1. *The Luminosity of Protostars*

We derived in the previous lecture the infall rate for a thermally supported sphere, is $\dot{M} \approx c_s^3/G$. Assume that a fraction f falls onto the central protostar (the other fraction, $1 - f$, might be carried off in an outflow). Also assume the central protostar has a mass M and a radius R. Then the luminosity generated by accretion is:

$$L_{acc} = f\frac{GM\dot{M}}{R} \tag{1}$$

The total luminosity is the sum of the accretion luminosity and the intrinsic luminosity of the source.

$$L_{tot} = L_{acc} + L_{int} \tag{2}$$

The accretion luminosity for a typical source, $\dot{M} = 5 \times 10^{-6}$ M$_\odot$ yr^{-1}, $M \sim 0.5$ M$_\odot$ and $R \sim 3$ R$_\odot$, then $L_{acc} = 16$ L$_\odot$.

2. *Starlight Reprocessed Dust Shell*

Imagine a star with a radius $R_\star$ and temperature $T_\star$ surrounded by an optically thick shell of dust at a radius R_{shell}. Assuming that the shell is in temperature equilibrium, i.e. it is emitting as much power as it is absorbing, then.

$$L_{shell} = L_\star \tag{3}$$

which can be written as

$$4\pi R_\star^2 \sigma T_\star^4 = 4\pi R_{shell}^2 \sigma T_{shell}^4 \tag{4}$$

where

$$\frac{T_{shell}}{T_\star} = \left(\frac{R_\star}{R_{shell}}\right)^{1/2} \tag{5}$$

3. *The Spectral Energy Distribution of a Protostar*

(From 5.2 in Hartmann, a very good read on this topic). In reality, a protostar is not one optically thick shell, but a series of concentric shells with decreasing density. At a given wavelength, most of radiation can be considered to come from the $\tau = 2/3$ surface, just as in stellar photospheres (Eddington-Barbier relationship). Unlike a stellar photosphere, the radius that the radiation comes from varies strongly with wavelength (this is because a stellar atmosphere is very thin with a very sharp rise in density - while an infalling protostellar envelope has a density that decreases with $r^{-3/2}$.

Let us assume spherical symmetry. Then

$$\rho(r) \approx \frac{\dot{M}}{4\pi r^2 v_{ff}} = \frac{\dot{M}}{4\pi(2GM)^{1/2}} r^{-3/2} \tag{6}$$

where $v_{ff} = \sqrt{2GM/r}$. As in the lecture, this can be integrated to find the optical depth integrating from infinity down to a radius of r.

$$\tau_\lambda = \frac{\kappa_\lambda \dot{M}}{2\pi(2GM)^{1/2}} r^{-1/2} \tag{7}$$

where κ_λ is the absorption per mass. Now, we can determine the radius r_λ where $\tau_\lambda = 2/3$.

$$r_\lambda = \frac{9\kappa_\lambda^2 \dot{M}^2}{32\pi^2 GM} \tag{8}$$

Using Wien's law, $\lambda_m[\mu m] = 2900/T_m[K]$, we can approximate the luminosity of the protostar as a blackbody.

$$L = 4\pi r_{\lambda m}^2 \sigma T_{\lambda m}^4 \tag{9}$$

Now, if we use Wien's relationship to relate λ to T, and we approximate $\kappa_\lambda = \kappa_0(\lambda/\lambda_0)^{-\beta}$, we can solve for

$$\frac{\lambda_m}{\lambda_o} = \left(\frac{2900}{\lambda_0}\right)\left(\frac{4\pi\sigma}{L}\right)^{1/(4+4\beta)} \left(\frac{9\dot{M}^2\kappa_o^2}{32\pi GM}\right)^{1/(2+2\beta)} \tag{10}$$

or, by adopting the extinction law $\kappa_\lambda = 0.2(\lambda/100\mu m)^{-2}$

$$\lambda_m[\mu m] \approx 30 \left(\frac{L}{L_\odot}\right)^{-1/12} \left(\frac{\dot{M}}{2\times 10^{-6} M_\odot\ yr^{-1}}\right)^{1/3} \left(\frac{M}{M_\odot}\right)^{-1/6} \tag{11}$$

4. *The Emission from a Disk*

See lecture.

5. *The Temperature of an Irradiated Flat Disk*

See lecture.