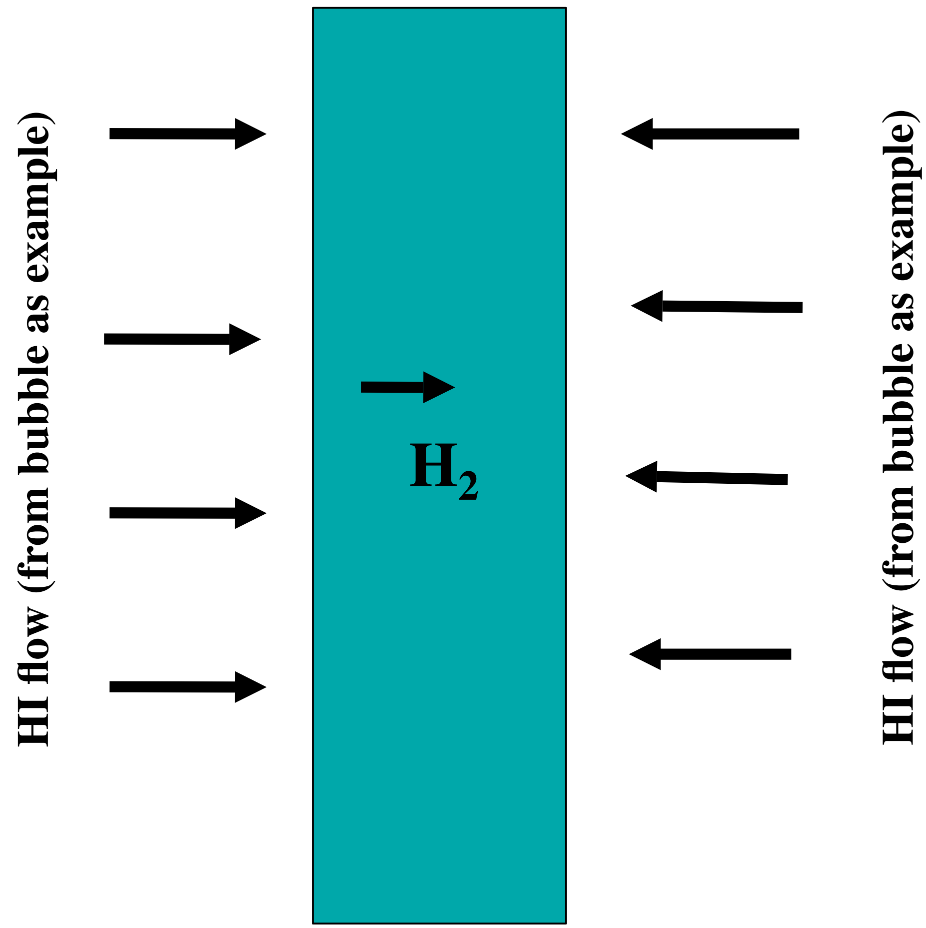


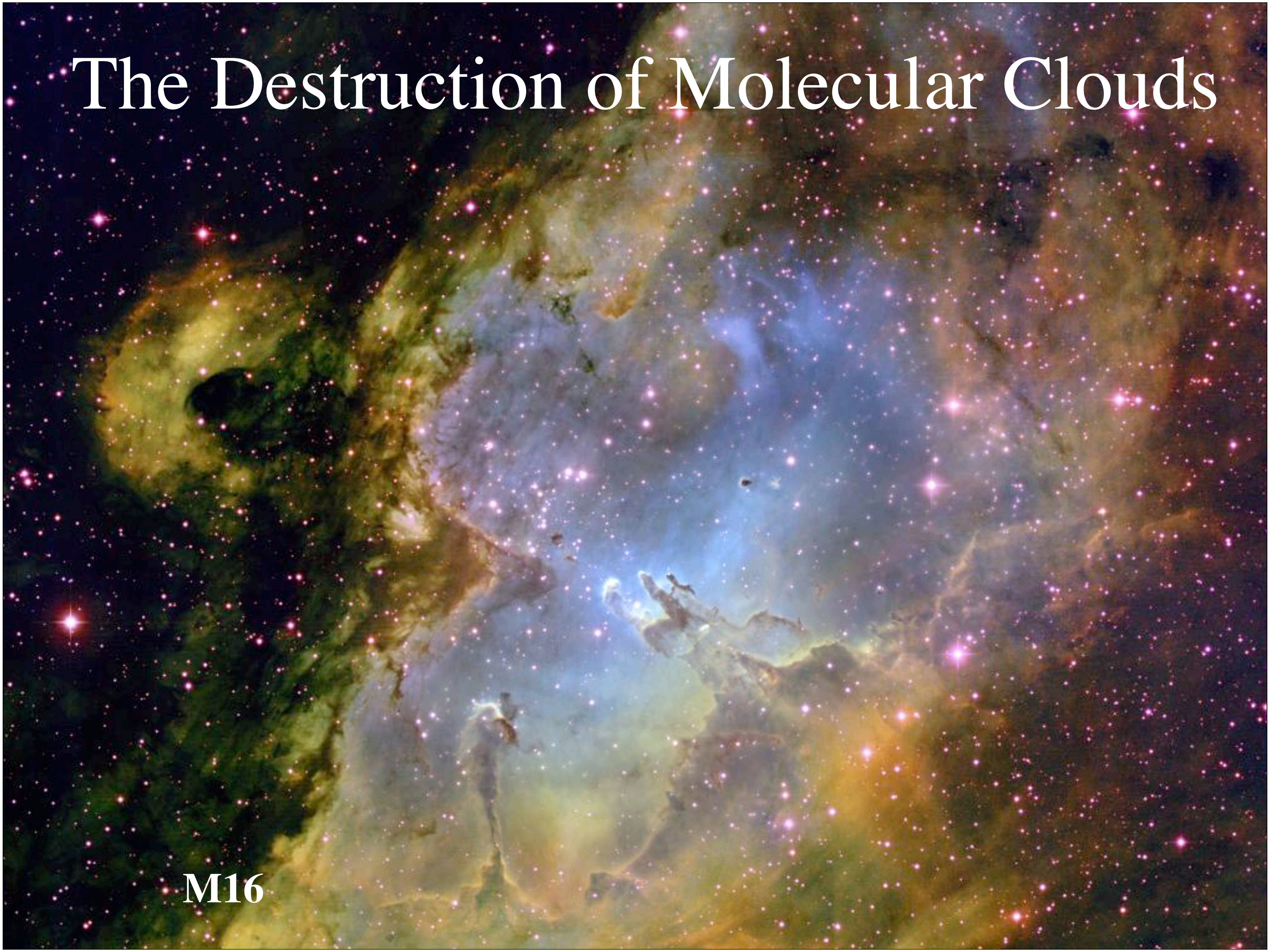
FIG. 2.— Time sequence of logarithmic column density maps for models Hf1 (top), Gf1 (center) and Gf2 (bottom), seen perpendicular to the inflow direction. The full computational domain is shown, measuring 22 × 44 pc.

Converging Flows

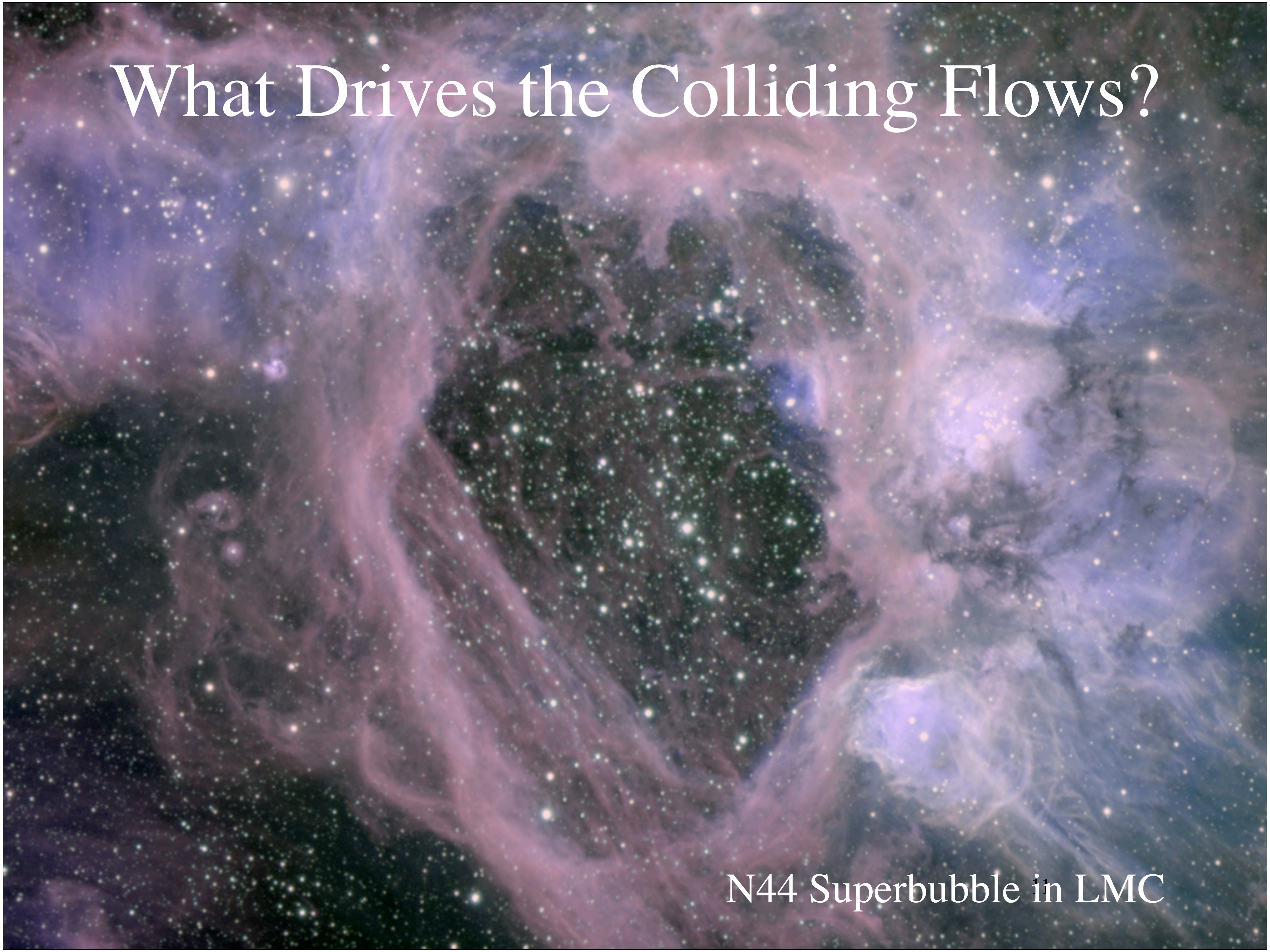


Heitsch et al. In press

The Destruction of Molecular Clouds



What Drives the Colliding Flows?



N44 Superbubble in LMC

The Virial Theorem for Molecular Clouds

Last semester, we derived the Virial theorem from essentially considering a series of particles which attract each other through gravitation. The result was that

$$\frac{d^2I}{dt^2} = 2K + \Omega \tag{1}$$

where the Virial $I = \int r^2 dm$, Ω is the gravitational potential energy and K is the total kinetic energy In the case of equilibrium where $\frac{d^2I}{dt^2} = 0$, we find that:

$$K = -\frac{1}{2}\Omega \tag{2}$$

which is very useful when considering stars in hydrostatic equilibrium. Unlike stars, for molecular clouds, there may be a significant external pressure. The virial theorem can be modified to take into account external pressure. In molecular cloud studies, a common way of writing the virial theorem is:

$$\frac{1}{2}\frac{d^2I}{dt^2} = 3\int_V PdV + \int_S P\, \mathbf{r} \cdot d\mathbf{S} + \Omega \tag{3}$$

Virial Theorem with External Pressure

We define $K_s =\equiv \frac{1}{2}\int_S P\ \mathbf{r}\cdot d\mathbf{S}$. If the external pressure is a constant value P_0, then we can replace:

$$K_s = \frac{1}{2}P_0\int_S \mathbf{r}\cdot d\mathbf{S} = \frac{3}{2}P_0 V \tag{4}$$

and we can write:

$$\frac{1}{2}\frac{d^2 I}{dt^2} = 3\int_V (P - P_0)dV + \Omega \tag{5}$$

Following Spitzer (1978), let's assume that a cloud is in equilibrium, i.e. $d^2I/dt^2 = 0$. The we rewrite equation 5 so that:

$$3P_0 V = 3\int_V P dV + \Omega \tag{6}$$

assuming a homogenous sphere with sound speed c_s

$$4\pi P_0 R_{cl}^3 = 3c_s^2 M_{cl} - \frac{3}{5}\frac{GM_{cl}^2}{R_{cl}} \tag{7}$$

Comparing the internal and external pressure of molecular clouds

$$\frac{1}{2}\frac{d^2I}{dt^2} = 3(P - P_0)V_{cl} + \Omega$$

- A self gravitating gas with $A_V > 2$ has an internal pressure of $P/k > 2 \times 10^4$ cm^{-3} K.

- $P/k = 100$ cm^{-3} $\sigma^2 \mu m_H = 3 \times 10^4$ cm^{-3} K ($\sigma = 1$ km s^{-1})

- Typical external ISM gas pressure is $P/k = 2 \times 10^4$ cm^{-3} K

- Molecular clouds with $A_V > 2$ and $n > 100$ cm^{-3} are not pressure confined.

Could a thermally supported cloud be stable?

Could a thermally supported cloud be stable?

To examine the stability of the cloud to changes in external pressure, we solve the above equation for P_0 and then take the derivative with R

$$P_0 = \frac{1}{4\pi}\left(\frac{3c_s^2 M_{cl}}{R_{cl}^3} - \frac{3}{5}\frac{GM_{cl}^2}{R_{cl}^4}\right) \tag{8}$$

and

$$\frac{dP_0}{dR_{cl}} = \frac{1}{4\pi}\left(-\frac{9c_s^2 M_{cl}}{R_{cl}^4} + \frac{12}{5}\frac{GM_{cl}^2}{R_{cl}^5}\right) \tag{9}$$

Now let's consider what happens if we increase pressure. If the right side of the equation is negative, i.e.

$$R > R_{crit} = \frac{4}{15}\frac{GM_{cl}}{c_s^2} \tag{10}$$

then the increase in pressure will cause the cloud to shrink, but the value of P_0 required for equilibrium goes up as well. Eventually, the cloud comes into equilibrium, unless $R \leq R_{crit}$. At this point, $dP_0/dR_{cl} = 0$ and P_0 is at a maximum. Increasing P_0 will imply that there is no stable equilibrium and the cloud will collapse.

Could a thermally supported cloud be stable?

$M = 10^4$ Msun
$T_K = 20$ K ($c_s = 0.25$ km s^{-1})
=> Rcrit = 200 pc

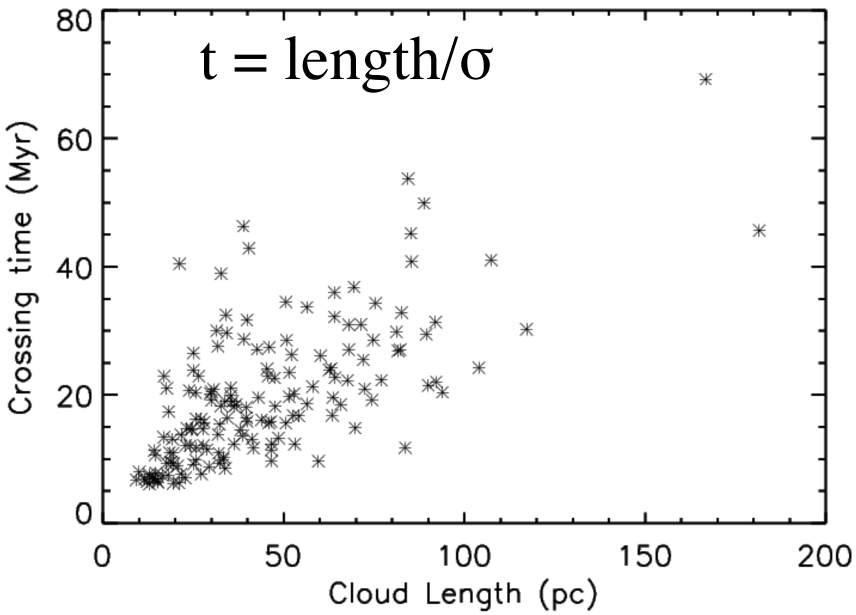
But most clouds are much smaller. Also, considering only thermal motions $\alpha = |\Omega/K| \sim 6$ - thus not virialized

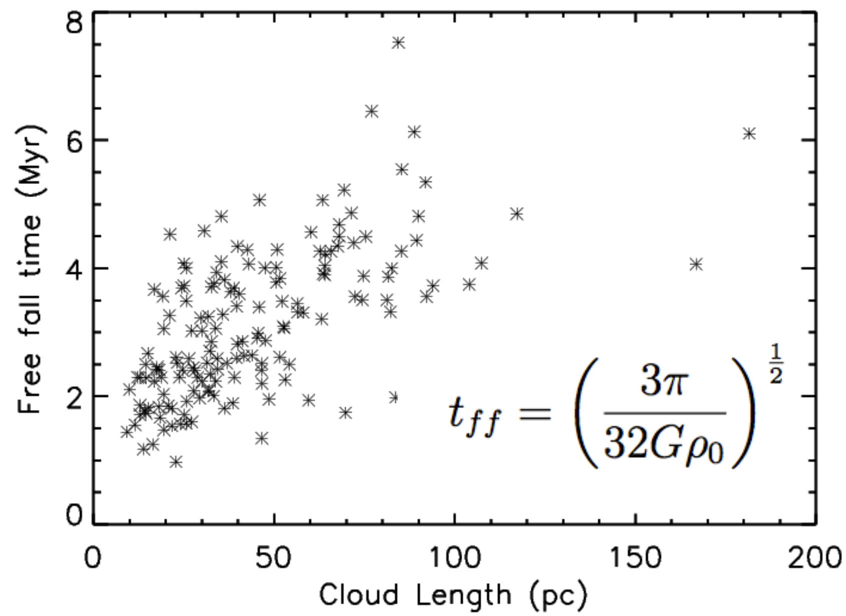
Furthermore Jeans criterion means that clouds are unstable to fragmentation

Timescales

- Star formation lifetime < 5 Myr

Star formation timescale comparable to cloud free-fall time and larger than crossing time.

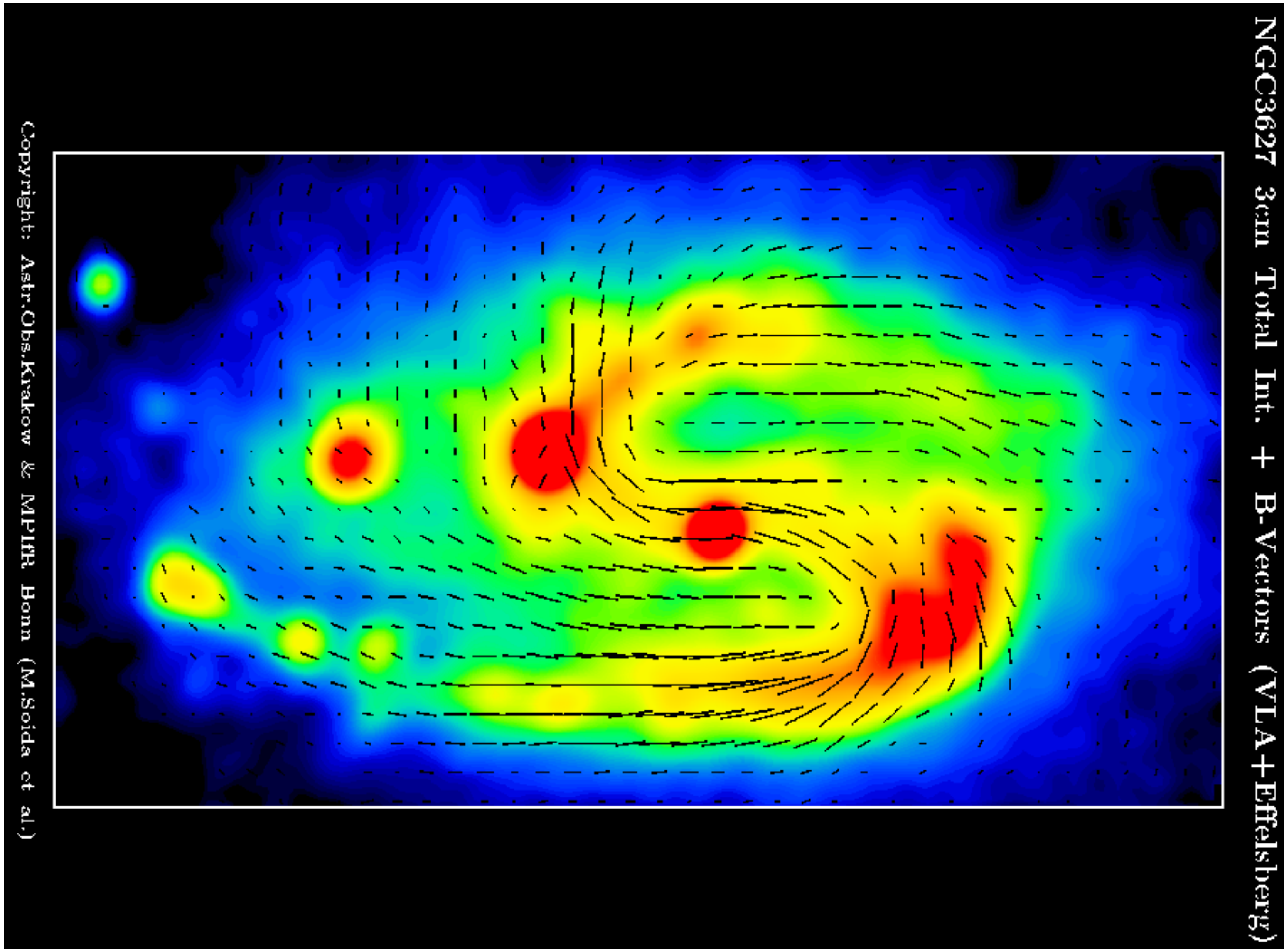


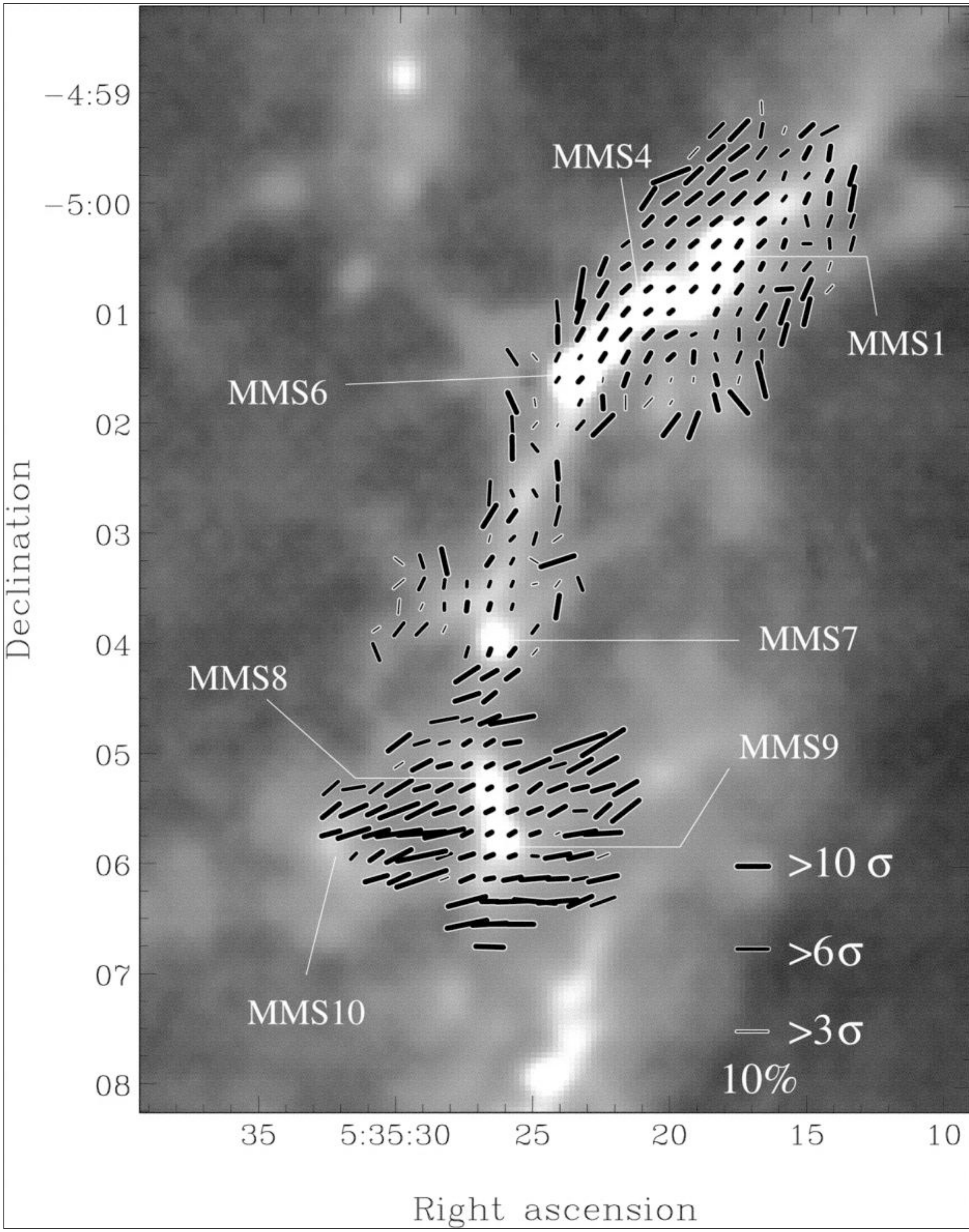


Star formation time > cloud free-fall time. This suggest some sort of support.

Could Magnetic Fields Support Clouds?

Magnetic Fields in Galaxies





Magnetic field Orientation in the Orion Molecular Cloud

Polarization of emission from grains at 850 microns. Polarization perpendicular to B

Matthews 2001 ApJ 562, 400.

Could Magnetic Fields Support Clouds?

Magnetic fields may have a stabilizing effect on clouds. Although the clouds are neutral, ionization by cosmic rays, and on the outer layers, UV radiation, may create a small number of electrons and ions. These would be coupled to the magnetic field, and in fact, freeze the magnetic field lines to the gas. Thus, the magnetic field pressure could exert a pressure.

$$\frac{1}{2}\frac{d^2 I}{dt^2} = 3\int_V (P - P_0)dV + \Omega + M_B \tag{11}$$

where

$$M_B = \frac{1}{8}\pi \int (B^2 - B_0^2)dV \tag{12}$$

where B_0 is the external field.

$$M_B = \frac{1}{8\pi}\int (B^2)dV \equiv \frac{1}{6}\bar{B}^2 R_{cl}^3 = \frac{1}{6\pi^2}\frac{\phi^2}{R} \tag{13}$$

where b is order unity and $\bar{B}$ is the mean magnetic field.

$$\phi = \int 2\pi r B dr \equiv \pi R^2 \bar{B} \tag{14}$$

Could Magnetic Fields Support Clouds?

An important value is the ratio of M_{cl}/ϕ where the graviational potential energy is equal to the magentic energy term, $\Omega = -M_B$. For a uniform sphere, this occurs when:

$$\frac{3}{5}\frac{GM_{cl}^2}{R} = \frac{1}{6}\frac{\phi^2}{R} \tag{15}$$

or when

$$\frac{M_{cl}}{\phi} = \left(\frac{5}{18}\right)^{1/2}\frac{1}{\pi G^{1/2}} = \frac{1}{2\pi G^{1/2}} \tag{16}$$

The ratio M_{cl}/ϕ is known as the mass to flux ratio. If

$$\frac{M_{cl}}{\phi} > \frac{1}{2\pi G^{1/2}} \tag{17}$$

What is the mass to flux ratio?

$$N(H_2) = \frac{M_{cl}}{\pi R_{cl}^2}, \ B = \frac{\phi}{\pi R_{cl}^2} \tag{18}$$

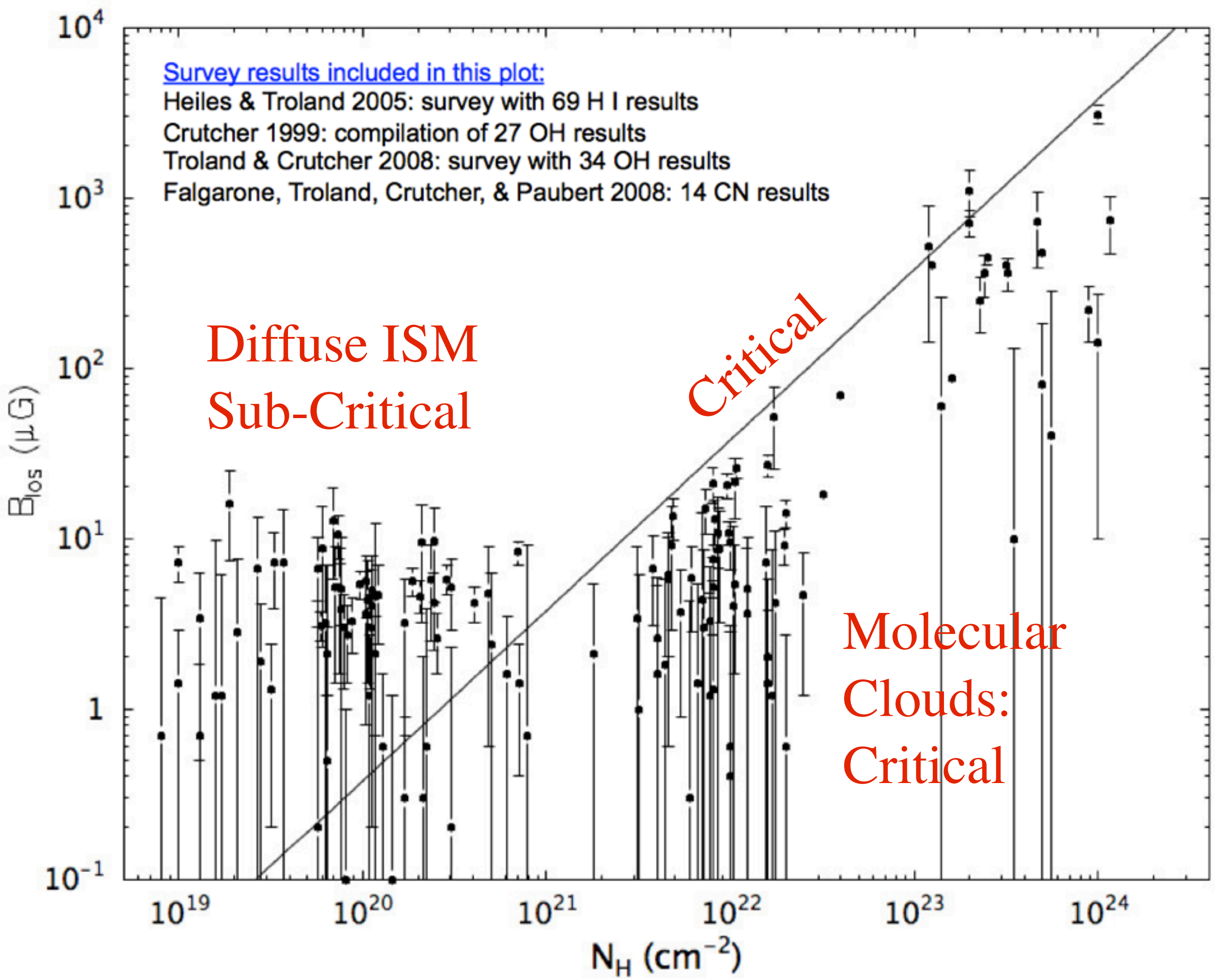
implies that

$$\frac{M_{cl}}{\phi} = \frac{N(H_2)}{B} \tag{19}$$

B can be measured by the Zeeman effect to OH or HI

$N(H_2)$ can be measured my molecular line or extinction measurements

The data: are molecular clouds supercritical or subcritical?

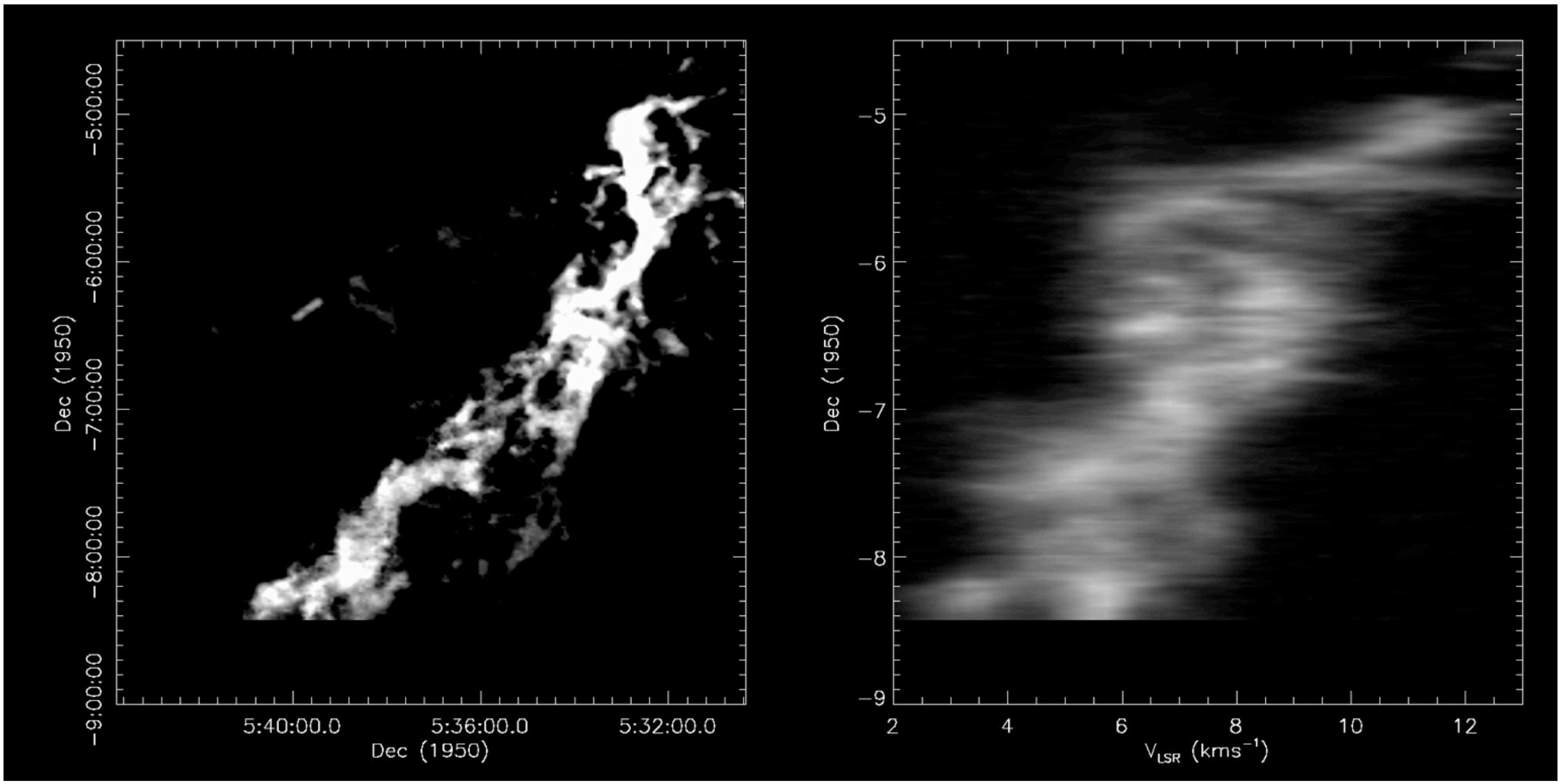


The data: are molecular clouds supercritical or subcritical?

- Current consensus is that magnetic fields do not provide support against collapse.
- This makes some sense, if magnetic fields dominated, then the clouds would expand.
- However, there are significant uncertainties in the data.
- However, magnetic fields may be strong enough to play some role in cloud evolution, this is an area of intense research.

Can Turbulence Stabilize Clouds?

Kinematics and Morphology of Orion Cloud



Turbulent Pressure

We now consider the addition of turbulent energy to the kinetic energy term. If we just consider the thermal motions of the gas, the kinetic energy is given by:

$$K = \frac{3}{2} M_{cl} c_s^2 \tag{20}$$

where c_s is the sound speed. Remember that the total 3-D velocity dispersion is

$$\sigma^2 = \sigma_x^2 + \sigma_y^2 + \sigma_z^2 = 3c_s^2 \tag{21}$$

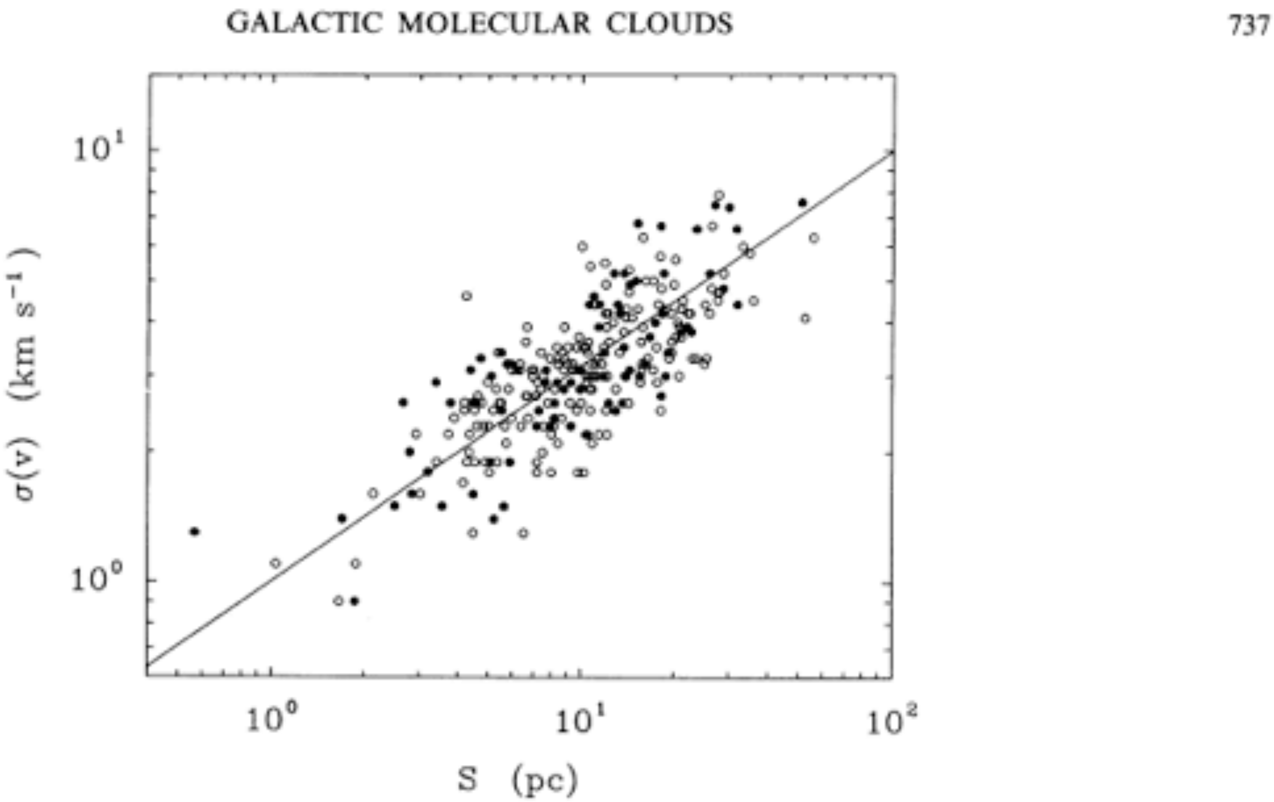
We can then define a pressure term:

$$P = \frac{3}{2} M_{cl} (c_s^2 + \sigma_{turb}^2) \tag{22}$$

where σ_{turb} is the 1-D turbulent dispersion. We can take this from the measured linewidth of the cloud, which is $\sigma_{tot}^2 = c_s^2 + \sigma_{turb}^2$.

$$P_{tot} = P_{th} + P_{turb} = \frac{3}{2} M_{cl} (\sigma_{tot}^2) \tag{23}$$

Supersonic Motions in Clouds



Sound speed in molecular clouds

$c_s = (kT/\mu m_H)^{1/2}$

$c_s = 0.25$ *km* s^{-1} *for* $T = 20$ *K*

Can Turbulence Stabilize Clouds?

- $M = 10^4$ Msun
  $\sigma = 1 \text{km s}^{-1}$
  => Rcrit = 12 pc
- $\alpha = |\Omega/K| \sim 1$ (2 needed for virial)

- This looks good, but there could still Jeans fragmentation
- Plus, turbulence may aid fragmentation (turbulent fragmenation)

Shock Waves

- Occur when two parcels of gas collide at supersonic velocities.
- Collision results in the compression of the gas.
- In sub-sonic case, compression would result in pressure (sound) waves.
- Since motion is faster than speed of sound, a dense layer of shock gas is produced.
- If gas cools rapidly, then shock will dissipate energy.

Shock Waves

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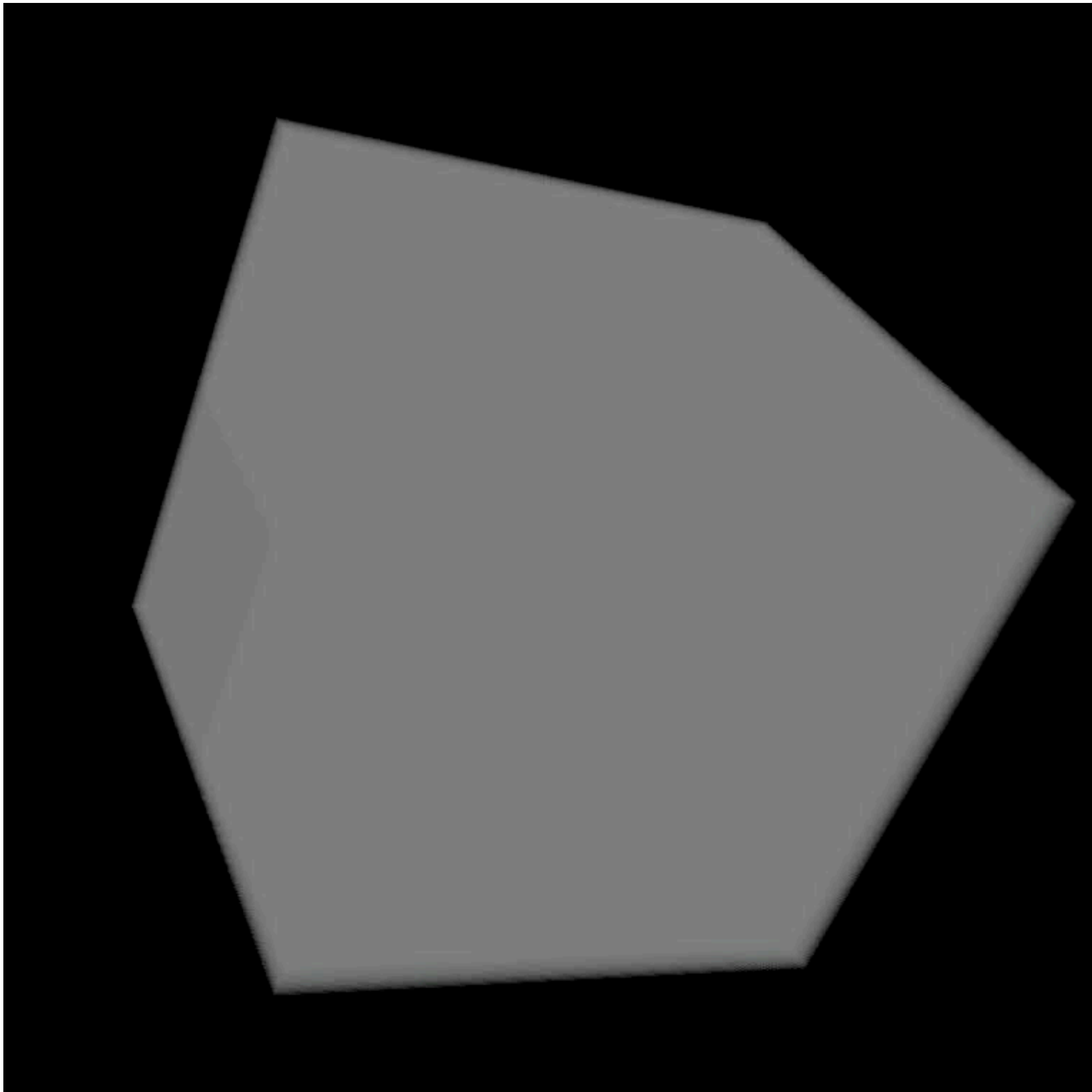
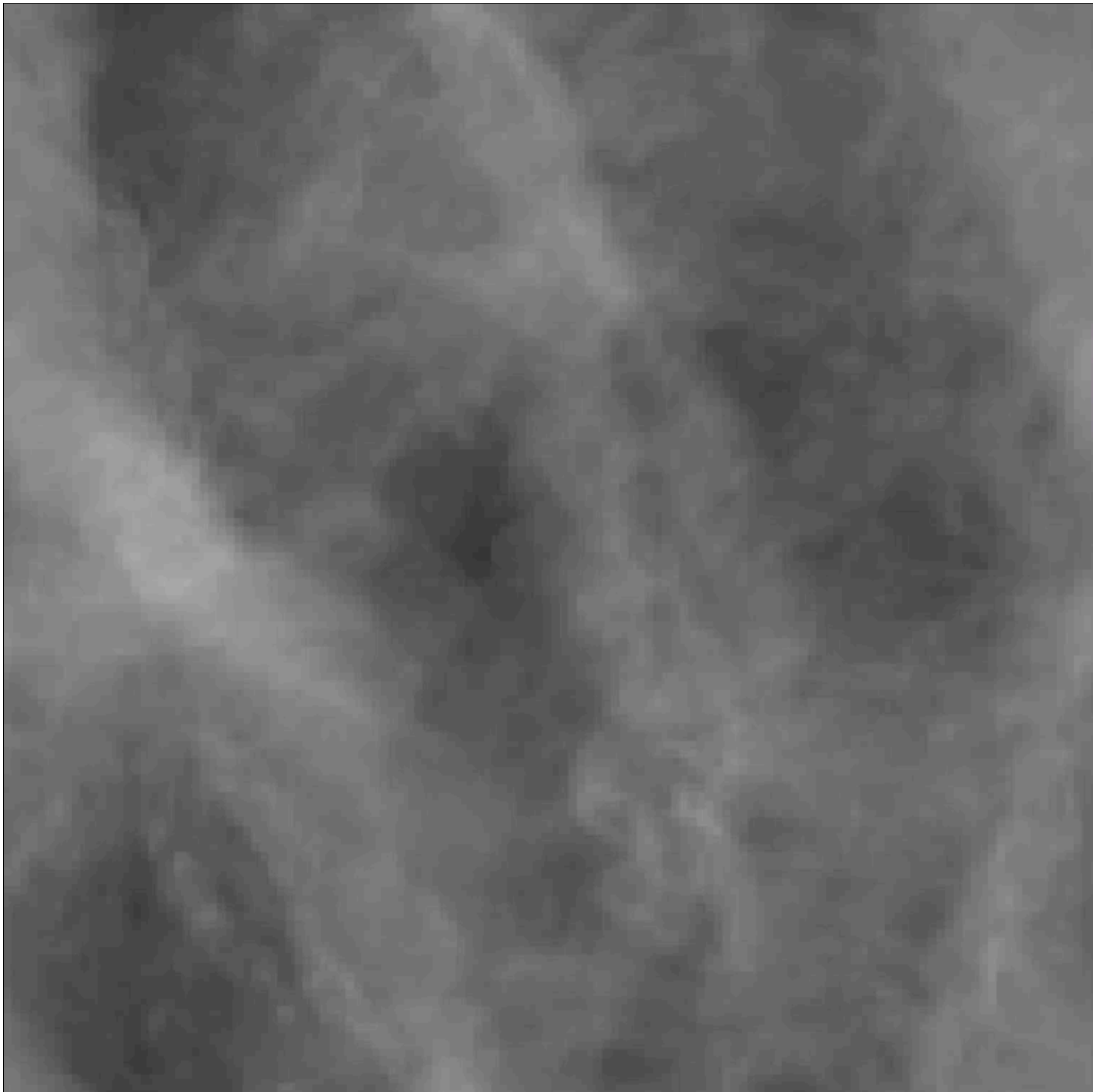


Driven Turbulence in a Periodic Box

Frrom Paolo Padon’s website: http://cass246.ucsd.edu/~ppadoan/new_website/astrophysical.php

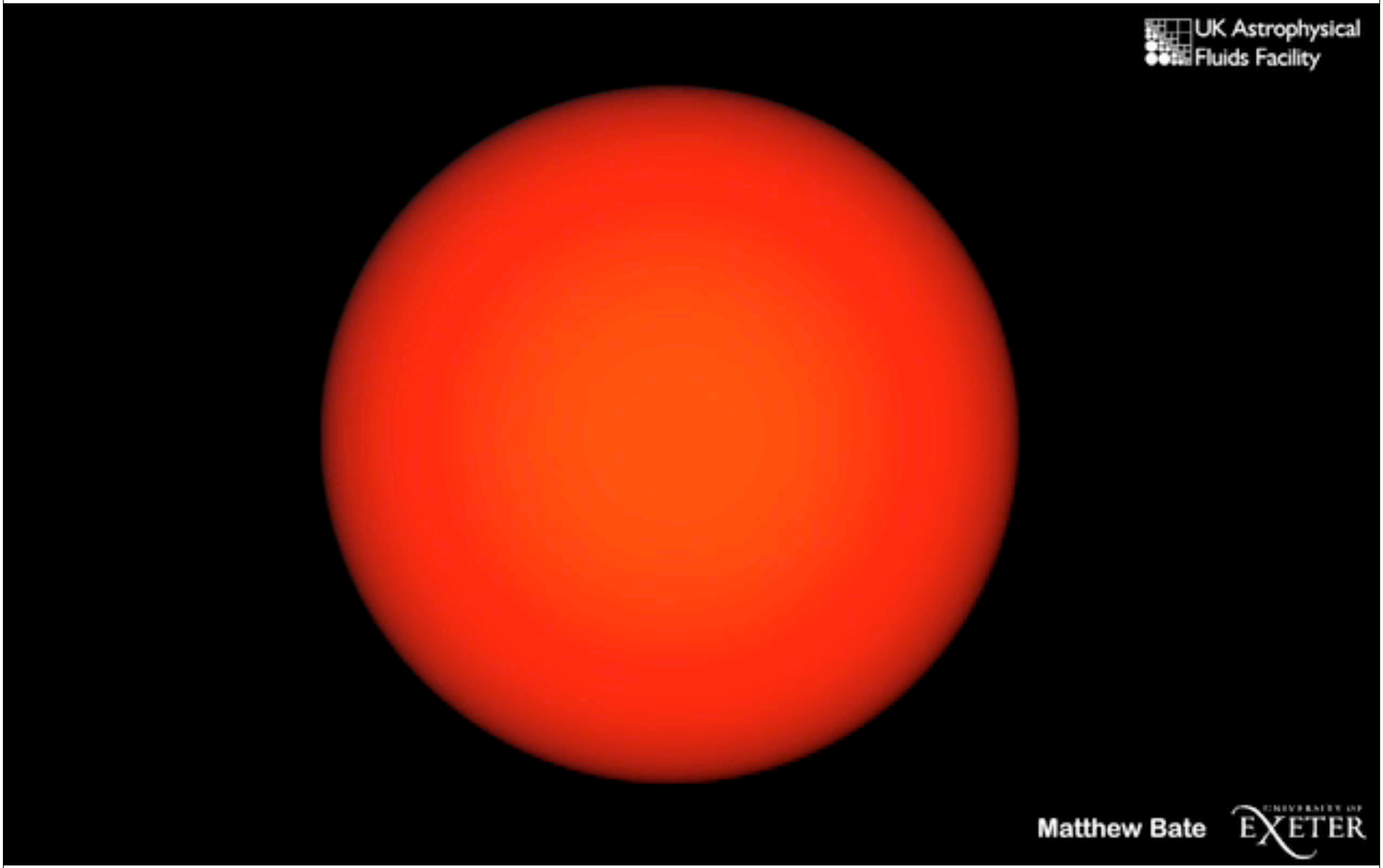


Self-gravitating turbulent gas with periodic boundary condition

Density projection (column density?)

Paolo Padoan’s website: http://cass246.ucsd.edu/~ppadoan/new_website/astrophysical.php

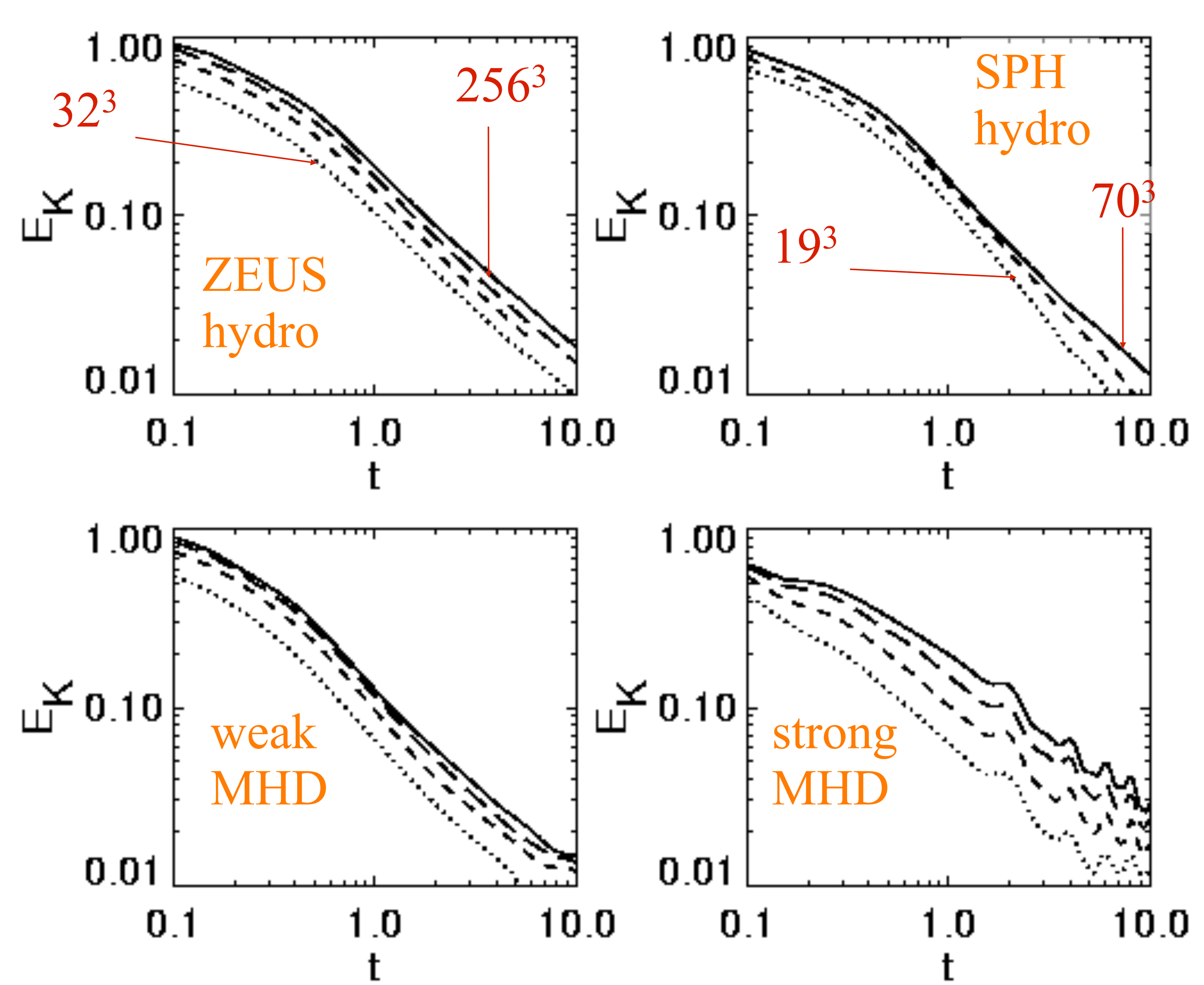
Cluster Formation in a Turbulent Cloud



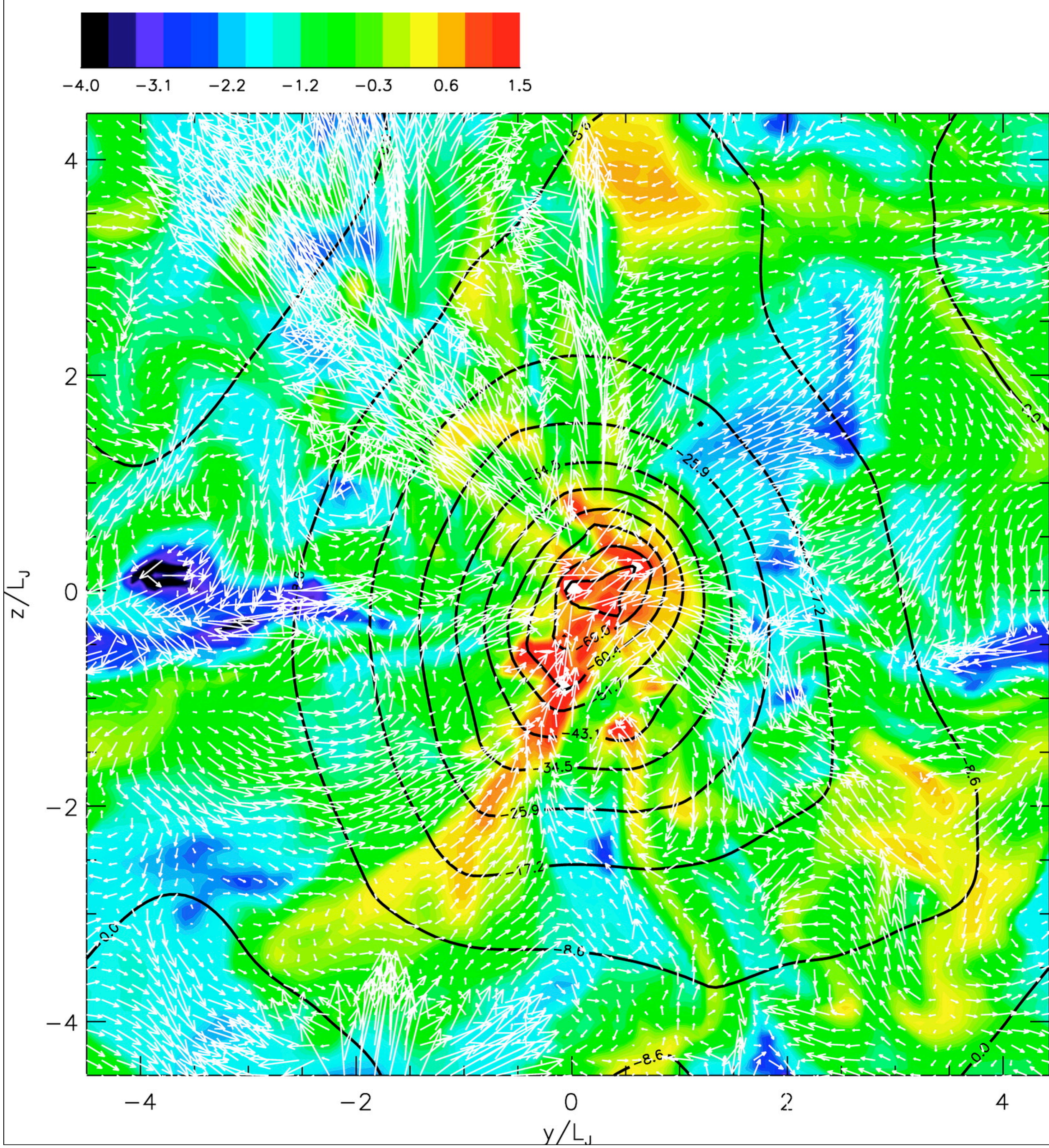
Can Turbulence Stabilize Clouds?

- The problem is that turbulence can decay rapidly.
- The decay timescale is a fraction of a crossing time.

Kinetic Energy Decay



ML, Klessen, Burkert, Smith (1998, Phys. Rev. Lett.)



Outflow Driven Turbulence

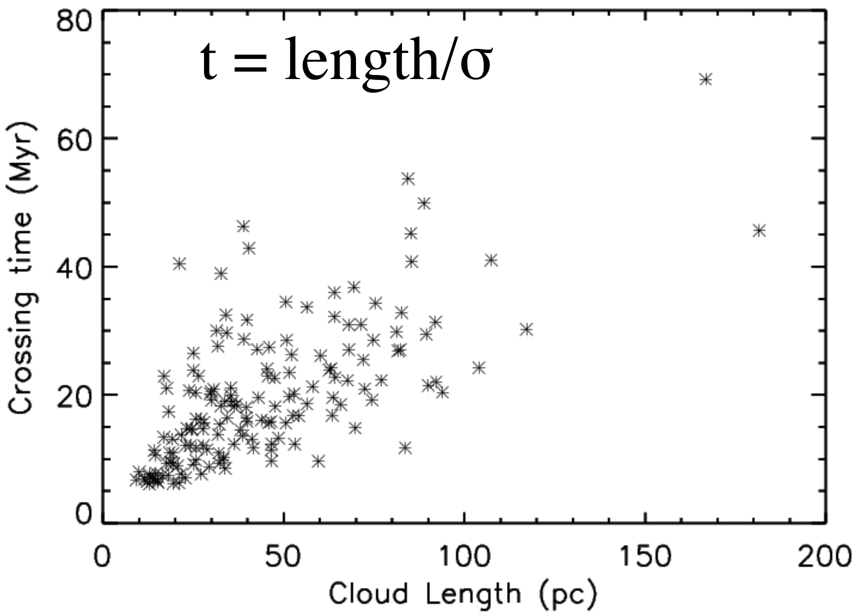
Simulation of a cluster forming clump within cloud. Outflows from stars in cloud eject gas, gas falls back on clump and stirs the turbulence.

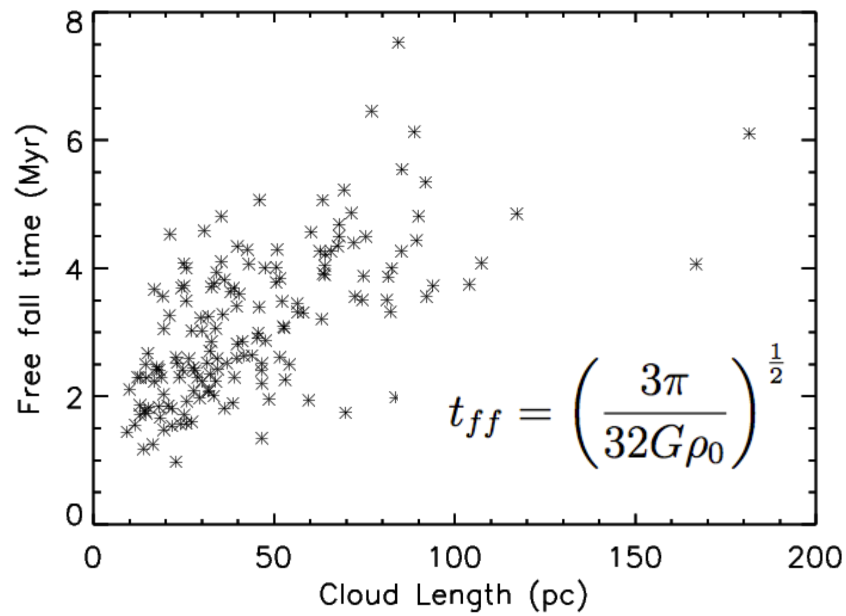
Nakamujra & Li 2007

Timescales

- Star formation lifetime < 5 Myr

Star formation timescale comparable to cloud free-fall time and larger than crossing time.





Star formation time > cloud free-fall time. This suggest some sort of support.

What is the role of turbulence?

- Decay of turbulence may happen, but perhaps decay on the order of a crossing time, which can be 10 Myr on the longest scale, is OK.
- Turbulence may delay collapse to more than one free fall time.
- Although turbulence may slow the collapse of the cloud, it will also enhance fragmentation and lead to the fragmentation of the gas into individual stars.
- Stars that are formed by turbulence can stir up gas (feedback), but will also destroy clouds

Summary

1. Molecular clouds are confined by gravity and not external pressure.
2. Thermally supported clouds are unstable
3. Molecular fields are unlikely to stabilize cloud
4. Turbulence may support clouds on large scales, but cause fragmentation on much smaller scales.