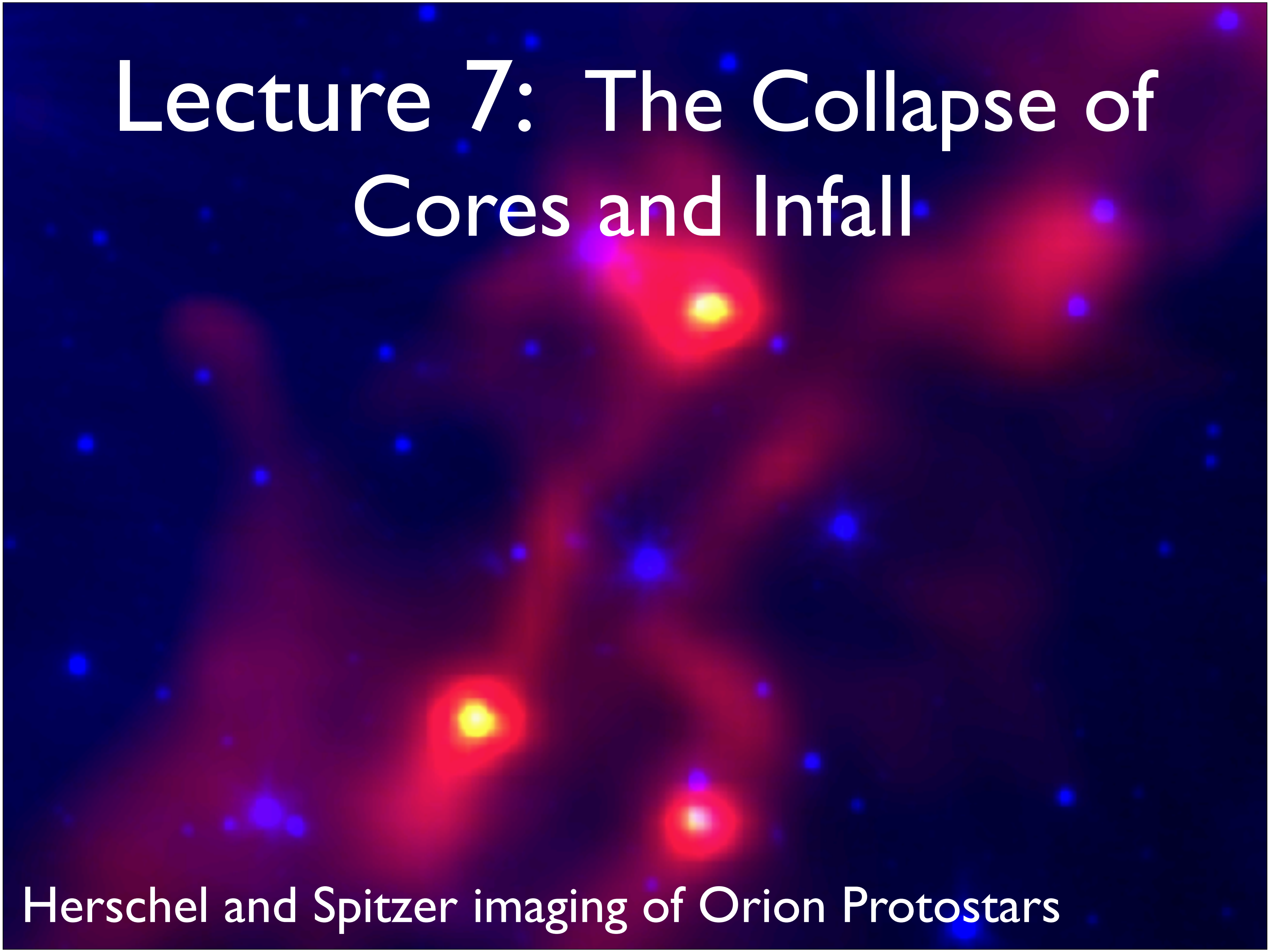
Lecture 7: The Collapse of Cores and Infall



Herschel and Spitzer imaging of Orion Protostars

Review: Dense Cores in Hydrostatic Equilibrium

We begin by estimating the size of a thermally supported, cold (10 K), one solar mass globule of gas. The equation for hydrostatic equilibrium is

$$\frac{dP}{dr} = -\rho G \frac{M}{r^2} \tag{1}$$

or if we approximate $dP/dr = (P_c - P_0)/R$ where P_c is the central core pressure, P_0 is the outer pressure and R is the core radius, then:

$$P_c = -\rho G \frac{M}{r} \tag{2}$$

where we assume $P_0 << P_c$, and then by applying the ideal gas law ($P = c_s^2 \rho$):

$$c_s^2 \approx G \frac{M}{R} \approx \frac{kT}{\mu m_H} \tag{3}$$

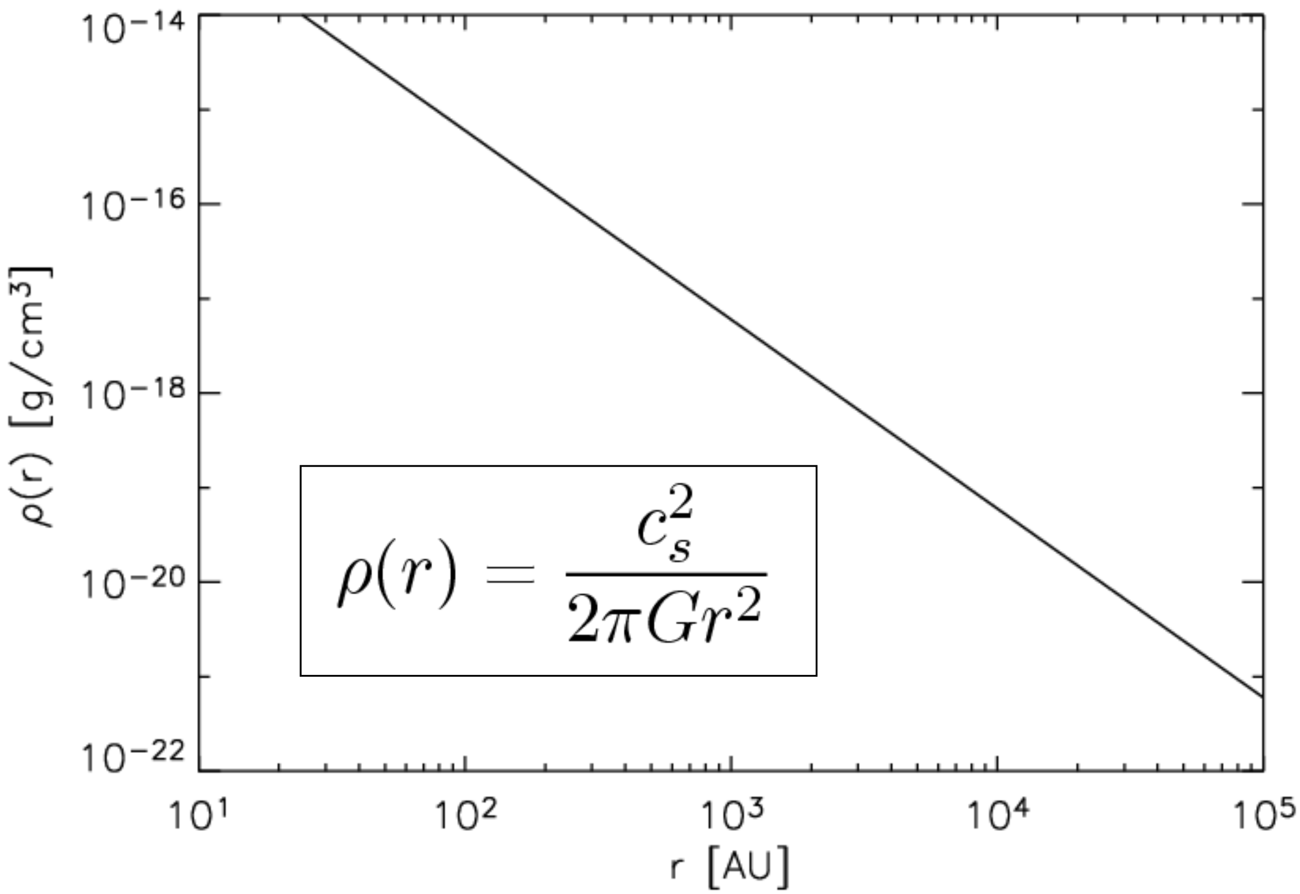
Note, this is very similar to the virial equation. For a core with $M = 1 M_\odot$ and $T = 10K$, we find $R = 0.15$ pc. This is very similar to the radii of molecular cores in low mass stars.

The Singular Isothermal Sphere

As a boundary condition, we must adopt a value for ρ(0).
If ρ(0) -> infinity, we get a singular isothermal sphere

<u>Numerical solutions:</u>

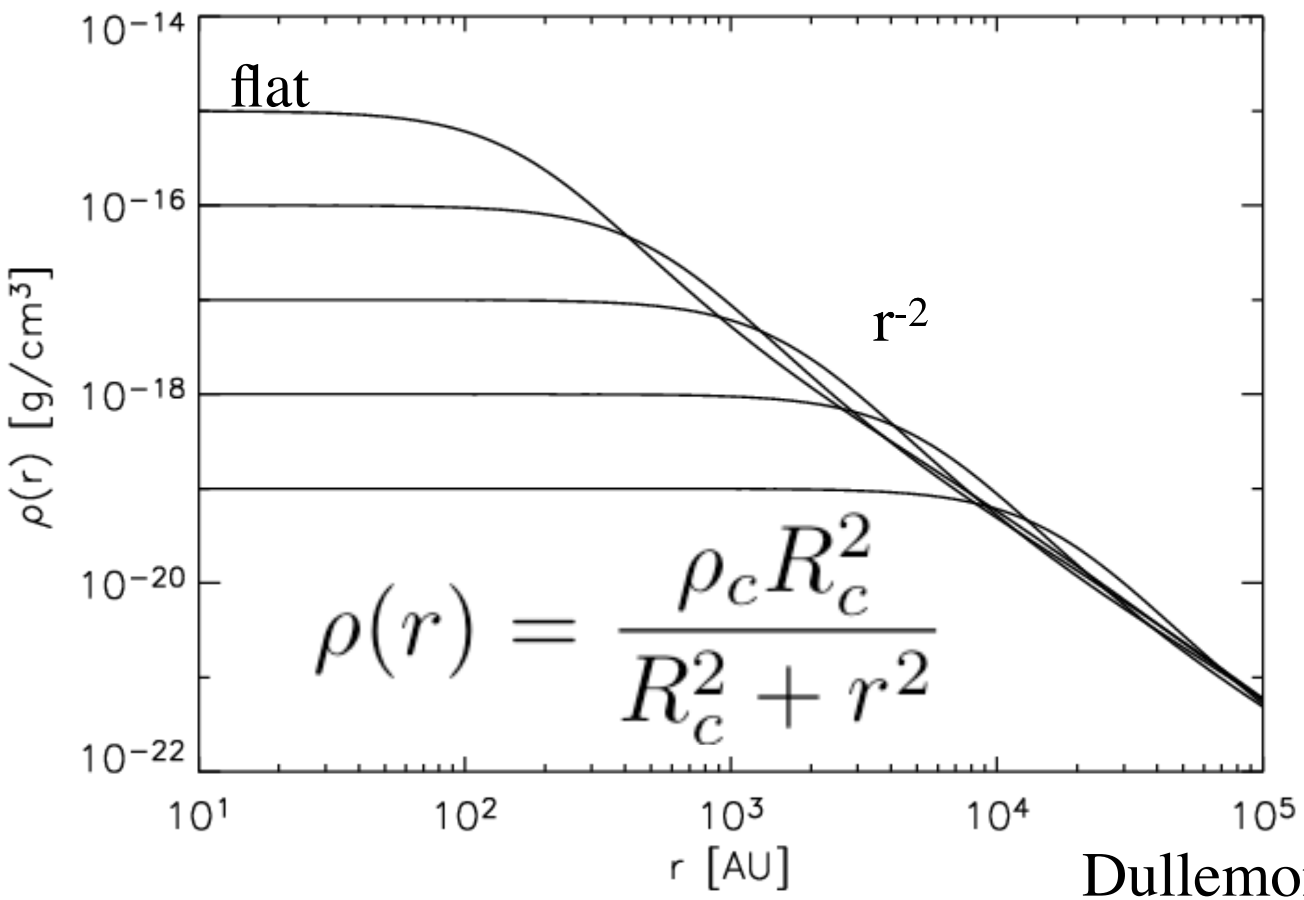
Singular isothermal sphere
(limiting solution)



The Bonnor-Ebert Sphere

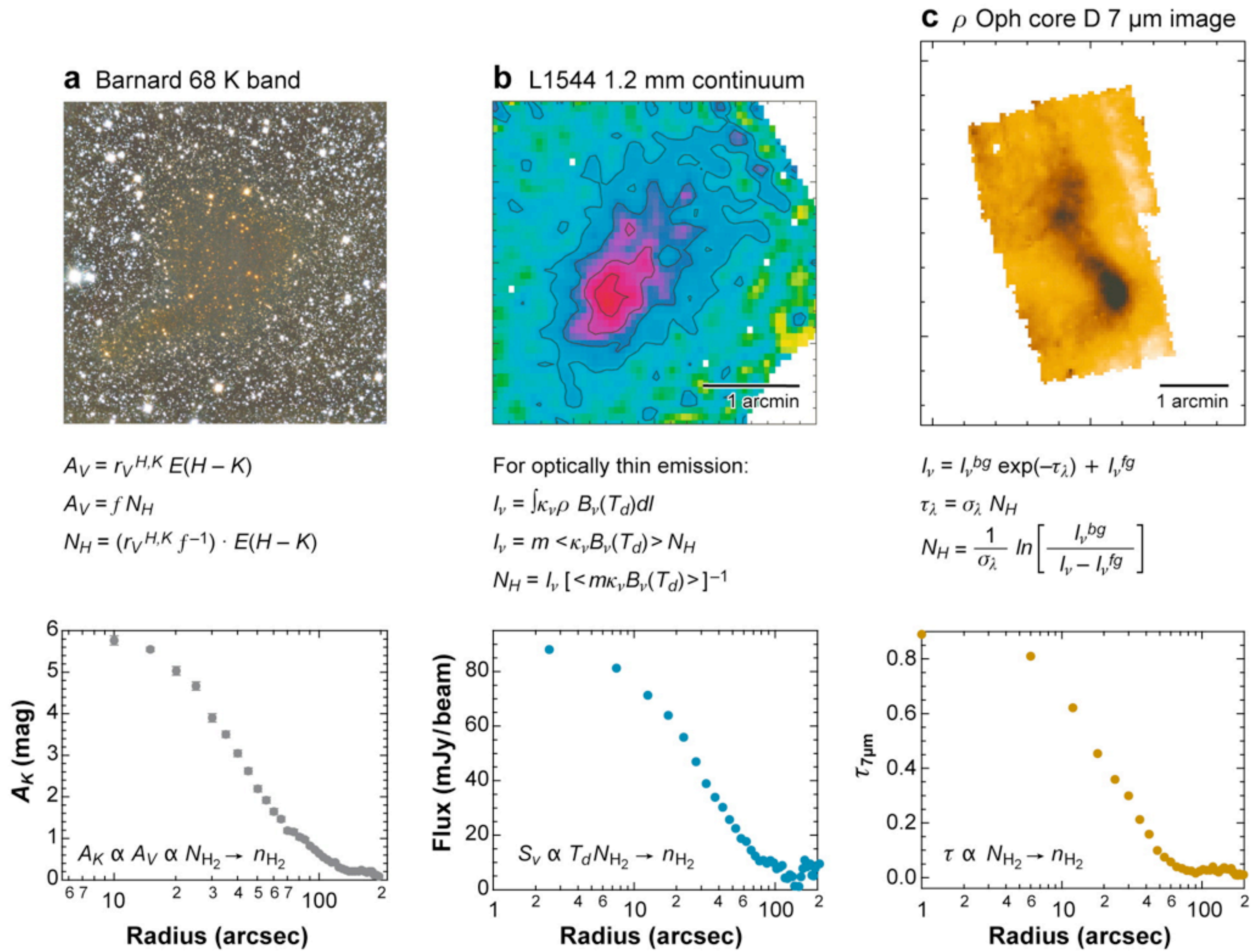
Numerical solutions:

Different starting ρ_o :
a family of solutions



$$\rho(r) = \frac{\rho_c R_c^2}{R_c^2 + r^2}$$

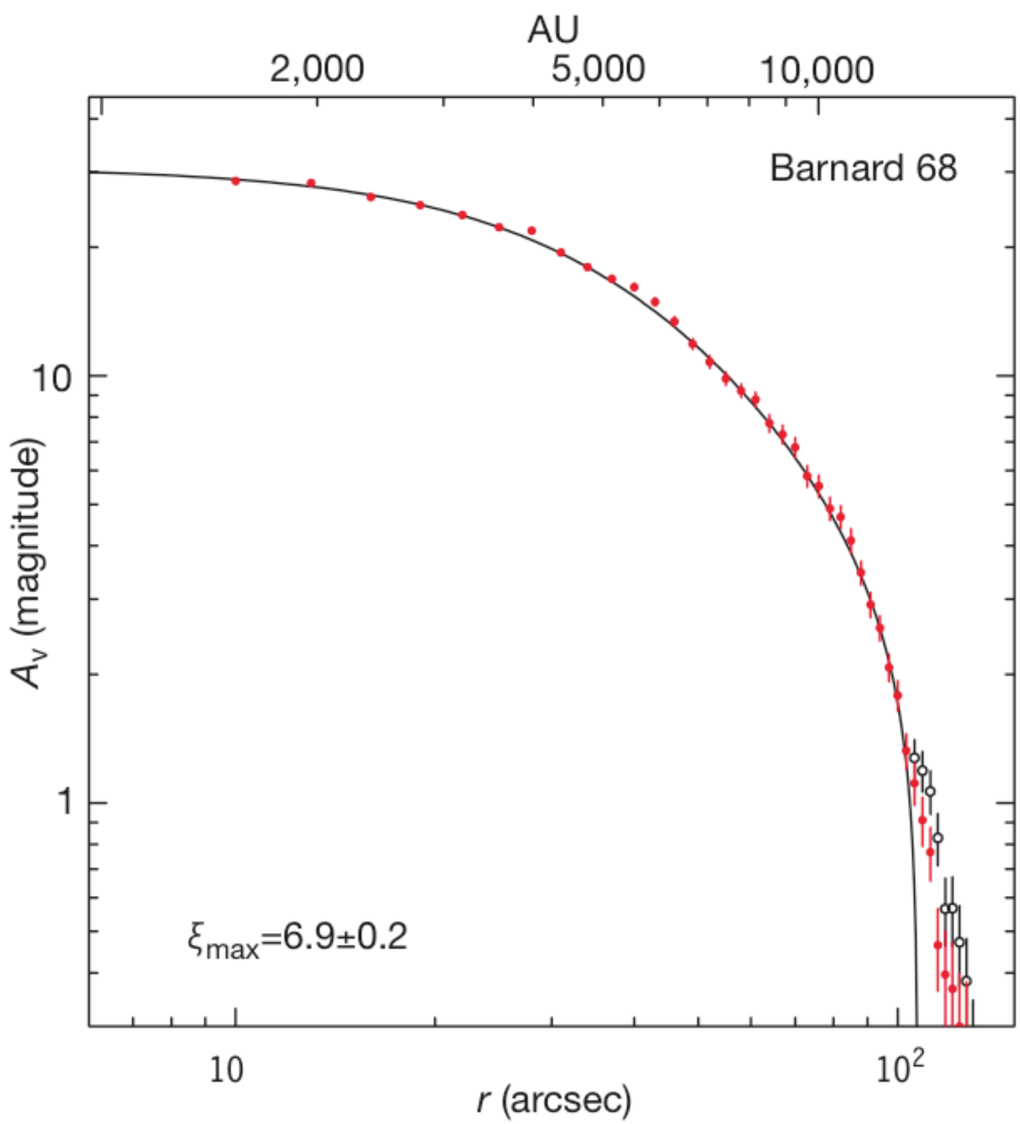
Dullemond



Bergin EA, Tafalla M. 2007.
Annu. Rev. Astron. Astrophys. 45:339–96

Alves et al 2001, Ward Thompson et al.. 1999, Bacman et al. 2000

B68: A real Bonnor-Ebert Sphere in Nature??



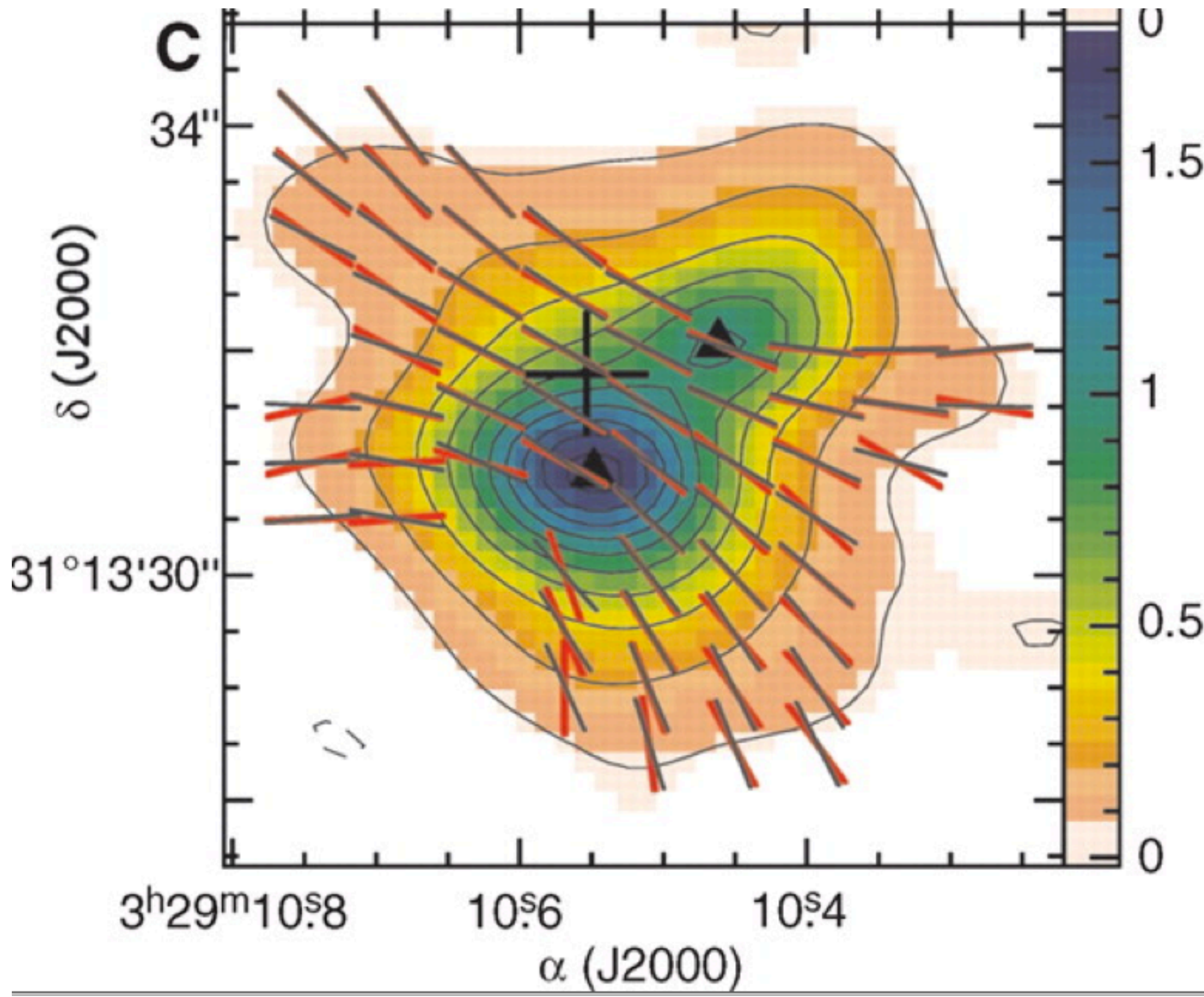
Collapse

What Initiates Collapse?

Possibilities:

- Formation of core (cores are never stable)
- External pressure increase (shock or pressure wave in turbulent medium)
- Mass accretes onto source
- Ambipolar Diffusion

Magnetic Support of Cores
(“Historical Diversion”)

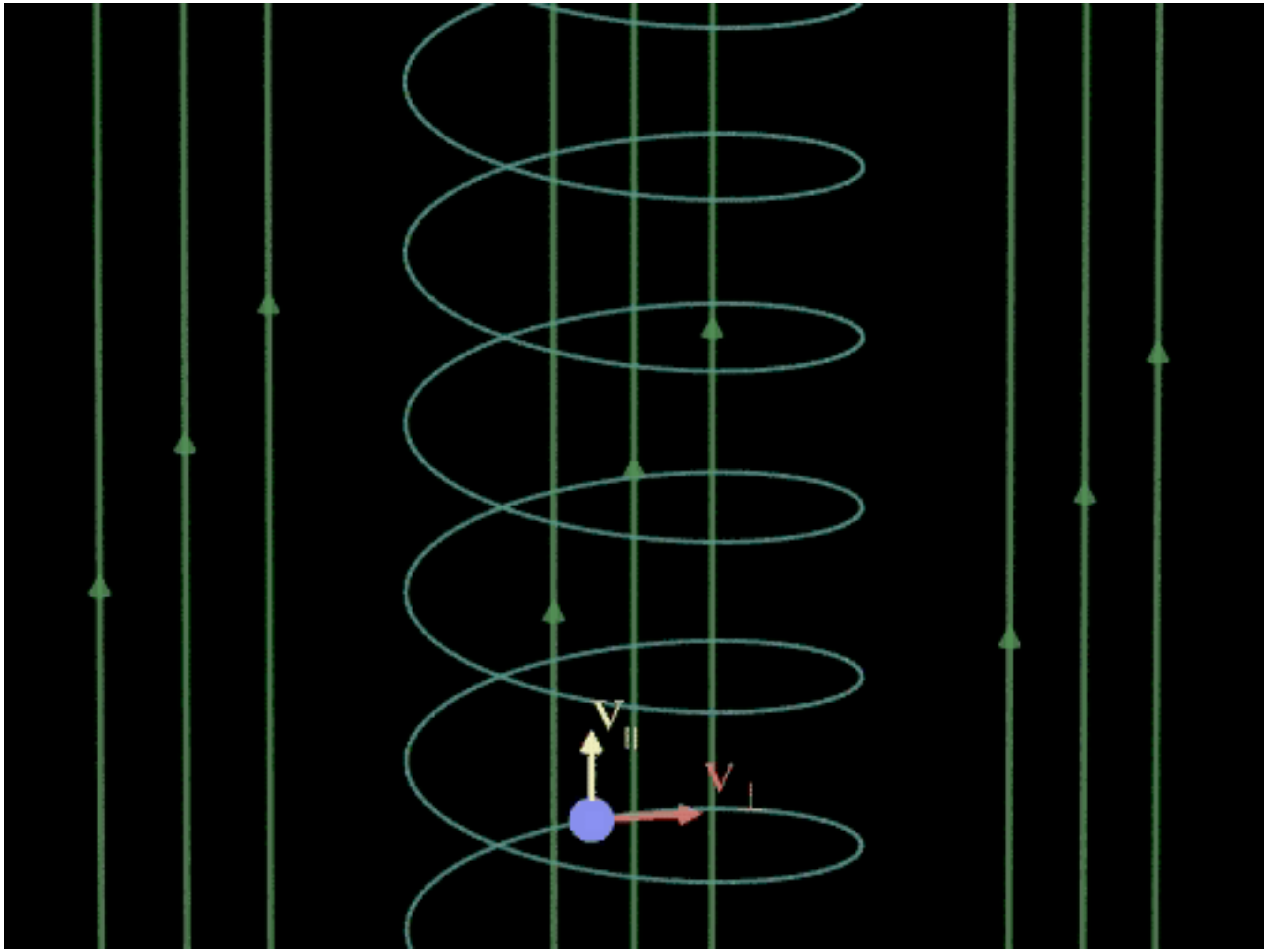


IRAS 4a/4b protostar

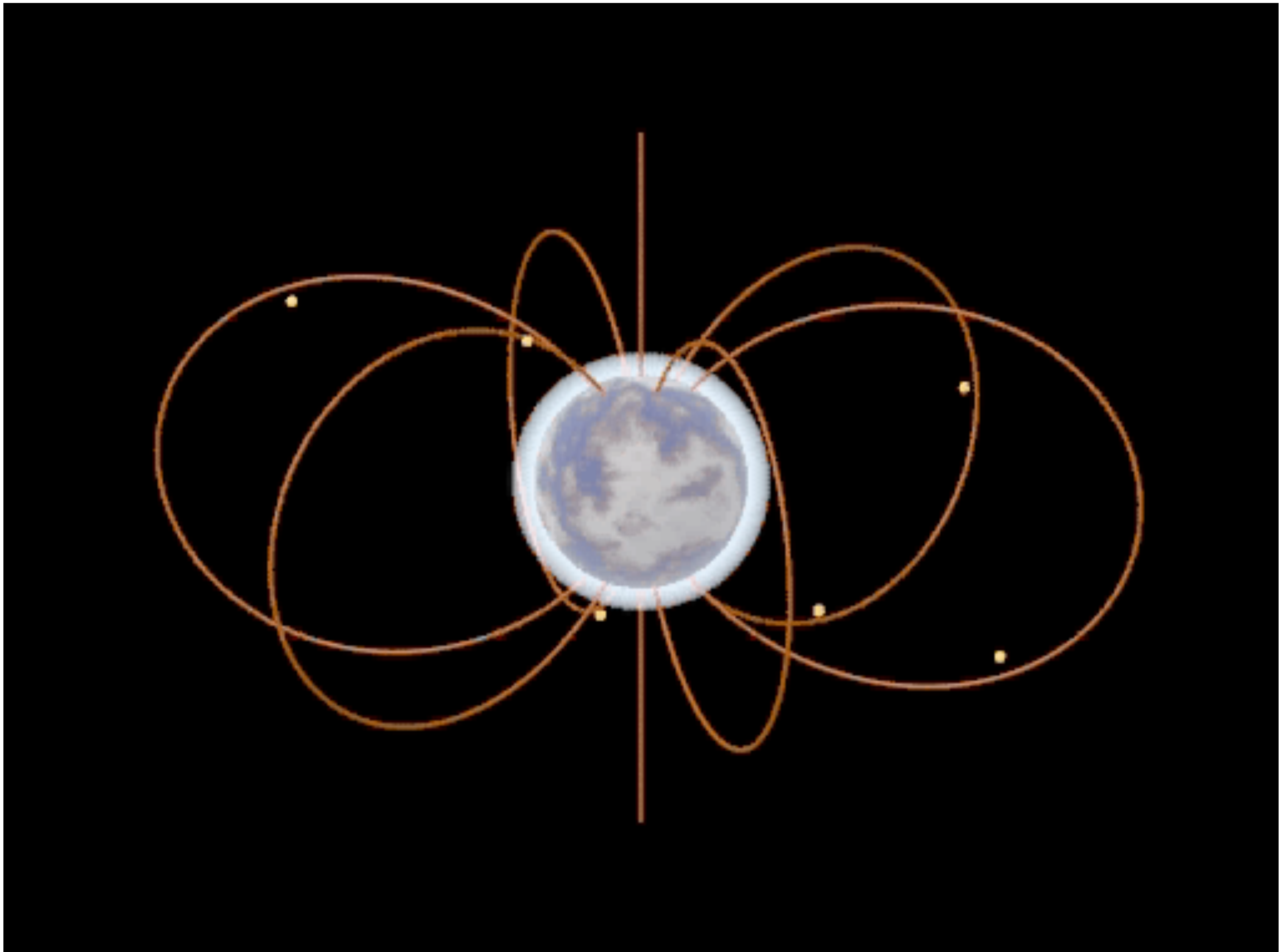
Magnetic field lines determined through sub-millimeter polarimetry

Girart et al. 2007

Ambipolar Diffusion



Charges in B-Fields



This animations illustrates charged particles frozen to the Earth's field

Collapse Initiated by Ambipolar Diffusion

- Ions & electrons are stuck to magnetic field
- Neutrals are not stuck to magnetic fields
- Magnetic field is strong enough to resist collapse.
- Neutrals are pulled in by gravity.
- Ions & electrons remain stuck to field
- Relative to the contracting neutral gas, the magnetic field, ions and electrons diffuse outward (hence *ambi*polar), reducing magnetic flux to mass ratio.
- Eventually cloud becomes supercritical and collapses

Timescale for Ambipolar Diffusion

Let's consider a gas where the number density of ions is n_i, where $n_i << n(H_2)$. The collision rate with neutrals for a given ion is

$$R_{col} = n_i < \sigma f(\mathbf{v})\mathbf{v} > \tag{1}$$

where $\mathbf{v}$ is the typical velocity difference between the particles, $f(\mathbf{v})$ is a Maxwellian distribution, σ is the cross section for collisions between ions and neutrals and $\rho/\mu m_H$ gives the density of neutrals.

The change in momentum per time (i.e. the acceleration) is then given by:

$$\frac{dP}{dt} = \rho n_i < \sigma f(\mathbf{v})\mathbf{v} > v_D \tag{2}$$

To simplify the derivation, *Spitzer* originally assumed a cylindrical geometry aligned with the magnetic field. This gives a gravitational force of:

Taken from Hartmann

To simplify the derivation, *Spitzer* originally assumed a cylindrical geometry aligned with the magnetic field. This gives a gravitational force of:

$$-\nabla\phi = 2\pi RG\rho \tag{3}$$

where R is the distance to the axis of the cylinder. We equate the force of gravity to the drag force of the ions and we get:

$$\rho\, n_i < \sigma f(\mathbf{v})\mathbf{v} > v_D = 2\pi RG\rho^2 \tag{4}$$

or

$$v_D = \frac{2\pi RG\rho}{n_i < \sigma f(\mathbf{v})\mathbf{v} >} \tag{5}$$

Finally, we can define a timescale for the diffusion of the magnetic field as $t_{amb} = R/v_D$ where:

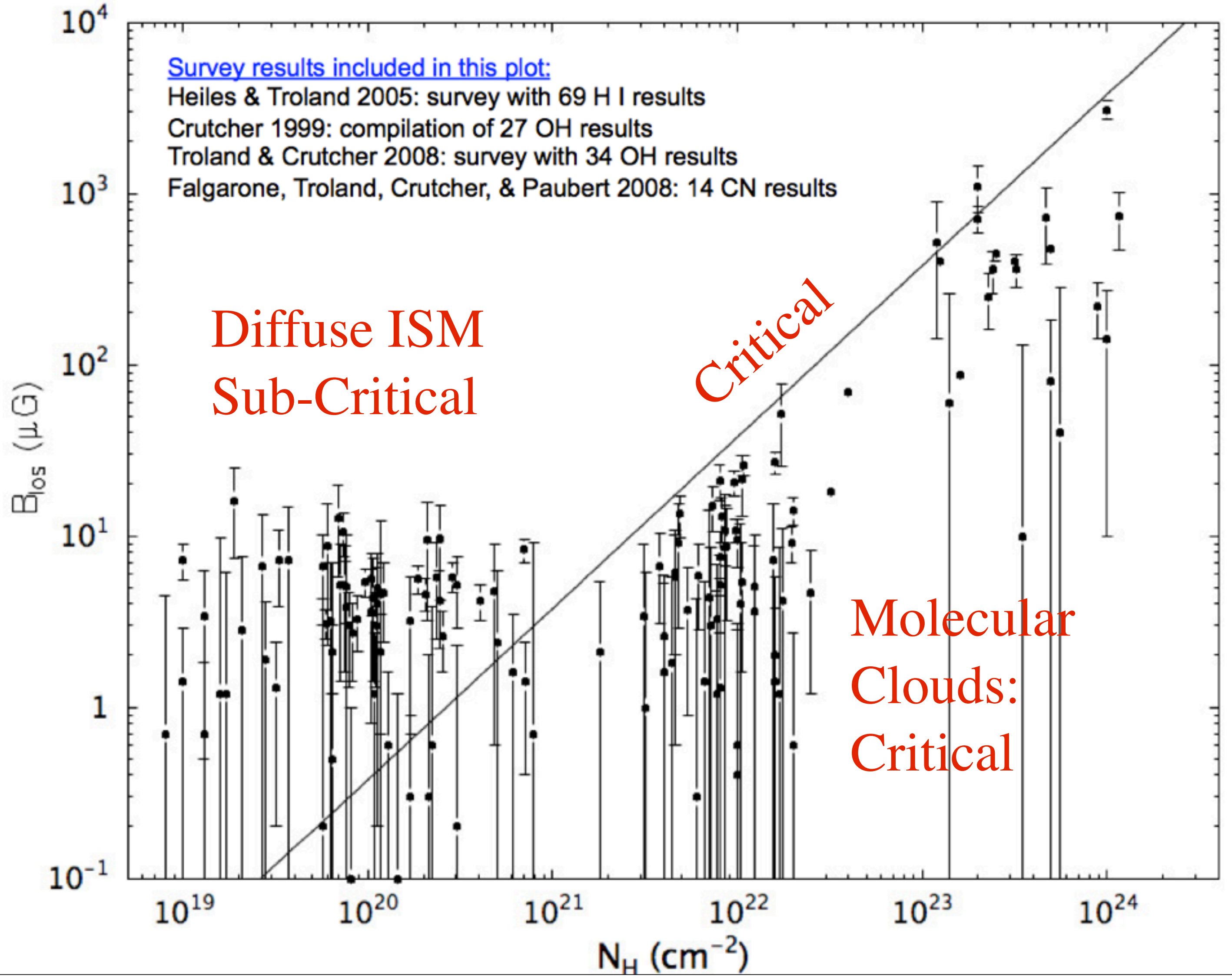
$$t_{amb} = \frac{< \sigma f(\mathbf{v})\mathbf{v} >}{2\pi G \mu m_H} \frac{n_i}{n_H} \tag{6}$$

where $< \sigma f(\mathbf{v})\mathbf{v} > \approx 2 \times 10^{-9} cm^3 s^{-1}$. This timescale depends strongly on the ratio of ions to neutrals in the cloud. We can write it approximately as:

$$t_{amb} \approx 5 \times 10^{13} \frac{n_i}{n_{H_2}} \tag{7}$$

Given estimates of $n_i/n_{H_2} = 10^{-7}(n_{H_2}/10^4 cm^{-3})$ by (McKee 1989), this gives $t_{amb} = 10^7$ years. This is longers than the lifespan of molecular clouds. Measurements also show that $\frac{N(H_2)}{B} \geq 1$, suggesting that magnetic fields do not support clouds (or cores) against collapse.

Current Consensus: Clouds and Cores are already Supercritical.



Collapse Calculations

Imaging the free fall of a core with mass M and radius R. We from the virial theorem, we showed last week for a thermally supported core:

$$c_s^2 \approx \frac{GM}{R} \tag{8}$$

Now consider the typical free fall velocity for an infalling core. Let's derive the velocity of a parcel of gas on the outside of the gas which has fallen from $R \rightarrow R/2$. The change in potential energy is:

$$\Delta U = G\frac{2mM}{R} \tag{9}$$

where m is the density of the gas. Equating this to the kinetic energy, the infall velocity is

$$v_{in} \approx 2\sqrt{\frac{GM}{R}} \approx 2c_s \tag{10}$$

Now consider the mass accretion rate. Define $t_{in} = R/c_s$. Then the mass accretion rate is given by:

$$\frac{dM}{dt} = \frac{M}{t_{in}} = \frac{Mc_s}{R} = \frac{c_s^3}{G} \tag{11}$$

Taken from Hartmann

You can derive this in another way. Take the Jeans mass:

$$m_j = \left(\frac{\pi c_s^2}{G}\right)^{3/2} \rho_0^{-1/2} \tag{12}$$

and divide it by the free fall time (Schmeja & Klessen 2004)

$$t_{ff} = \sqrt{\frac{3\pi}{32G\rho}} \tag{13}$$

to get

$$\frac{dM}{dT} \approx \frac{m_J}{t_{ff}} = \sqrt{\frac{32}{3}}\pi\frac{c_s^3}{G} \tag{14}$$

Spherically symmetric free falling cloud

Free fall velocity:

$$v_{\mathrm{ff}} = \sqrt{\frac{2GM(r)}{r}}$$

If stellar mass dominates:

$$v_{\mathrm{ff}} = \sqrt{\frac{2GM_*}{r}}$$

Continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial (r^2 \rho v)}{\partial r} = 0 \quad \xrightarrow{\text{Stationary free-fall collapse}} \quad \frac{\partial (r^2 \rho v_{\mathrm{ff}})}{\partial r} = 0$$

$$\boxed{\rho(r) \propto r^{-3/2}}$$

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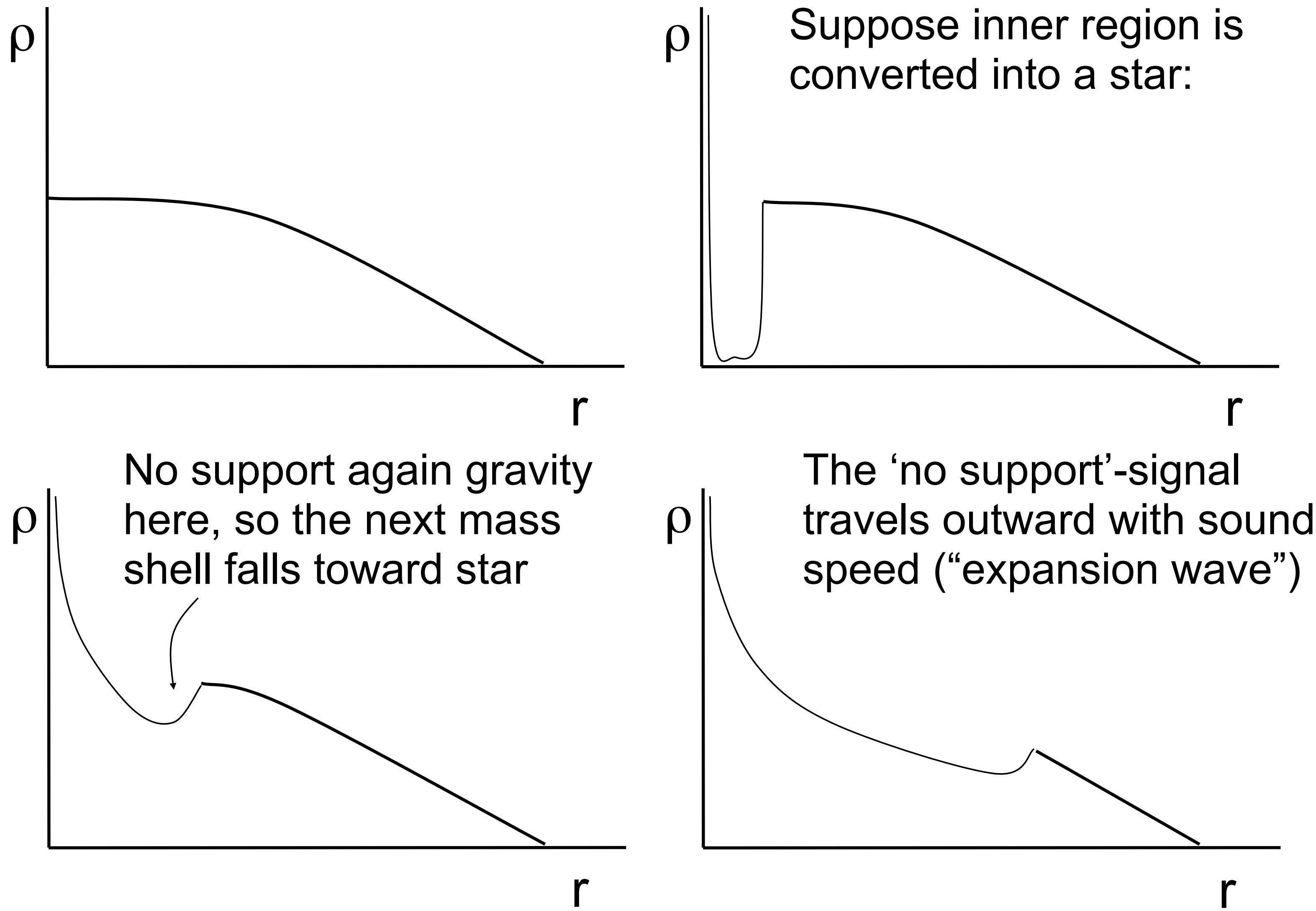
Inside-out collapse of metastable sphere



(warning: strongly exaggerated features)

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Hydrodynamical equations

Continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial (r^2 \rho v)}{\partial r} = 0$$

Comoving frame momentum equation:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} - \frac{GM(r)}{r^2}$$

$$M(r) \equiv \int_0^r 4\pi\, r'^2\, \rho(r')\, dr'$$

Equation of state:

$$P = \rho c_s^2$$

$$c_s^2 \equiv \frac{kT}{\mu m_{\mathrm{H}}} = \mathrm{const.}$$

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Inside-out collapse model of Shu (1977)

- The analytic model:
  - Starts from singular isothermal sphere
  - Models collapse from inside-out
  - Applies the `trick' of self-similarity

- Major drawback:
  - Singular isothermal sphere is unstable and therefore unphysical as an initial condition

- Nevertheless very popular because:
  - Only existing analytic model for collapse
  - Demonstrates much of the physics

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Inside-out collapse model of Shu (1977)

Expansion wave moves outward at sound speed.
So a dimensionless coordinate for self-similarity is:

$$x = \frac{r}{c_s t}$$

If there exists a self-similar solution, then it must be of the form:

$$\rho(r,t) = \frac{\alpha(x)}{4\pi G t^2} \qquad M(r,t) = \frac{c_s^3 t}{G} m(x)$$

$$v(r,t) = c_s u(x)$$

Now solve the equations for $\alpha(x)$, $m(x)$ and $u(x)$

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Inside-out collapse model of Shu (1977)

Solution requires one numerical integral. Shu gives a table.

An approximate (but very accurate) 'solution' is:

$$g \equiv \frac{1}{1.43\,x^{3/2}} \qquad h \equiv \frac{2}{x}$$

$$\alpha(x) = \left(g(x)^{7/2} + h(x)^{7/2}\right)^{2/7}$$

$$u(x) = \left(h(x)^{5/9} - 2^{5/9}\right)^{9/10}$$

$$m(x) = 1.025\,x^2 + 0.975 + 0.075\,x\,(1-x)$$

For any t this can then be converted into the real solution

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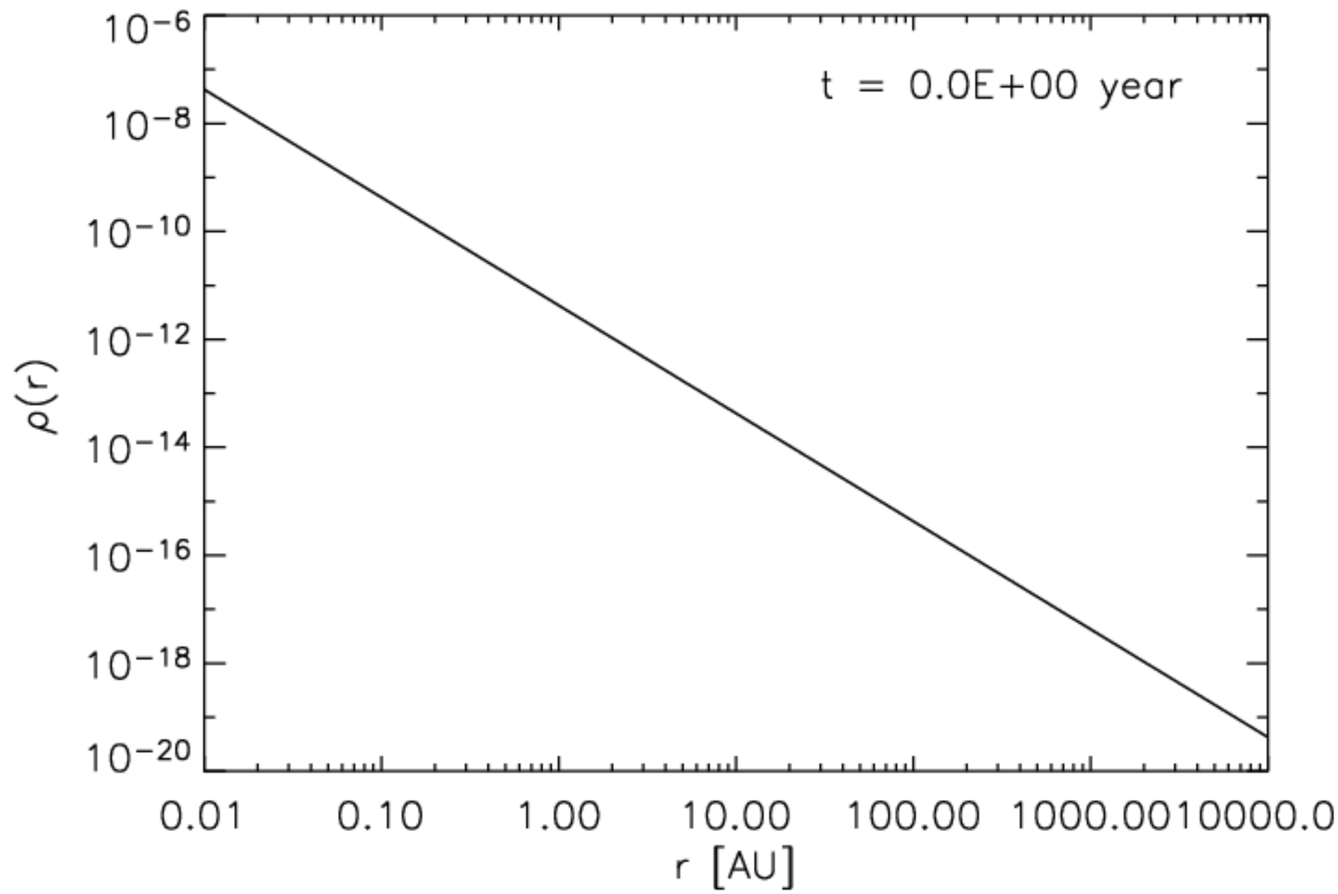
Inside-out collapse model of Shu (1977)



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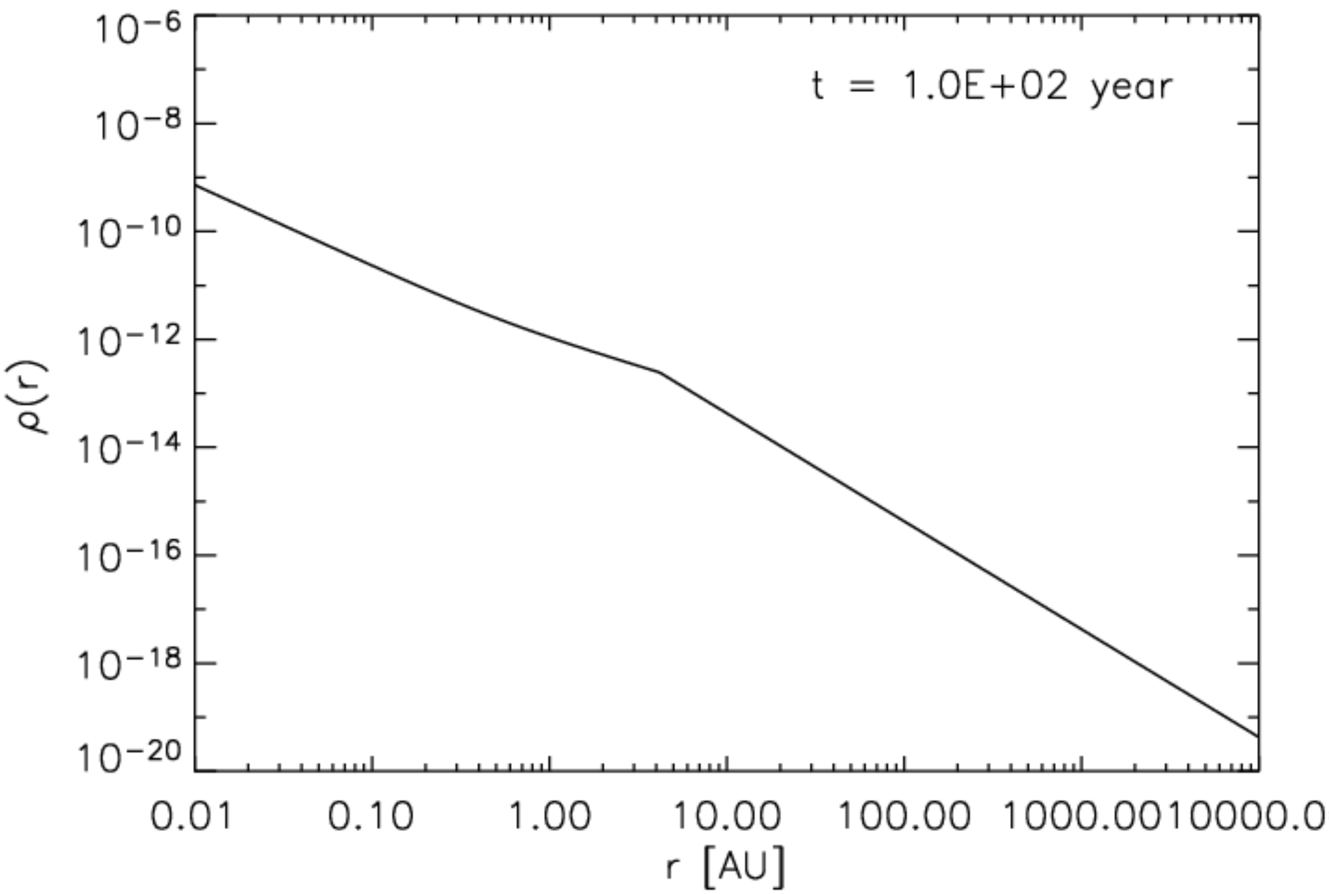
Inside-out collapse model of Shu (1977)



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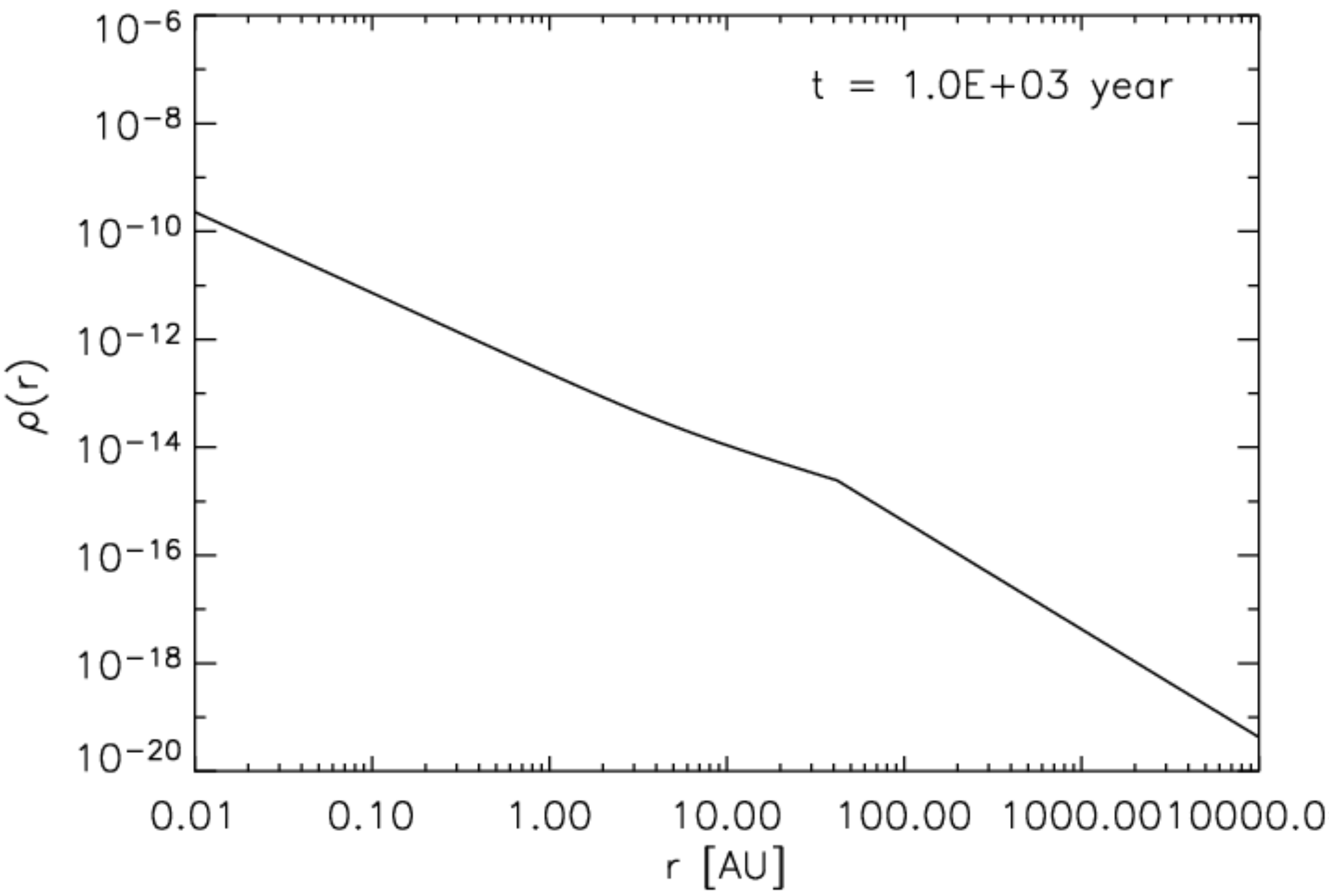
Inside-out collapse model of Shu (1977)



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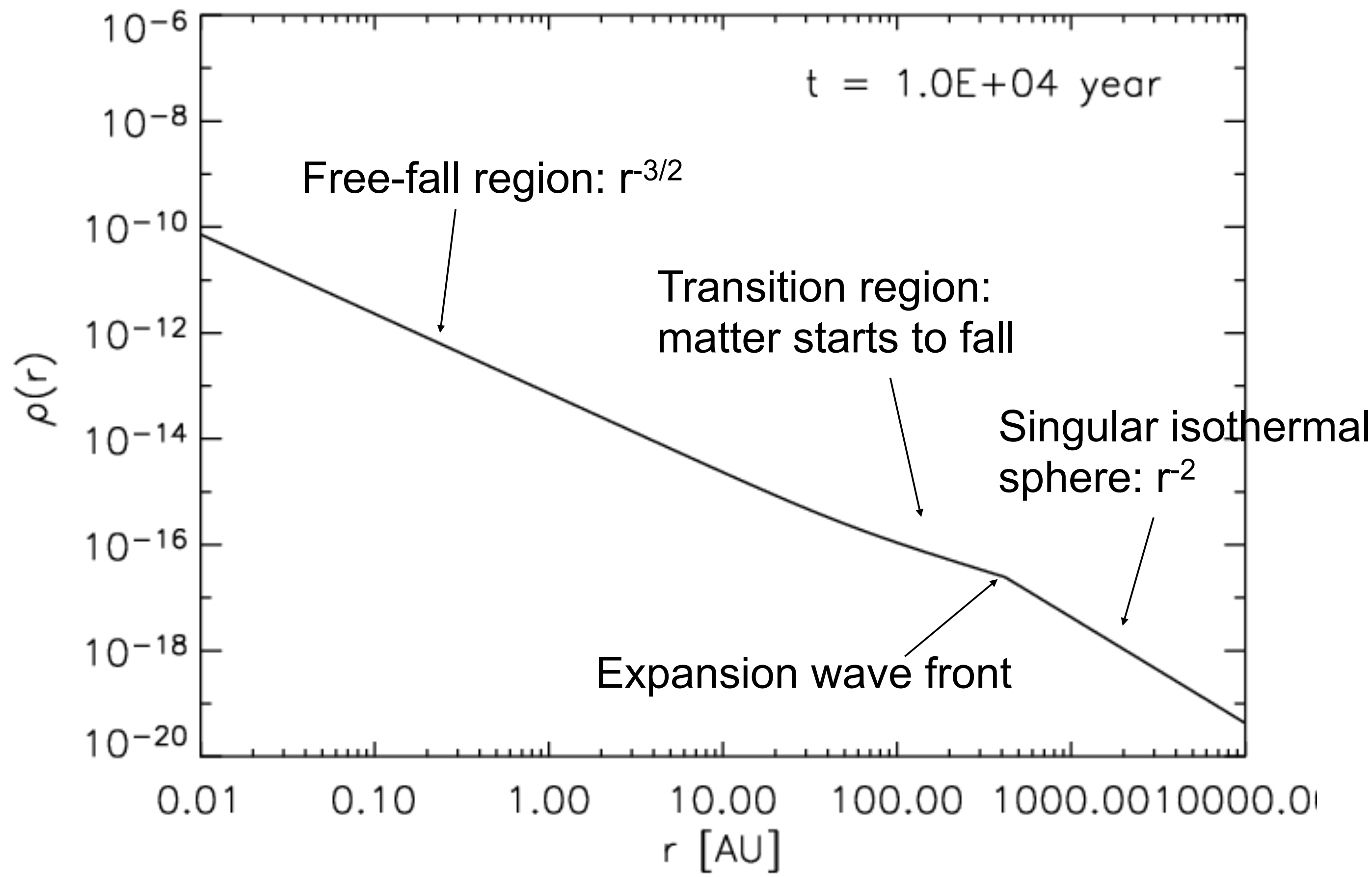
Inside-out collapse model of Shu (1977)



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Inside-out collapse model of Shu (1977)

Deep down in free-fall region ($r \ll c_s t$):

$$\rho(r,t) = \frac{c_s^{3/2}}{17.96\,G} \frac{1}{\sqrt{t}} \frac{1}{r^{3/2}} \qquad v(r,t) = \sqrt{\frac{2GM_*(t)}{r}}$$

Accretion rate is constant: $$\dot{M} = \frac{c_s^3 m_0}{G} = 0.975 \frac{c_s^3}{G}$$

Stellar mass grows linear in time

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The Collapse of a Bonnor-Ebert Sphere

Foster & Chevalier 1993

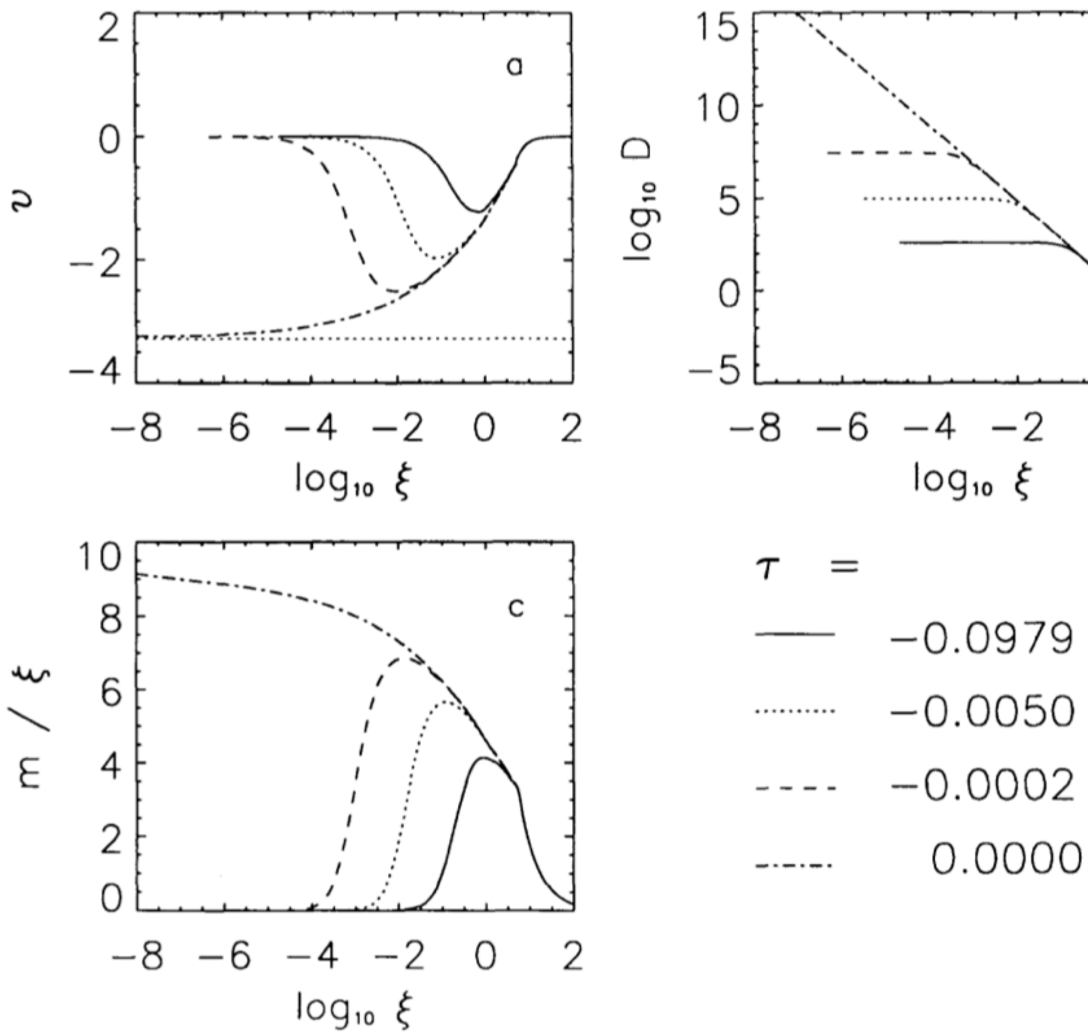


FIG. 1.—Radial profiles of the velocity, density, and enclosed mass over radius for the standard case are presented, at several times prior to core formation. The profiles shown are for a run with 200 zones. The horizontal line in plot *a* represents the Larson-Penston solution at core formation. A 250 zone run is identical to the 200 zone case, for the range shown in the figure.

τ = free fall time for ρ_c
v = v/c_s^2

$$\xi \equiv \left(\frac{4\pi G\rho_c}{c_s^2}\right)^{1/2} r$$

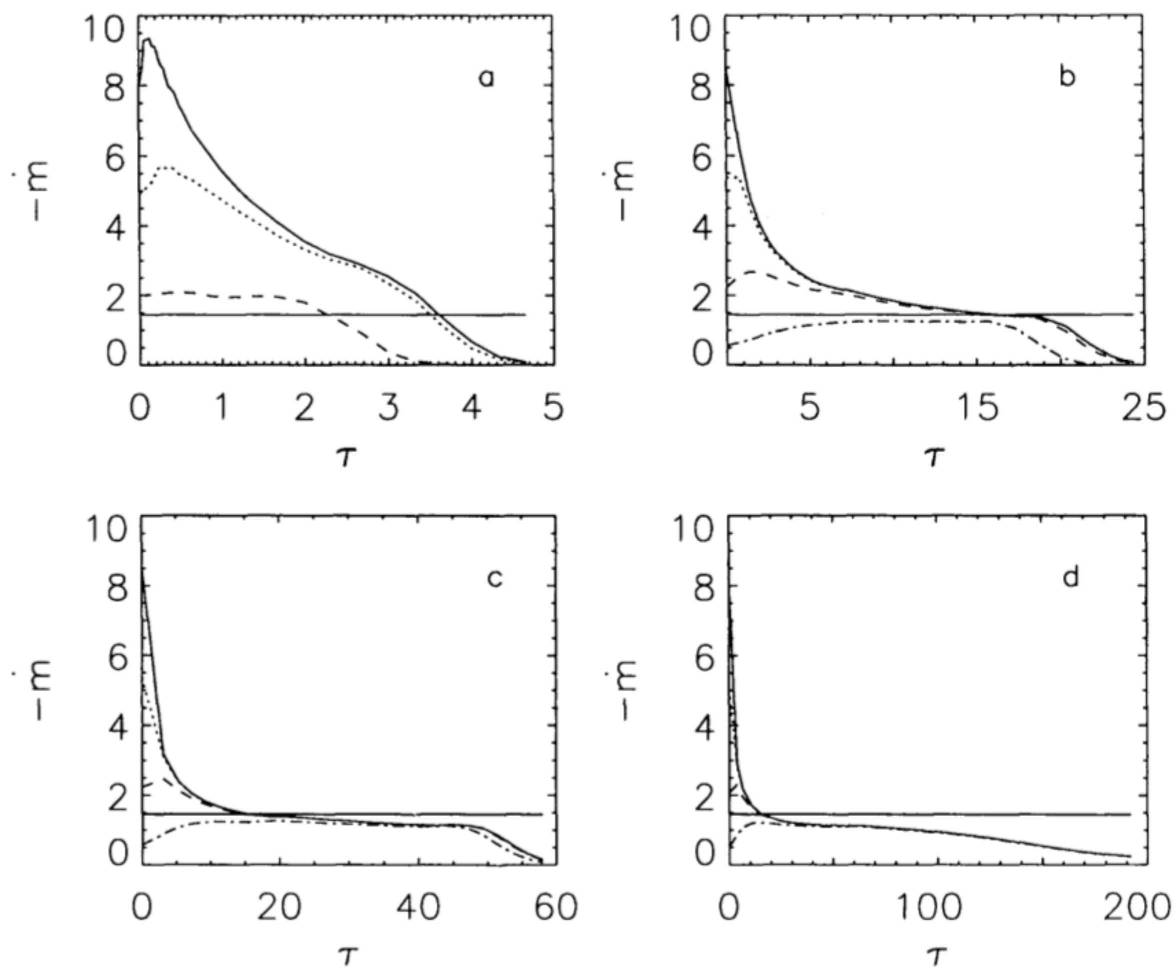


FIG. 3.—The mass accretion rate at four different radii is shown as a function of time. The quantity $\dot{m}$ is calculated as the product of $\xi^2 Dv$. $\xi \sim 0.3$ is the solid line, $\xi \sim 1.0$ is the dotted line, $\xi \sim 3.0$ the dashed line, and $\xi \sim 10$ the dot-dash line. The horizontal line is $\dot{m} = 1.45$, corresponding to the Shu solution $A = 2.2$ case. Fig. 3*a* is for the standard run and Figs. 3*b*, 3*c*, and 3*d* are for runs with $\xi_{max} = 20$, 40, and 100, respectively. The $\xi = 0.3$ line is missing from Fig. 3*d* because it is less than ξ_{min} for this case.

τ = free fall time at central density

$$m \equiv \frac{P_0^{1/2} G^{3/2} M}{c_s^4}$$

Self Gravitating Sheets

The Collapse of an Isothermal Sheet

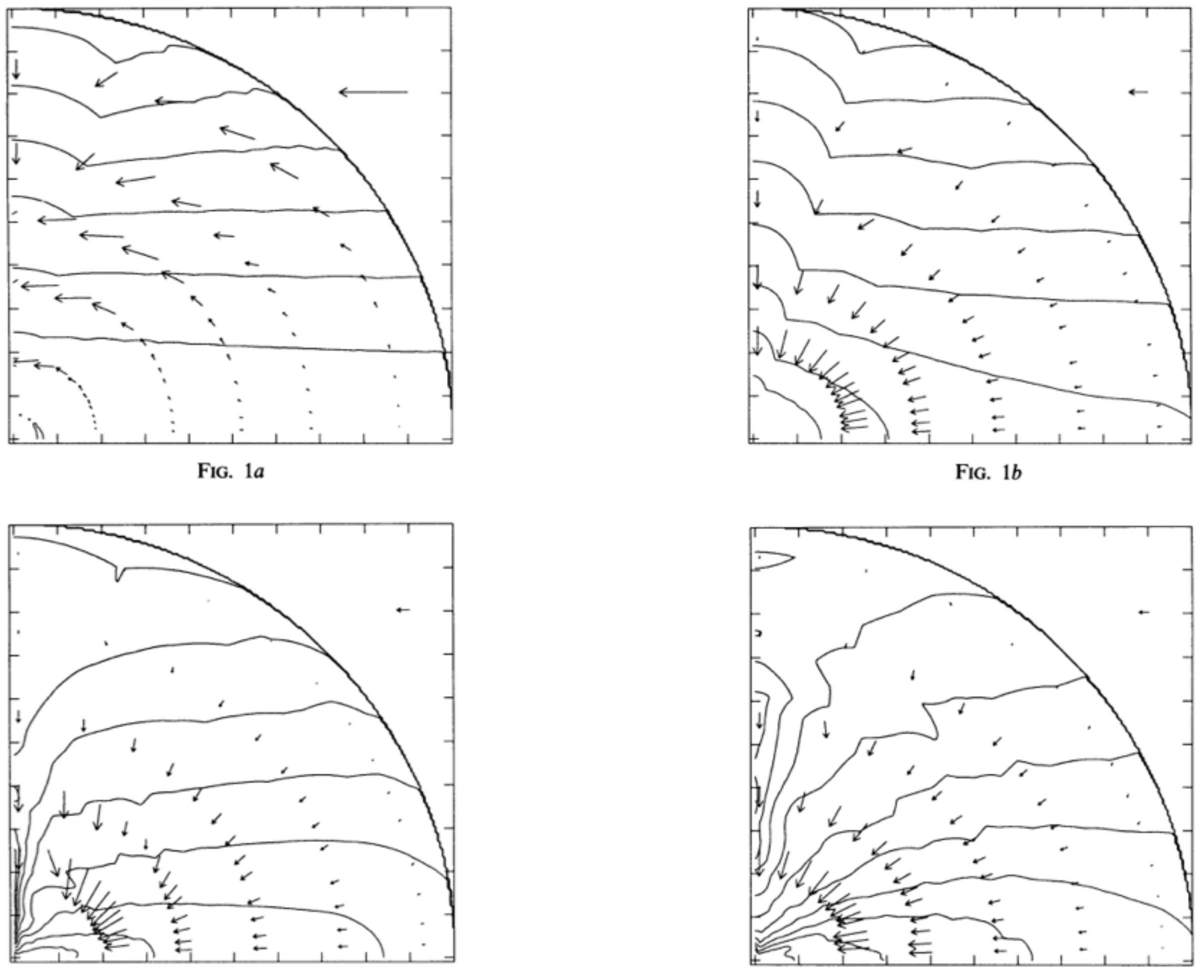


FIG. 1.—Velocity fields and density contours separated by factors of $10^{1/2}$) at selected times for the collapse model described in the text: (*a*) $t = 2.7t_{ff}$; (*b*) $t = 6.0t_{ff}$; (*c*) $t = 6.5t_{ff}$; (*d*) $t = 7.0t_{ff}$. The left-hand border is the symmetry axis; the bottom border is $z = 0$. The entire cloud is shown (radius = 5400 AU). The r and θ grids are nonuniform and resolve the cloud outside 7 AU (not visible here). The highest (central) density contours denote a molecular hydrogen number density of 3.16×10^6 cm^{-3} in all except (*a*), where the highest contour is 3.16×10^5 cm^{-3}. Velocity vectors are plotted for only a small fraction of the 50 radial × 22 angular grid points. The lengths of the velocity vectors are proportional to the speeds, with the length of the arrow in the upper right-hand corner denoting the sound speed = 0.2 km s^{-1}.

Hartmann, Boss, Calvet & Whitney 1994 ApJ 430, L49

The Collapse of an Isothermal Sheet

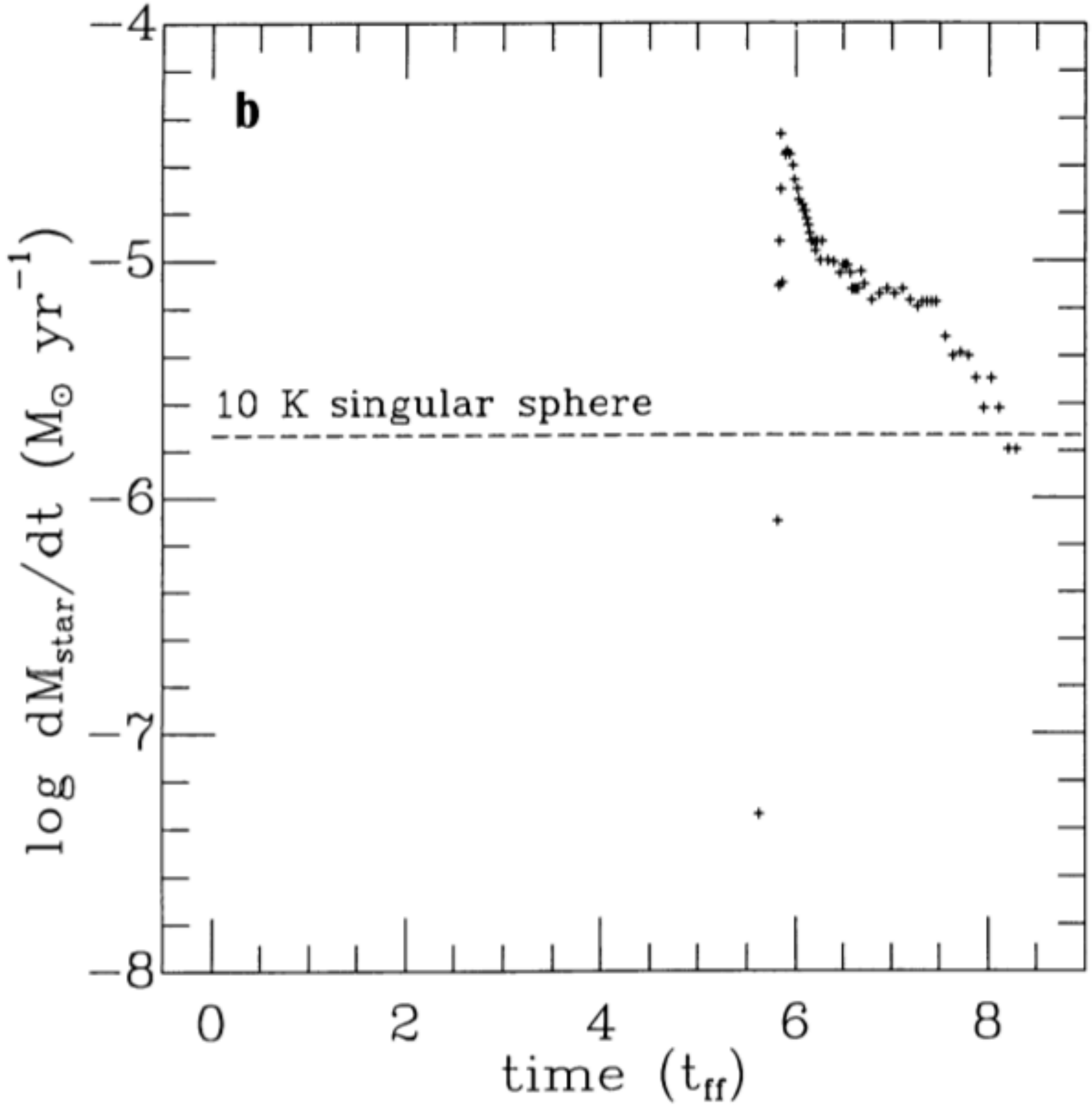
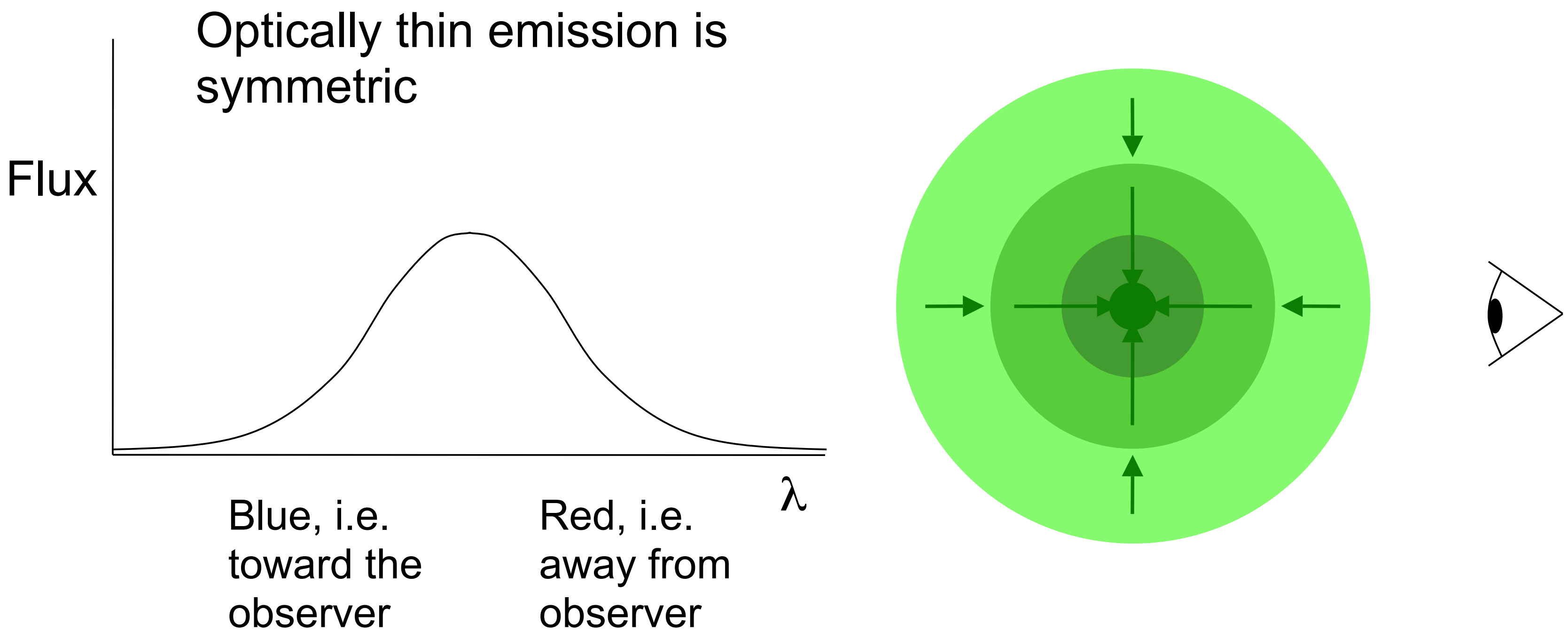


FIG. 2.—Mass accretion rate of the central protostar (sink cell of radius of 7 AU) as a function of time.

Hartmann, Boss, Calvet & Whitney 1994 ApJ 430, L49

Evidence for Collapse and Infall

Line profile of collapsing cloud



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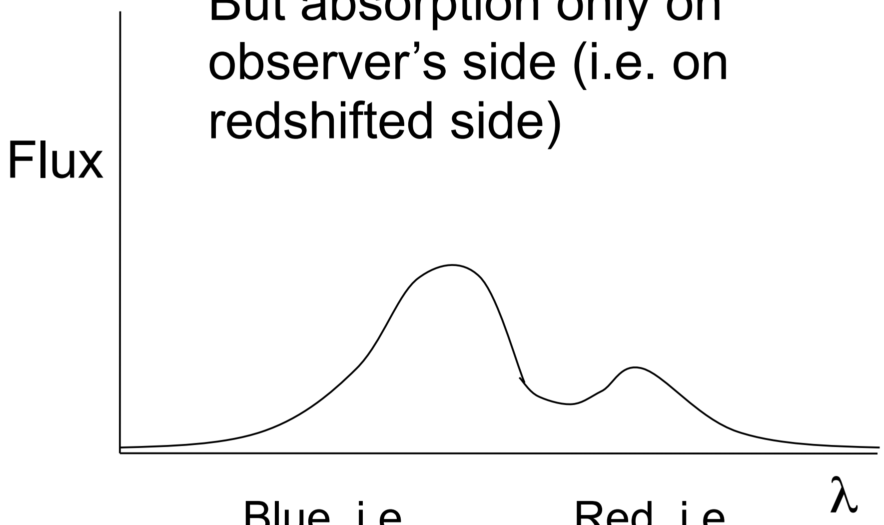
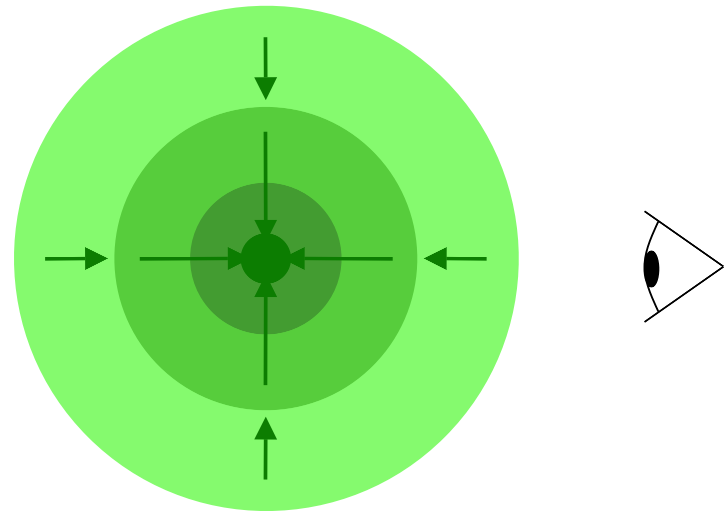
Line profile of collapsing cloud

But absorption only on observer’s side (i.e. on redshifted side)

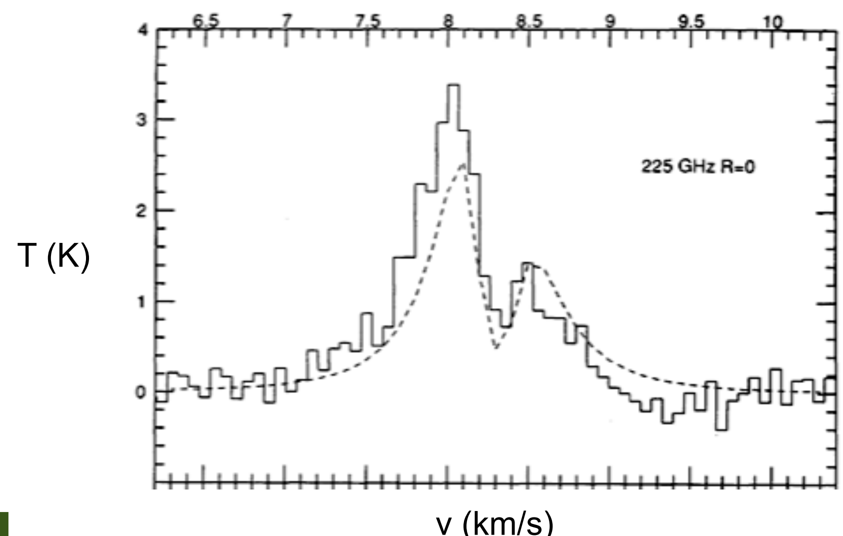


Blue, i.e. toward the observer

Red, i.e. away from observer



Example:
Observations of B335 cloud.
Zhou et al. (1993)



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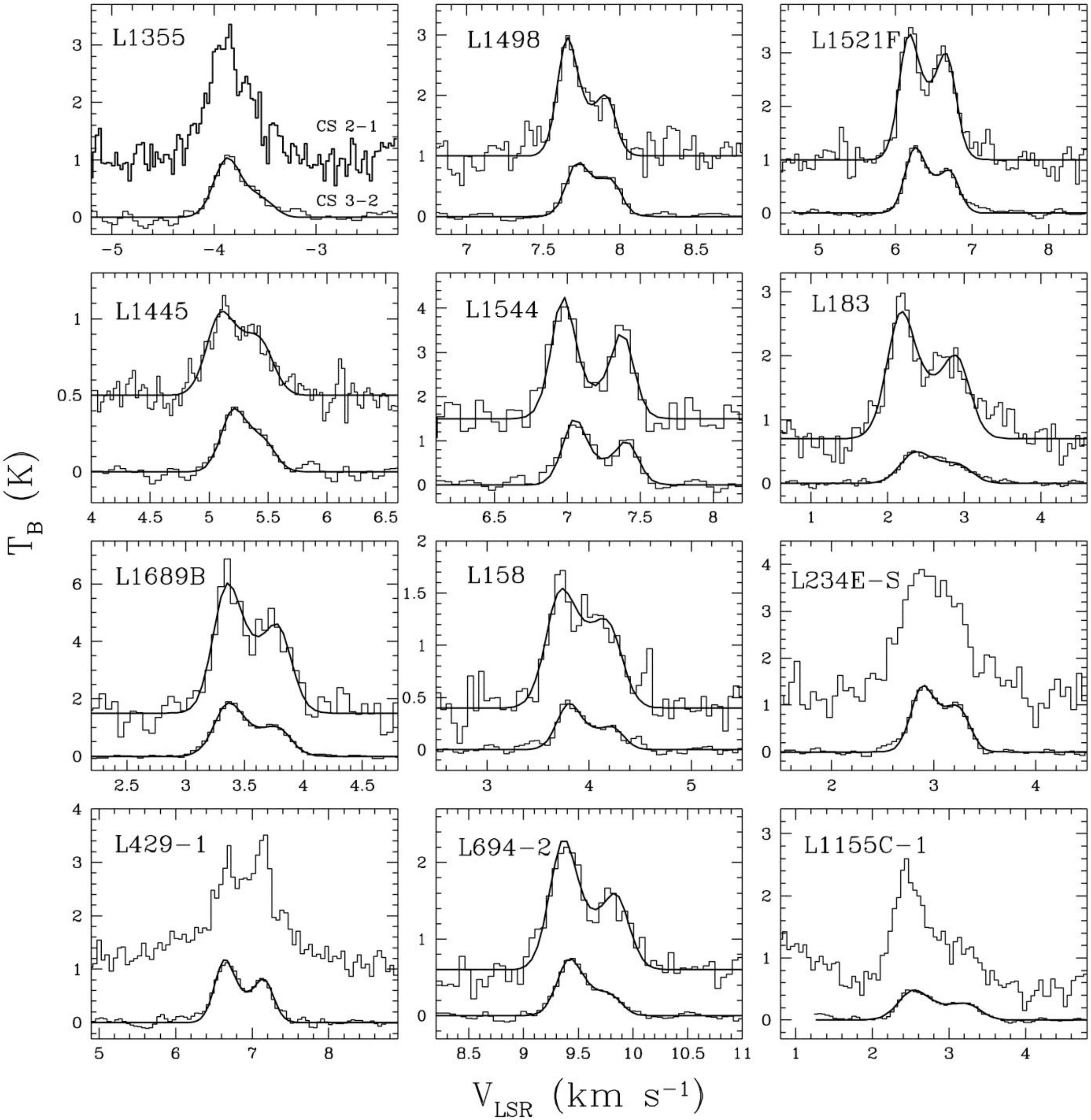
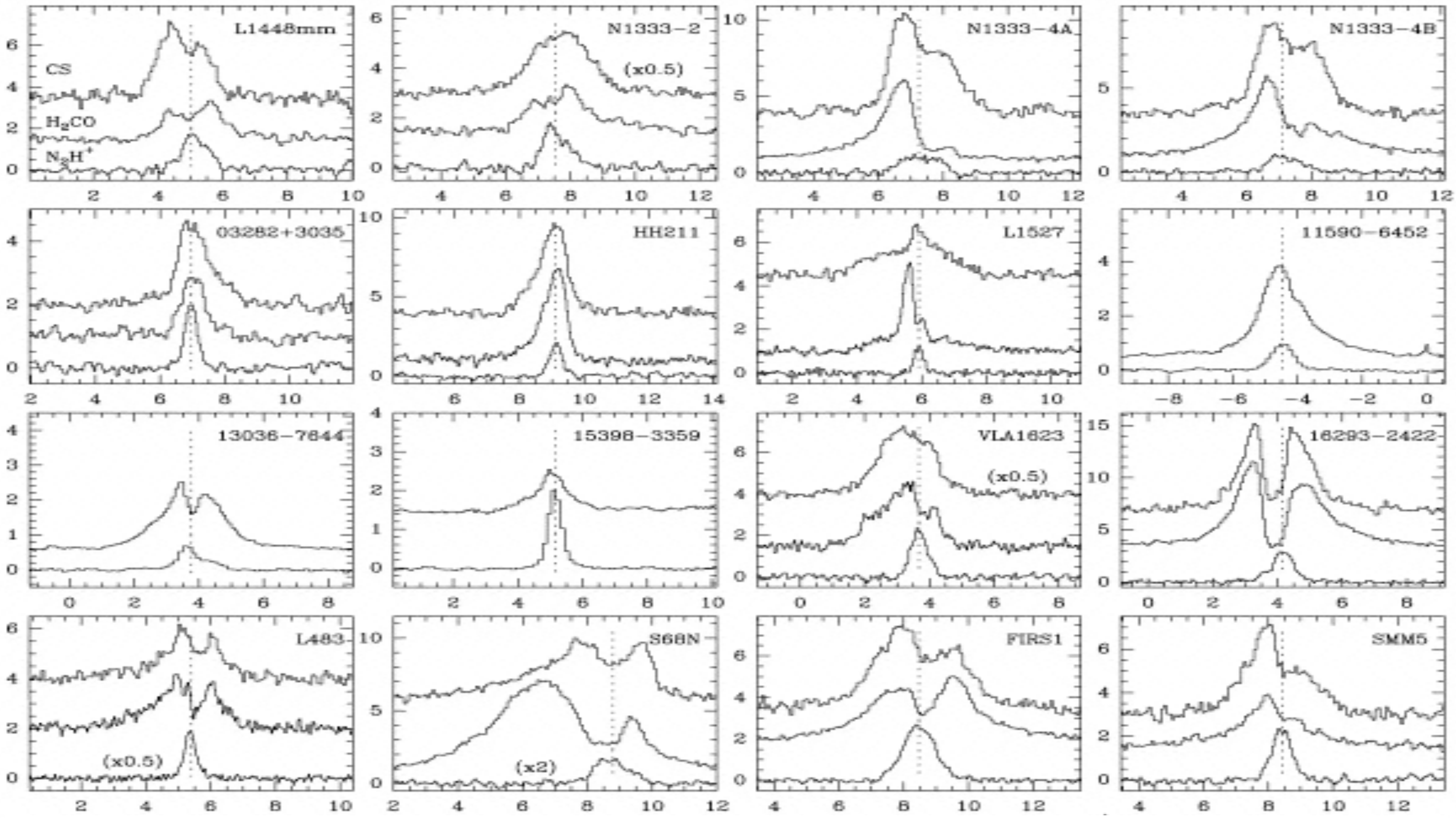


Infall in starless cores:

Lee, Myers & Plume 2004 ApJ 153, 523

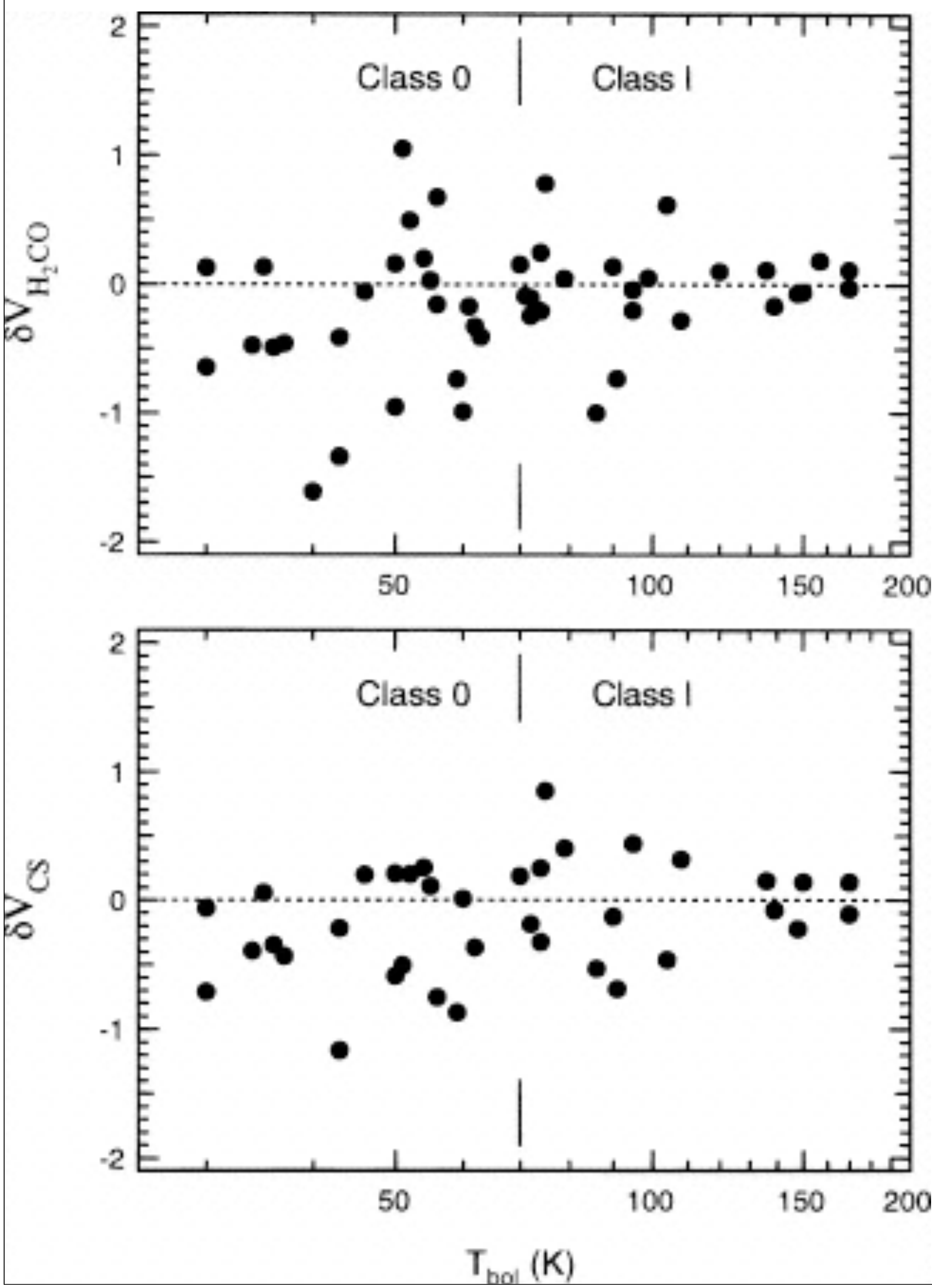
Clear infall signature for 18 of 94 starless



Infall in protostellar cores:

Lee, Myers & Plume 2004 ApJ 153, 523

Clear infall signature in 15 of 47 sources



Further evidence for infall: Optically thick lines are blueshifted for the youngest protostars.

$$\delta V = (V_{thick} - V_{thin})/V_{thin}$$

Infall Speeds

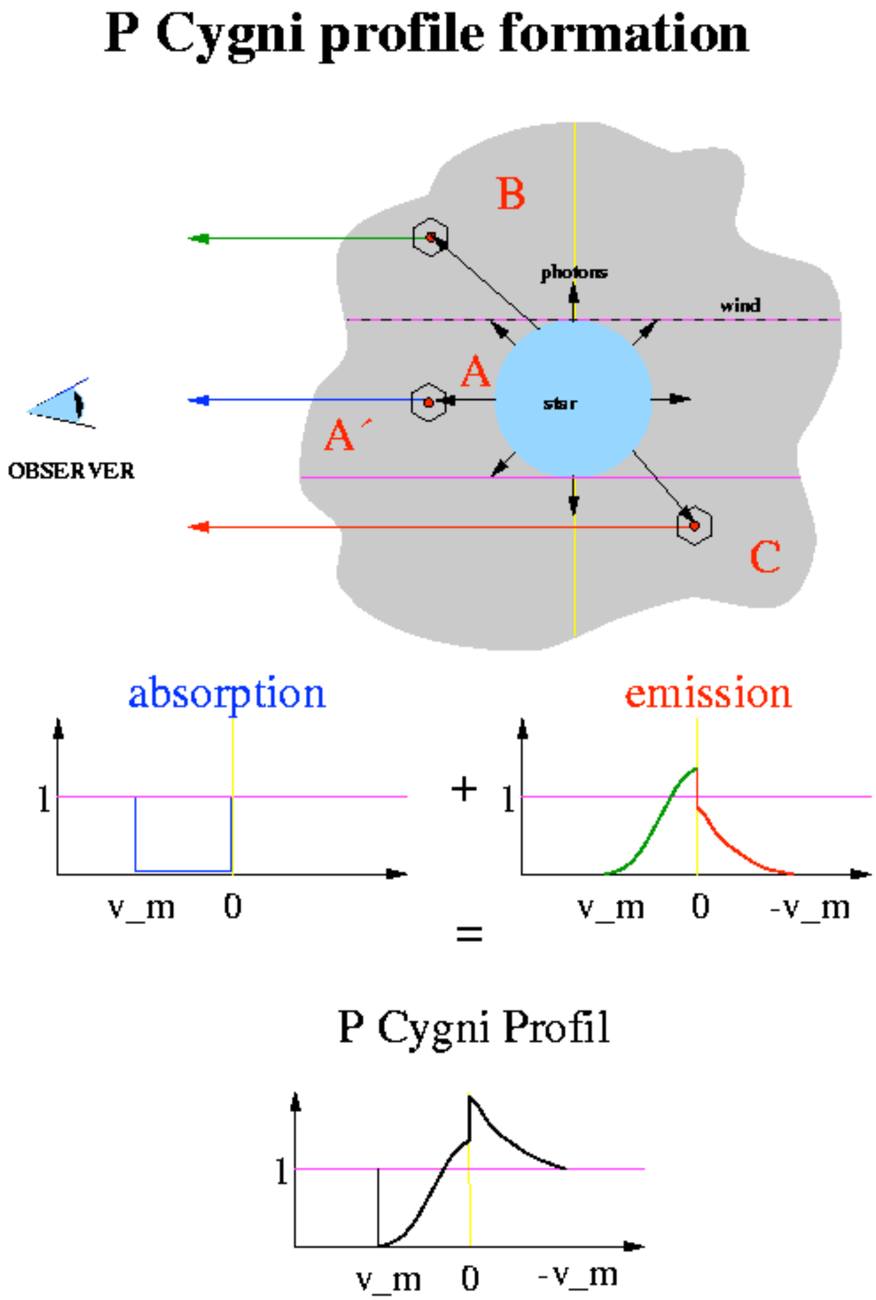
Infall Speeds 0.04 - 0.07 kms^{-1}

Times of a few million years

Longer than free fall time

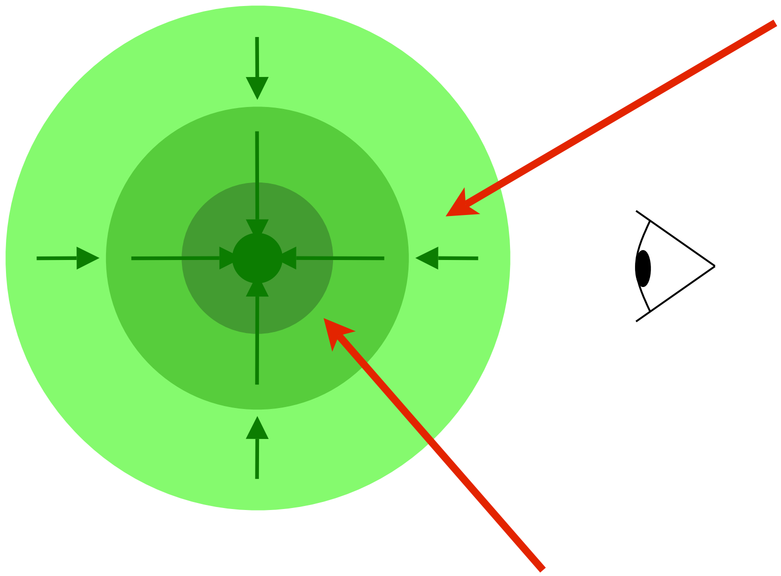
Shorter than ambipolar diffusion time

P-Cygni profiles for winds around evolved stars



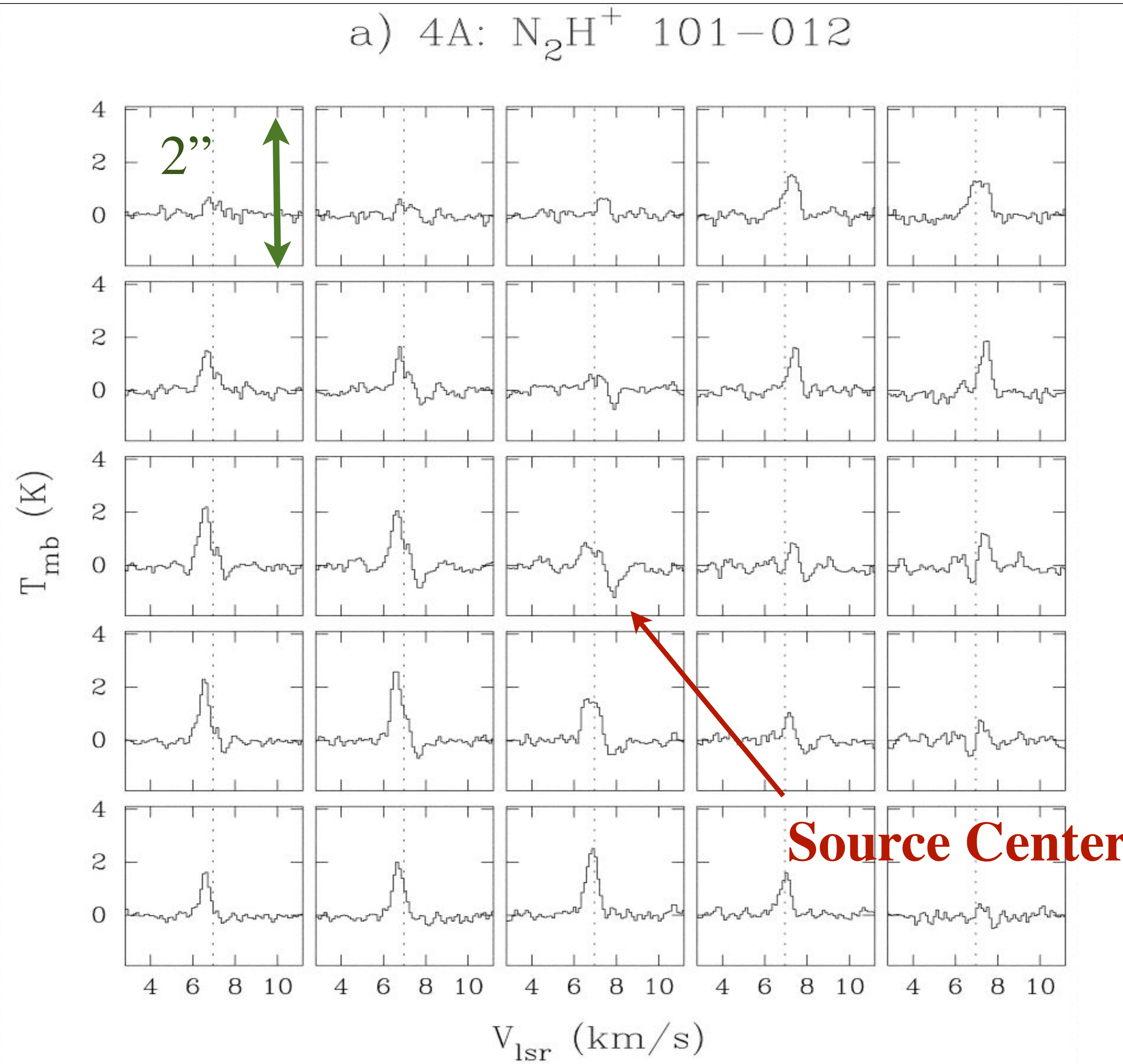
http://www.daviddarling.info/images/P_Cygni_profile.gif

Inverse P-Cygni Profiles



Cold infalling gas generates absorption line.

Continuum emissiom from hotter inner core.



Inverse P-Cygni Profiles toward the Protostar IRAS 4A

Infall velocities of ~ 0.5 km s^{-1}

DiFrancesco et al. ApJ 2001 562, 770

Summary

- Mechanisms for initiating collapse in hydrostatically supported cores
  - Ambipolar Diffusion
  - Pressure wave compresses a critical Bonnor-Ebert sphere (increases ϱ_c/ϱ_o to unstable regime)
  - Mass accretion onto a Bonnor-Ebert sphere
- Solution for infall
  - Infall in a isothermal sphere
  - Solutions for collapse of sheet and Bonnor-Ebert sphere
- Evidence for infall
  - Redshifted self-absorption
  - Inverse P-Cygni