1. The Luminosity of Protostars

We derived in the previous lecture the infall rate for a thermally supported sphere, is \( \dot{M} \approx c_s^3/G \). Assume that a fraction \( f \) falls onto the central protostar (the other fraction, \( 1 - f \), might be carried off in an outflow). Also assume the central protostar has a mass \( M \) and a radius \( R \). Then the luminosity generated by accretion is:

\[
L_{\text{acc}} = f \frac{GM\dot{M}}{R} \tag{1}
\]

The total luminosity is the sum of the accretion luminosity and the intrinsic luminosity of the source.

\[
L_{\text{tot}} = L_{\text{acc}} + L_{\text{int}} \tag{2}
\]

The accretion luminosity for a typical source, \( \dot{M} = 5 \times 10^{-6} M_\odot \, \text{yr}^{-1} \), \( M \sim 0.5 M_\odot \) and \( R \sim 3 R_\odot \), then \( L_{\text{acc}} = 16 L_\odot \).

2. Starlight Reprocessed Dust Shell

Imagine a star with a radius \( R_\star \) and temperature \( T_\star \) surrounded by an optically thick shell of dust at a radius \( R_{\text{shell}} \). Assuming that the shell is in temperature equilibrium, i.e. it is emitting as much power as it is absorbing, then.

\[
L_{\text{shell}} = L_\star \tag{3}
\]

which can be written as

\[
4\pi R_\star^2 \sigma T_\star^4 = 4\pi R_{\text{shell}}^2 \sigma T_{\text{shell}}^4 \tag{4}
\]

where

\[
\frac{T_{\text{shell}}}{T_\star} = \left( \frac{R_\star}{R_{\text{shell}}} \right)^{1/2} \tag{5}
\]
3. The Spectral Energy Distribution of a Protostar

(From 5.2 in Hartmann, a very good read on this topic). In reality, a protostar is not one optically thick shell, but a series of concentric shells with decreasing density. At a given wavelength, most of radiation can be considered to come from the $\tau = 2/3$ surface, just as in stellar photospheres (Eddington-Barbier relationship). Unlike a stellar photosphere, the radius that the radiation comes from varies strongly with wavelength (this is because a stellar atmosphere is very thin with a very sharp rise in density - while an infalling protostellar envelope has a density that decreases with $r^{-3/2}$).

Let us assume spherical symmetry. Then

$$
\rho(r) \approx \frac{\dot{M}}{4\pi r^2 v_{ff}} = \frac{\dot{M}}{4\pi (2GM)^{1/2} r^{-3/2}}
$$

where $v_{ff} = \sqrt{2GM/r}$. As in the lecture, this can be integrated to find the optical depth integrating from infinity down to a radius of $r$.

$$
\tau_{\lambda} = \frac{\kappa_{\lambda} \dot{M}}{2\pi (2GM)^{1/2} r^{-1/2}}
$$

where $\kappa_\lambda$ is the absorption per mass. Now, we can determine the radius $r_{\lambda}$ where $\tau_{\lambda} = 2/3$.

$$
r_{\lambda} = \frac{9\kappa_\lambda^2 \dot{M}^2}{32\pi^2 GM}
$$

Using Wien’s law, $\lambda_m[\mu m] = 2900/T_m[K]$, we can approximate the luminosity of the protostar as a blackbody.

$$
L = 4\pi r_{\lambda m}^2 \sigma T_{\lambda m}^4
$$

Now, if we use Wien’s relationship to relate $\lambda$ to $T$, and we approximate $\kappa_\lambda = \kappa_0 (\lambda/\lambda_0)^{-\beta}$, we can solve for

$$
\frac{\lambda_m}{\lambda_0} = \left( \frac{2900}{\lambda_0} \right) \left( \frac{4\pi \sigma}{L} \right)^{1/(4+4\beta)} \left( \frac{9\dot{M}^2 \kappa_0^2}{32\pi GM} \right)^{1/(2+2\beta)}
$$
or, by adopting the extinction law $\kappa_\lambda = 0.2(\lambda/100\mu m)^{-2}$

$$\lambda_m[\mu m] \approx 30 \left( \frac{L}{L_\odot} \right)^{-1/12} \left( \frac{\dot{M}}{2 \times 10^{-6} M_\odot \ yr^{-1}} \right)^{1/3} \left( \frac{M}{M_\odot} \right)^{-1/6}$$ (11)

4. *The Emission from a Disk*

See lecture.

5. *The Temperature of an Irradiated Flat Disk*

See lecture.